

Optimal *Relative* Performance Evaluation

Thomas Hemmer*

October, 2012

Abstract

I study the properties of contracts that optimally condition an agent's compensation both on his own performance and on how well he fared relative to a peer (group). A key point of the paper is that the predictions of a negative coefficient on positively correlated peer performance typically tested in the empirical literature on relative performance evaluation are obtained from the highly special case where the evaluator actually doesn't need to know individual performance and its relation to peer performance; an aggregate performance measure obtained as a weighted sum of individual and peer performance will suffice. In a model where this is not the case I show the expected coefficient on peer performance is exactly zero. Instead in the optimal contract is linear in own performance and its *correlation* with peer performance.

*Third preliminary and rough draft. Please do not circulate further or quote. I am grateful to Tod Cox for assistance with the graphical illustrations, to participants in the 2012 Stanford Summer Camp, Shiva Shivaramakrishnan and George Drymiotis for comments on an earlier and rougher draft, and to Eva Labro and Karen Nelson for comments on an even earlier and rougher draft than that second one. All remaining deficiencies are exclusively of my own doing.

1 Introduction

I study optimal relative performance evaluation using the dynamic approach of Holmström and Milgrom (1987). The main distinguishing feature of my model is that normality as well as the (equilibrium) covariance of the relevant performance measures are derived rather than imposed. Several key insights are obtained. First, what constitutes a manager's peers is not simply determined by the sign of the (equilibrium) covariance with own performance. Instead, groups of peers are optimally formed based on the marginal effect of the agent's second-best action on the correlation of his own performance with that of his peer(s). Second, for any random groups of peers, an agent's compensation will be independent of (aggregate) peer performance. Peer performance will instead enter an agent's optimal contract through realized *correlation* between own and peer performance. For a group of peers where an increase in the agent's (second best) action increases the covariation with the performance of his peers, the agent's compensation increases linearly in the coefficient of correlation. For a group of peers where increased effort leads to a decrease in covariance the effect of the correlation between own and peer performance, the effect is the opposite.

Relative performance evaluation permeates almost all aspects of human endeavour. In sports, in personal life, sports, politics and in business it is more or less a truism that "achieving" or/and "performing" are at least partially judged in relative terms. Winning the Indy 500, keeping up with the Joneses, being the first man on the moon, beating an earnings forecast, loosing a debate or, in general, outperforming your competitors on some desirable dimension are purely relative in nature and key to how we assign credit. Surely, lap-times matters in racing, doing well financially is of value too even if the Joneses are doing slightly better, playing golf on the moon is cool even if it had not been a first and delivering profits can be good even if other managers win the profitability contest. But I'd argue that it is indisputable that when humans evaluate each other's performance, in particular when we evaluate those that are stewards of our well being such as the managers of companies in which we are stakeholders, comparisons virtually always play a key part.

More than two and a half decades have passed since the publication of the pioneering work of Antle and Smith (1986) opened the empirical inquiry into the prevalence of relative performance evaluation for managers of public corporations. Based on the work of Holmström (1979, 1982) they proposed and tested several versions of what was perceived to be the central agency theory prediction with regards to relative performance evaluation: that economy wide shocks are optimally (partially) removed from the performance related compensation of firm managers via a negative coefficient on the performance of their peers. Despite the thoroughness of their study, Antle and Smith (1986) found only what can be characterized as quite modest indications that some form of relative performance evaluation might have been at work for some part of their sample. This somewhat disappointing message in turn was the seed that gave rise to the relative performance evaluation puzzle that persists to this day.

The relative performance evaluation puzzle can perhaps best be described as the lack of cooperation by almost any set of data following Antle and Smith's (1986) with the prediction of firms taking out common shocks from individual performance by deducting out some fraction of market or peer performance. The robustness of Antle and Smith's (1986) discouraging empirical results has persisted despite the (loosely speaking) countless studies aimed at refining both the theoretical guidance and the empirical approaches used to test particular versions of this prediction. Because of the vastness of the relative performance evaluation literature I'll refrain from attempting to do all the idiosyncratic contributions justice here. In the spirit of this particular literature, however, what I will do instead is to try and focus on the role and implications of the (in my mind) most common component of the many studies in this literature: the model(s) and their key prediction of taking out the common noise or risk component of individual performance by deducting (positively correlated) peer performance somehow (appropriately) scaled from own performance, and basing the compensation on the resulting net.

While the theoretical references used to motivate the specific empirical relative perfor-

mance evaluation (*RPE* hereafter) hypotheses/tests have evolved over time, the starting points for this evolution, I would argue, is Holmström (1979) and, in particular, Holmström (1982). These papers developed versions of the so-called "sufficient statistic condition" and showed that measures other than measures of an agent's own performance should be used if and only if the condition is violated. The two last results in the second of these papers go a bit further. Specifically Holmström's (1982) Theorem 8 demonstrates for two types of normally distributed production functions, that an aggregate measure of peer performance such as a market index obtained as a weighted average of the peers' individual performances will capture all the relevant information needed for the optimal contract.

Holmström (1982) is careful to point out, however, that sufficiency of a weighted average index is not the general case. Moreover, that even when a weighted average index *will* suffice, it does not imply that the optimal contract generally is going to be based simply on a linear aggregate of own performance and this index; just that the contract will be a non-trivial function of both pieces of information. In Theorem 9, however, he goes on to show that if there is a large number of firms exposed to the same common uncertain component, then one can approximate the solution obtained if the common uncertainty component was entirely absent. Interesting, then, the case provided by Holmström (1982) where the optimal contract *can* be based exclusively on a linear aggregate of own and a weighted average index of peer performance is one where the aggregation weight on the peer performance index is exactly minus one.

Of course, the set of standard predictions in the ensuing literature does not include a negative weight of "one" on peer performance. Rather, predictions tend to come either in the so-called weak form which simply predicts that *some* negative weight be assigned to peer performance, or in the strong form, which provides predictions on both the sign and the determinants of the magnitude of the weight assigned to peer performance.¹ The specific

¹See Janakiraman et al. (1992) for a fuller discussion. Table 1 in Albuquerque (2009) provides a nice overview of the empirical relevance of these predictions. Notice the absence of a column for a prediction of "minus one."

strong-form predictions are typically obtained from the standard properties of the normal distribution, either relying on the analysis of Banker and Datar (1989) or (more recently) the so-called LEN model which is motivated by (but generally not consistent with) the work of Holmström and Milgrom (1987, 1991).² The common component of these studies is that removing the common noise component in the optimal contract implies deducting from an agent's own performance a positive fraction of peer performance proportional to the covariance between own and peer performance.

The lure of the normal distribution is the mathematical simplicity it brings combined with sometimes being a reasonably descriptive way to represent standard managerial performance metrics such as stock returns. Reliance on the normal distribution is not without significant drawbacks though. Avoiding that the problem disintegrate into triviality (Mirrlees non-existence), as standard agency models based on normals normally do, requires that additional constraints must be imposed.³ When the strong form predictions are obtained with reference to Banker and Datar (1989) the standard approach is to suggest that the distribution is somehow not quite normal after all, but miraculously truncated *just* enough to rule out superiority of penalty contracts over those obtained using the first-order approach. In addition, to maintain the ability to aggregate own and peer performance linearly, attention must also be confined to the case where the covariance between own and peer performance is completely exogenous and independent of managerial actions. When the LEN model is used the approach is of course to altogether abandon optimal contracts and simply impose linearity exogenously.⁴

What is important to keep in mind, however, is that absent exogenous restrictions on the contract in models with the normal/independence specification of the production function discussed above, approximate first-best is always obtainable in which case *RPE* is of no meaningful economic relevance. In other words the so-called strong form prediction that

²See for example Albuquerque (2011) and Dikoli et. al. (2011b).

³See Mirrlees (1974) and Holmström (1982), Section 2.

⁴Effort independence of the variance-covariance matrix is of course a standard LEN assumption also.

has been the predominant prediction guiding the empirical *RPE* inquiry does not arise from the basic underlying model itself but rather is entirely accounted for by the arbitrary restrictions placed on this model. Further, while restricting attention to effort independent noise terms is mathematical expedient and key to the linear aggregation result (except, of course, when linearity is just exogenously imposed) it is arguably not particularly appealing from an economic vantage point either. Besides from being a “measure zero” type of restriction from a technical perspective, such independence seems highly implausible from an economic perspective as well. Indeed I would suggest that in most situations where peer performance is relevant, an individual’s actions have a direct impact on the degree of exposure to uncontrollable events that also are facing his peer(s).

Consider for example what seems to be close peers such as McDonald’s and Burger King. By locating on opposite street corners of the same intersection, next to the same malls or freeway ramps, clearly their exposure to factors driving local demand is increased. A retailer such as Target, on the other hand, by choosing to locate in suburban area malls away from the inner cities where many Sears stores traditionally were to be found clearly should expect to reduce their exposure the uncontrollable events Sears is facing. And similarly in many non-business situations that involves competing peers. Take Tour de France for example. A cyclist working hard to constantly keep within close distance to his main peers will be exposed to the exact same weather and congestion conditions as the peer at the same points of the route, while one that doesn’t won’t. The examples are endless.

Somewhat related, using a single shot standard Principal-Agent model as the theoretical vehicle to study *RPE* at a minimum seems somewhat less appropriate than perhaps for other issues. Just consider the nature of the examples above. When the cyclist in Tour de France competes he is provided very frequent, if not continuous, feedback about own and peer performance, and can obviously choose to alter his actions in response. McDonald’s and Burger King can monitor each other’s pricing, product mix, promotional actions, store openings etc. and casual empiricism certainly suggests that they do follow each others actions

closely. Moreover, if indeed assumed normality is to be at least partially justified based on the behavior of stock returns, which is the central focus of the empirical *RPE* literature, it is clear that own and peer stock-returns too are providing the evaluatees more or less continuous feedback on which they can condition their actions. Frequent feedback seems almost part and parcel to many real world situations where *RPE* is relevant, and results obtained from a single shot model likely fails to account for the implications hereof.

Finally, the special case where linear aggregation is optimal arguably is the case where *relative* performance is the least relevant aspect of peer performance. Sure the standard *RPE* models do predict that there be an adjustment to own performance for peer performance. But the magnitude of this adjustment is exclusively tied to *absolute* peer performance and does not depend on how own performance measures up *relative* to peer performance. That feature is of course specific to the special case where linear aggregates are sufficient. Conversely, when peer performance optimally matters but when linear aggregation will not suffice, how a given level of peer performance influences the optimal contract will necessarily depend on who outperformed who, i.e., their relative performance. Because it is only when linear aggregates won't suffice that relative performance plays a role in calibrating the role of *absolute* peer performance, it would make sense to consider predictions about the nature of optimal *RPE* that obtains in such cases.

The alternative approach I take in this paper is intended to address the above issues related to the standard approach, while maintaining the desirable features of normality.⁵ At first my results may seem somewhat counter-intuitive being in sharp contrast to the conventional wisdom of the standard model(s). The key reason why my approach generates fundamentally different predictions about optimal *RPE* works like this however: as pointed out by Banker and Datar (1989), in the case of normally distributed performance measures aggregate performance is sufficient *only* if the covariance between own and peer performance

⁵Other papers in the theoretical literature that question the general validity of the strong form prediction for entirely different reasons than those advanced in this paper include Dye (1992), Hemmer (2004) and Celentani and Loveira (2006).

is independent of the agent's action. As part of what I show in this paper however, when one develops the normal distributions directly using the dynamic approach of Holmström and Milgrom (1987), rather than simply assuming performance to be normal with the specific properties that allow for linear aggregation, it becomes evident that the covariance is independent of effort only in highly specific cases. In general, the covariance (at the second best level of effort) is either increasing or decreasing in the agent's effort.

Consider first the case where the covariance is increasing in the agent's effort. In this case, while on average higher own performance is a good thing, mismatched performance is actually a bad thing! For very low levels of peer performance the penalty for mismatched performance therefore implies that the agent's compensation is decreasing in own performance, while for very high levels of peer performance the agent's compensation is increasing (more so) in own performance. The opposite happens in the case where an increase in effort from the second best level decreases the covariance. As I show, for the continuous/normal version of the Holmström and Milgrom set-up, the implication is that the optimal contract is linearly increasing in own performance and for peers where effort is increasing (decreasing) the covariance, linearly increasing (decreasing) in the realized *correlation* between own and peer performance. The expected *OLS* coefficient on peer performance in a standard "*RPE* regression" is on the other hand exactly zero either way!

The remainder of this paper is dedicated to introducing the model, deriving the core properties of *RPE* under broad conditions where a simple aggregate performance measure won't suffice but where knowing relative performance in addition to own performance actually matters. I'll then derive the empirical predictions of this model and show that while they differ fundamentally from those obtained in the special case where aggregates are sufficient, they match the bulk of the empirical evidence accumulated to date perfectly. I will go on to discuss the economic intuition of this model's predictions relative to the intuition of those models common in the literature and end with a few concluding remarks.

2 Model

This section develops a model where *i*) the focal agent's own performance and peer performance are distributed joint normal and *ii*) performance feedback is frequent and *iii*) the effect of the agent's action on the variance-covariance matrix is allowed to arise endogenously. The approach taken is based on that developed in the seminal paper by Holmström and Milgrom (1987).⁶ Accordingly, rather than assuming that the focal agent acts only once at the start of the period in question and that nothing happens until at the very end where performance then appears, effort is assumed to be supplied and performance is assumed to evolve continuously over the period covered by the model.⁷

As in Holmström and Milgrom (1987), I start by sub-dividing the fixed-length contracting period into m identical sub-periods where m is a positive integer. Normalizing the length of the contracting period to one, the length of each of the sub-period is denoted by

$$\theta = 1/m.$$

In each sub-period of length θ , performance is assumed to follow a multinomial distribution. Specifically, to make sure that the agent's own performance becomes distributed normal in the limit as $m \rightarrow \infty$, I restrict attention to a case where the measure of the focal agent's performance in each sub-period, ω_θ , can be either positive or negative so that:⁸

$$\omega_\theta = \omega_\theta^i, \quad i = +, -.$$

The agent controls his own performance through his choice of his action which is taken to be the choice of the probability of a positive outcome. Similar to Hellwig and Schmidt

⁶Using their approach to develop the predictions also allow me to make the point as starkly as possible, that the predictions one would derive using the LEN model are fundamentally different from, and in inherent conflict with those obtain here.

⁷Of course, the particular length of the period and the confinement of attention to just one period is completely inconsequential for the results using this model set-up.

⁸Notice that with more outcomes per sub-period, it is not the agent's performance that becomes distributed normal in the limit. Rather it is the Holmstrom and Milgrom (1987)- type account balances that are distributed normal which is something entirely different. I will return to the accounts in the next section.

(2002) I use wlog the following normalization:

$$p^* \omega_\theta^+ + (1 - p^*) \omega_\theta^- = 0,$$

where p^* is the optimal second-best action for each of the m sub-periods.⁹ That is, in equilibrium the expected value of (the noisy component of) own performance is normalized to zero. The history of realized own performance from the first to the last sub-period is denoted $\vec{\omega}$, while aggregate realized own performance for the entire m sub-periods is denoted Ω .

For the correlated peer performance measure I assume that it evolves in the same fashion as own performance, but independently of the actions of the focal agent. Thus, for each sub-period of length θ let peer performance also be either positive or negative:

$$\pi_\theta^+ > 0 > \pi_\theta^-$$

As in the case of own performance the expected value of (the noisy component of) peer performance is normalized to zero so that

$$q\pi_\theta^+ + (1 - q)\pi_\theta^- = 0,$$

where q is the probability that peer performance in any of the sub-periods is π_θ^+ which, again, does not depend on the focal agent's actions.¹⁰ The history of realized peer performance from the first to the last sub-period is denoted $\vec{\pi}$, while aggregate realized peer performance for the entire m sub-periods is denoted Π .

Of course, in any set-up like this, the use of peer performance in the evaluation of an agent is optimal only if the sufficient statistic condition is violated which implies that the covariance between peer performance and own performance, here denoted $\sigma_{\omega\pi}$, is non zero. To introduce non-zero covariance into the model I rely on the variable $\gamma \in [0, 1]$ which here represents the conditional probability of $\pi_\theta = \omega_\theta$. A γ of 1 then represents the case where the random component of performance is always matched where as $\gamma = 0$ represents the case

⁹ $p \in [0, 1]$ here represents all the potential actions available to the focal agent.

¹⁰The normalizations are done to de-emphasize any implications of the relative magnitudes of p^* and q for the relative performance of the firm and the peer(s).

where the random component is always mismatched. Of course, $\gamma = .5$, then, represents the case of independence and thus $\sigma_{\omega\pi} = 0$.

I will also utilize the convenient but at first glance perhaps somewhat stark assumption that the evolution of peer performance, just like own performance, is observed as it happens by the two parties relevant to my analysis. In the case of just one sub-period it is obviously non-controversial. In case of, for example, within-firm *RPE*, sporting events (again, think Tour de France or Indy 500) and arguably the key between-firm case (think stock-prices/returns), it may be not just a reasonable but also an accurate and desirable representation in general.¹¹ In cases where peer performance is obtained from periodic measures compiled by units exogenous to the firm (say quarterly earnings) and where there are many sub-periods, this assumption is obviously more circumspect. However, after utilizing the simplicity and tractability it brings to it's fullest, I will return later to demonstrate its benign nature in terms of its impact on the overall conclusions of this analysis.¹²

Finally, for simplicity and wlog I assume the principal to be risk-neutral and normalize the agent's coefficient of absolute risk-aversion to one so that the agent's utility function takes the form

$$u(S, c(p)) = -e^{-[S - \Sigma_m \theta c(p)]},$$

where $\theta c(p)$ is the agent's personal cost of implementing p in one of the m sub-periods.¹³ c' is used to represent the positive derivative of $c(p)$ and S represents the aggregate payment under the optimal contract. The payments associated with the possible performance measure realizations in each sub-period are denoted by s^{ij} , $i, j \in \{-, +\}$, where index i is associated with the realization of own performance and index j the realization of peer performance.

¹¹Core et al. (2003) suggest that stock prices account for by far the largest share of the variation in executive compensation. Accounting measures appear to have only a minimal role. Also, as can be gleaned from, for example, Table 1 in Dikolli et al. (2011a), empirical relative performance evaluation studies are based on stock prices.

¹²Much later - like not in this paper.

¹³Technically, the agent's choice can differ across the sub-periods, but since by Holmström and Milgrom (1987) Theorem 5, in equilibrium p^* is constant over time, the simpler cost representation used here will suffice.

3 Optimal Relative Performance Evaluation

Before getting started on characterizing the optimal *RPE* contract, I'll provide a technical result that is key to understanding why the dynamic Holmström and Milgrom (1987) approach that I rely on here generates results fundamentally different from the standard strong form prediction discussed above.

Lemma 1 *For the dynamic model specified in the previous section, $\sigma_{\omega\pi}$ is independent of the focal agent's choice, p , if and only if $\sigma_{\omega\pi} = 0$.*

Proof of Lemma 1. Given that I focus on a setting where the agent can only impact the distribution of own performance through his action in each of the sub-periods, γ must satisfy:

$$p\gamma + (1 - p)(1 - \gamma) = q.$$

It is then straight forward to verify that γ is independent of p if and only if p (and thus q) is equal to .5. For $p = .5$, $\sigma_{\omega\pi}$ is easily calculated to be 0, while for $p \neq .5$, $\sigma_{\omega\pi} \neq 0$ and is monotonic increasing in γ . Then noting that for $p \neq .5$,

$$\gamma = \frac{q + p - 1}{2p - 1},$$

so that

$$\frac{d\gamma}{dp} = \frac{1 - 2q}{(1 - 2p)^2}$$

completes the proof. ■

The key take-away from Lemma 1 is this: when normally distributed own and peer performance arise from the dynamic Holmström and Milgrom (1987) model as I use here, having both the covariance between own and peer performance be positive *and* having this covariance be independent of the focal agent's actions is logically inconsistent. That very assumption-combination is of course at the heart of the standard strong form prediction, whether it originates from Baker and Datar (1989) or from the LEN framework. Since a necessary condition for *RPE* to be optimal is that $\sigma_{\omega\pi} \neq 0$, in my analysis I only consider parameter values for p^* and γ different from .5 in which case $\sigma_{\omega\pi}$ is non-zero, but also not independent of the agent's actions.

Now, to identify the optimal properties of the incremental payments under the optimal contract generated by the realizations of the two available performance measures in any of the m sub-periods, for simplicity consider the strategies of the peers as "fixed" in the sense that each peer's strategy is pre-determined and unaffected by the unfolding of events (would be easy and therefore unnecessary to build a Nash Equilibrium as such!), and the principal wishes to implement p^* for (each of the sub-periods). His contract design problem can then be described as:¹⁴

$$\begin{aligned}
\min_{s^{ij}} \quad & p^* \gamma s^{++} + p^* (1 - \gamma) s^{+-} + (1 - p^*) \gamma \delta s^{--} + (1 - p^*) (1 - \gamma) s^{-+} \\
& \widehat{\lambda}_\theta \{ p^* \gamma u(s^{++}) + p^* (1 - \gamma) u(s^{+-}) + (1 - p^*) \gamma u(s^{--}) + (1 - p^*) (1 - \gamma) u(s^{-+}) \} \\
& + \mu_\theta \{ \theta c' p^* \gamma u(s^{++}) + \theta c' p^* (1 - \gamma) u(s^{+-}) \\
& + \theta c' (1 - p^*) \gamma u(s^{--}) + \theta c' (1 - p^*) (1 - \gamma) u(s^{-+}) \\
& + u(s^{++}) \left. \frac{d(p\gamma)}{dp} \right|_{p=p^*} + u(s^{+-}) \left. \frac{d(p(1-\gamma))}{dp} \right|_{p=p^*} \\
& + u(s^{--}) \left. \frac{d((1-p)\gamma)}{dp} \right|_{p=p^*} + u(s^{-+}) \left. \frac{d((1-p)(1-\gamma))}{dp} \right|_{p=p^*} \},
\end{aligned}$$

which has standard first-order conditions:

$$\frac{1}{-u(s_\theta^{++})} = \widehat{\lambda}_\theta + \mu_\theta \left\{ \theta c' + \frac{\left. \frac{d(p\gamma)}{dp} \right|_{p=p^*}}{p^* \gamma} \right\} \equiv \lambda_\theta + \mu_\theta L^{++}, \quad (1)$$

$$\frac{1}{-u(s_\theta^{+-})} = \widehat{\lambda}_\theta + \mu_\theta \left\{ \theta c' + \frac{\left. \frac{d(p(1-\gamma))}{dp} \right|_{p=p^*}}{p^* (1 - \gamma)} \right\} \equiv \lambda_\theta + \mu_\theta L^{+-}, \quad (2)$$

$$\frac{1}{-u(s_\theta^{--})} = \widehat{\lambda}_\theta + \mu_\theta \left\{ \theta c' + \frac{\left. \frac{d((1-p)\gamma)}{dp} \right|_{p=p^*}}{(1 - p^*) \gamma} \right\} \equiv \lambda_\theta + \mu_\theta L^{--}, \quad (3)$$

$$\frac{1}{-u(s_\theta^{-+})} = \widehat{\lambda}_\theta + \mu_\theta \left\{ \theta c' + \frac{\left. \frac{d((1-p)(1-\gamma))}{dp} \right|_{p=p^*}}{(1 - p^*) (1 - \gamma)} \right\} \equiv \lambda_\theta + \mu_\theta L^{-+}. \quad (4)$$

¹⁴By Theorem 5 in Holmström and Milgrom (1987), the optimal contract and the agent's resulting choice can be found by solving the problem for any one of the m sub-periods.

It is then straight-forward, using the (single) *IR*-constraint and one of the m identical *IC*-constraints, to verify that both λ_θ and μ_θ are decreasing in m and are approaching zero from above as $m \rightarrow \infty$.

Consider now introducing the " $N - 1$ accounts" as in Holmström and Milgrom (1987), here corresponding to each of the three different " $s^{ij} - s^{--} \neq 0$ " events that can take place in each of the m sub-periods. That is an account for the event $++$, an account for the event $+-$ and one for $-+$. In each sub-period credit the account for $++$ with θ if event $++$ occurs and with zero if that event does not occur and similarly for the two other accounts. Let A^{++}, A^{+-} and A^{-+} denote the respective account balances at the end of the contracting horizon. Then I have:

Theorem 1 *The agent's optimal RPE contract can be written as*

$$S(\vec{\omega}, \vec{\pi}) = m \times (s_\theta^{--} + A^{-+} \times [s_\theta^{-+} - s_\theta^{--}] + A^{+-} \times [s_\theta^{+-} - s_\theta^{--}] + A^{++} \times [s_\theta^{++} - s_\theta^{--}]). \quad (5)$$

To gain some initial insights into what (5) implies for the relation between the agent's pay and realized performances, simply ignore for the moment the natural logs embedded in (5) to obtain the following transformation of the optimal contract:¹⁵

$$\begin{aligned} \tilde{S}(\vec{\omega}, \vec{\pi}) &= m \times (\lambda_\theta + (1 - A^{-+} - A^{+-} - A^{++}) \times \mu_\theta L^{--} \\ &\quad + A^{-+} \times \mu_\theta L^{-+} + A^{+-} \times \mu_\theta L^{+-} + A^{++} \times \mu_\theta L^{++}), \end{aligned} \quad (6)$$

where $(1 - A^{-+} - A^{+-} - A^{++})$ would be the aggregate account balance for the "missing" N^{th} account, A^{--} .

Now consider the plots of the potential values of $\tilde{S}(\vec{\omega}, \vec{\pi})$ in Figure 1. Both plots are done for positive (expected) covariance between own and peer performance, that is $\gamma > .5$. Panel A represents the case then where $q > p^*$, while Panel B represents the case where

¹⁵I'll return to why doing so makes very good sense below.

$q < p^*$. What these panels reveal is by no means earth shattering: Panel A can easily be verified to correspond to the cases where the covariance between own and peer performance is increasing in p at p^* , while Panel B corresponds to the case where it is decreasing. And while in both cases own performance is rewarded *in expectation*, in the case of Panel A mismatched performance is penalized while in Panel B mismatched performance is instead rewarded. The implications of this for a standard regression are summarized by the following Lemma:

Lemma 2 *Using the transformed compensation, $\tilde{S}(\vec{\omega}, \vec{\pi})$, as the dependent variable in the following OLS-regression on aggregate own and peer performance,*

$$\tilde{S}(\vec{\omega}, \vec{\pi}) = \tilde{\beta}_0 + \tilde{\beta}_\Omega \Omega + \tilde{\beta}_\Pi \Pi + \tilde{\varepsilon},$$

for a large sample of identical firms, the coefficients one would expect to obtain satisfy

$$\tilde{\beta}_\Omega > \tilde{\beta}_\Pi = 0.$$

Proof of Lemma 2. The expected relative frequency of realized performance and incremental compensation in each of the m independent sub-periods are $A^{--} \equiv (1 - A^{-+} - A^{+-} - A^{++}) = m(1 - p^*)\gamma$, $A^{+-} = mp^*(1 - \gamma)$, $A^{-+} = m(1 - p^*)(1 - \gamma)$ and $A^{++} = mp^*\gamma$, where the first two accounts track relatively low peer performance while the last track relatively high. The OLS-regression then picks the \bar{s}^- and \bar{s}^+ that minimizes

$$m(1 - p^*)\gamma(\mu_\theta L^{--} - \bar{s}^-)^2 + mp^*(1 - \gamma)(\mu_\theta L^{+-} - \bar{s}^-)^2$$

and

$$m(1 - p^*)(1 - \gamma)(\mu_\theta L^{-+} - \bar{s}^+)^2 + mp^*\gamma(\mu_\theta L^{++} - \bar{s}^+)^2.$$

Using $\frac{d\gamma}{dp} = \frac{1-2q}{(1-2p)^2}$ it is easily verified that $\bar{s}^- = \bar{s}^+$ and accordingly that $\tilde{\beta}_\Pi = 0$. The proof that $\tilde{\beta}_\Omega > 0$ is mechanically identical and is thus left for the reader. ■

The intuition for Lemma 2 can be gleaned from the plots in Figures 2 and 3. Figure 2 collapses Figure 1 panel A into two two-dimensional plots, the first in the "own" dimension (Panel A) and the second in the "peer" direction (Panel B). The bold lines through the plotted areas in Panel B of both figures are the mean-squared-error-minimizing (*OLS*) lines for either "sample" taking into account the underlying joint density. Accordingly, the slope of these lines represent the *OLS* regression-coefficients one would expect to obtain from the

(standard *RPE*) regression in the above Lemma. Figure 3 does the same for Panel B of Figure 1 as Figure 2 does for Panel A of Figure 1. The key take-away from this hypothetical case is of course that absent the natural log transformation in (5), β_{Π} would be zero whether the covariance is increasing or decreasing in own effort. The skeptic can easily verify the same would be true if the correlation between own and peer performance had been negative. That β_{Ω} is also always positive here independent of the sign of the covariance is also straightforward to verify.

This brings me to the implications of the presence of the natural logs in the optimal contract detailed in Theorem 1. Again, there are no surprises here. The concave transformation of the natural log does exactly what it is supposed to do: it makes the low payments relatively lower than the higher ones when compared to the ones depicted in Figures 1 through 3. The consequence for the regression coefficient, β_{Π} , from the regression in the lemma above based on data generated by (5) rather than (6) then are as follows:

Theorem 2 *For a large sample of identical firms the coefficients one would expect to obtain from the OLS regression of compensation on aggregate own and peer performance,*

$$S(\vec{\omega}, \vec{\pi}) = \beta_0 + \beta_{\Omega}\Omega + \beta_{\Pi}\Pi + \varepsilon,$$

satisfy

$$\beta_{\Omega} > 0,$$

and

$$\beta_{\Pi} \begin{cases} < 0, & \text{if } \frac{d\sigma_{\omega\pi}}{dp} > 0, \\ > 0, & \text{if } \frac{d\sigma_{\omega\pi}}{dp} < 0, \end{cases}$$

where $\sigma_{\omega\pi}$ denotes the covariance between ω and π .

Proof of Theorem 2. The OLS-regression in the Theorem picks the values \bar{s}^{-} and \bar{s}^{+} that minimizes

$$m(1-p^*)\gamma(s^{--} - \bar{s}^{-})^2 + mp^*(1-\gamma)(s^{+-} - \bar{s}^{-})^2$$

and

$$m(1-p^*)(1-\gamma)(s^{-+} - \bar{s}^{+})^2 + mp^*\gamma(s^{++} - \bar{s}^{+})^2.$$

It is easily verified that for $\frac{d\gamma}{dp}$ positive, $\frac{d((1-p)\gamma)}{dp}\Big|_{p=p^*} > \frac{d((1-p)(1-\gamma))}{dp}\Big|_{p=p^*}$ and $\frac{d(p(1-\gamma))}{dp}\Big|_{p=p^*} <$

$\frac{d(p\gamma)}{dp} \Big|_{p=p^*}$. We can of course always find a pair of positive constants, $\{\tau, \phi\}$, such that

$$\ln L^{--} = \tau + \phi L^{--}$$

and

$$\ln L^{+-} = \tau + \phi L^{+-}.$$

However, since the natural log is a concave function, it follows from the two inequalities above that

$$\ln L^{++} < \tau + \phi L^{++}$$

and

$$\ln L^{-+} < \tau + \phi L^{-+}.$$

Then it follows directly from Lemma 1 that here $\bar{s}^+ < \bar{s}^-$ and accordingly that $\beta_{\Pi} < 0$. It is also straight forward to verify using the same approach that for $\frac{d\gamma}{dp}$ negative, $\beta_{\Pi} > 0$. The proof that $\beta_{\Omega} > 0$ is again left for the reader. ■

By Theorem 2 the key implication of the logs in the optimal contract, $S(\vec{\omega}, \vec{\pi})$, is that in case of Figure 2 Panel B β_{Π} turns negative while its counterpart in the case represented by Figure 3 turns positive. Without any way of preconditioning on whether $\frac{d\gamma}{dp}$ is positive or negative, the expected β_{Π} from a large sample regression of the Theorem 2 - type should still be zero. Fortunately, that is what the data keeps on saying it is. How is that for a successful and robust prediction of basic agency theory? The ironic part is, of course, that the very reason we have all the evidence in support of the prediction I have provided here is that a coefficient of zero never was the “strong form” prediction supplied.

To close the formal analysis, however, I want to return to the motivation for excluding the natural logs in (6) and the (initial) discussion of the empirical properties of the optimal *RPE* contract derived here. The simple reason for the exclusion is that the logical (no pun intended) implication of high frequency feedback (m growing very large) such as that provided by stock price data is that the relative increments in the optimal RPE contract converge to the relative increments of (6):

Theorem 3 *Let b^{ij} denote the increment associated with the enumeration account A^{ij} in*

(5) in the case where $m \rightarrow \infty$ and the accounts therefore are continuous on $[0, 1]$. Then

$$\frac{b^{ij}}{b^{kl}} = \frac{L^{ij}}{L^{kl}}, \quad k, l \in \{-, +\}.$$

Proof of Theorem 3. From the derivations underlying Theorem 1, the change in the agent's compensation associated with a change in A^{ij} is simply

$$\ln (\lambda_{\theta} + \mu_{\theta} L^{--} + \mu_{\theta} (L^{ij} - L^{--}))$$

Since in the limit as m grows large μ_{θ} becomes infinitesimal small, the above expression becomes identical to

$$\frac{\mu_{\theta} (L^{ij} - L^{--})}{s^{--}}$$

and the result thus follows. ■

The basic message here is that when the information flow on own and peer performance is frequent and timely as in the case of stock prices, the properties of the optimal RPE contract are best described by (6). While it does not fundamentally change any of the insights relative to those provided by (5), it does reinforce the prediction of a zero coefficient on peer performance in the standard RPE regression independent of the particular peer group that is being used. More importantly, perhaps, it facilitates a simple re-representation of the optimal contract in terms of two (relatively) easy to obtain independent variables. The final Theorem provides the details.

Theorem 4 *In the limiting case where $m \rightarrow \infty$, the optimal contract can be written on the form*

$$S(\vec{\omega}, \vec{\pi}) = \gamma_0 + \gamma_{\Omega} \Omega + \gamma_{\rho} \rho,$$

where ρ is the realized coefficient of correlation between own and peer performance over the time interval for which the compensation is awarded.

Proof of Theorem 4. Define $A^+ \equiv A^{++} + A^{+-}$ so that A^+ tracks own performance regardless of peer performance. Due to the "linearity in accounts" property of the optimal contract it is always possible to rewrite it as some linear function of A^+, A^{+-} and A^{-+} , hereafter denoted $\psi(A^+, A^{+-}, A^{-+})$. Then, for $A^+ = 1, \rho = 1$ if $A^{+-} = 0$ and $\rho = -1$ if $A^{+-} = 1$ so that for $A^+ = 1, \rho = 2 \left((1 - A^{+-}) - \frac{1}{2} \right)$. For $A^+ = 0, \rho = 1$ for $A^{-+} = 0$ and $\rho = -1$ for $A^{-+} = 1$ so that for $A^+ = 0, \rho = 2 \left((1 - A^{-+}) - \frac{1}{2} \right)$. Then, again due to the "linearity in accounts" the optimal compensation corresponding to a particular ρ , say $\bar{\rho}$, is

given as

$$\begin{aligned} & A^+ \times \psi\left(1, \frac{1-\bar{\rho}}{2}, 0\right) + (1-A^+) \times \psi\left(0, 0, \frac{1-\bar{\rho}}{2}\right) \\ &= \psi\left(0, 0, \frac{1-\bar{\rho}}{2}\right) + A^+ \left[\psi\left(1, \frac{1-\bar{\rho}}{2}, 0\right) - \psi\left(0, 0, \frac{1-\bar{\rho}}{2}\right) \right]. \end{aligned}$$

Further note that

$$\begin{aligned} & \psi\left(1, \frac{1-\bar{\rho}}{2}, 0\right) - \psi\left(0, 0, \frac{1-\bar{\rho}}{2}\right) \\ &= b^{+-} + \left(1 - \frac{1-\bar{\rho}}{2}\right) (b^{++} - b^{+-}) - \left(b^{--} + \frac{1-\bar{\rho}}{2} (b^{-+} - b^{--})\right) = b^{++} - b^{--} (> 0), \end{aligned}$$

and therefore the reward for own performance is independent of ρ .

Now also note that since the optimal compensation is linear in Ω and $dS/d\Omega$ is independent of ρ , if $dS/d\rho$ is dependent on Ω , it must be the case that $d(dS/d\rho)/d\Omega$ is a constant. Then given that

$$\left. \frac{dS}{d\rho} \right|_{A^+=0} = -\frac{b^{-+} - b^{--}}{2} = \frac{b^{++} - b^{--}}{2} = \left. \frac{dS}{d\rho} \right|_{A^+=1},$$

$d(dS/d\rho)/d\Omega = 0$ and thus

$$\frac{dS}{d\rho} = \frac{b^{++} - b^{--}}{2}$$

independent of own performance, Ω .

Next notice that for any $\{\Omega, \Pi\}$ pair, the feasible combinations of A^{+-} and A^{-+} correspond uniquely to a particular level of ρ . Since by Lemma 1 the optimal contract is independent of Π , this concludes the proof. ■

4 Discussion

The results obtained in the previous section differ fundamentally from those on which standard predictions in the *RPE* literature are based. A key difference is clearly (but in my mind entirely unsurprisingly) that in the general case it is not the sign of the covariance that drives optimal *RPE*; it is what the agent can *do* about the covariance that matters! From an empirical vantage point that is, at first glance, somewhat unfortunate as it moves the predictor from being an equilibrium property that can be easily estimated (covariance) to an out-of-equilibrium one (change in covariance) than cannot, at least not directly. If one steps

back from the technical aspects of the *RPE* results, both the common ones in the existing aggregate performance evaluation literature and those presented here, and instead considers the general economic intuition behind them I think the ones presented here make a lot more intuitive sense; both in terms of what they say about identifying the relevant peers and in terms of the observable variables that should be able to explain observed compensation.

To see why this is, consider first again the Holmström's (1982) Theorem 9 case. While insightful and elegant I don't see much enthusiasm in the literature for the case that the predicted weight on peer performance should be minus one.¹⁶ The reason, I suspect, is that as discussed in the introduction it corresponds to a case where *only* relative performance, not own performance, matters. Generally, I think, most people's experience with *RPE* is less extreme. Winning is not enough, for example. If you played well typically matters too. Doing slightly better than Enron is not likely to elicit much applause, having the fastest car on the block is not really that cool if it is a '87 Yugo GV, having the fewest publications in the department is a bit different if fewest means 20 than if it means zero and so on.¹⁷ In other words, a negative weight of one doesn't correspond to the way we intuitively think about *RPE* and it shouldn't be surprising either that it isn't what the data suggest.

I would argue that the same type of reality check on the standard (strong form) prediction in the existing *RPE* literature does not make most feel particularly convinced that it actually corresponds to their intuition or experiences either. I cannot think of (m)any good economic examples where independent of whether the individual did better or worse than his peers, individual performance is simply adjusted (down) by some fraction of average peer performance. Alternatively, I don't think that people exposed to *RPE* are ever satisfied just to be provided with some summary statistic without being able to somehow discern how they actually performed relatively to their peers either. Telling students that they got

¹⁶Allocation of fixed bonus pools or total raises available would fit as an example, but is in my mind not the kind of RPE we expect for executives that are typically the subjects of study in this literature.

¹⁷Furthermore, while pundits for many years have called for, for example, indexing Executive Stock Options thus effectively placing a negative weight of one on peer performance, that idea appears to have gained very little traction as a practical matter.

a B^+ is not going to be sufficient without also telling them the mean for their peers. Or telling a faculty member that their raise is 3% is not going to work that well, unless it is accompanied with information about what the peers got and why. However, if the conditions that underlie the standard “strong form” prediction are met, providing anything beyond the final aggregate should really be of no significance other than to verify that the aggregation was done correctly perhaps. And again the data agrees with the intuition/casual empiricism here: it is just not how *RPE* appears to be done in general.

The key insights generated by my analysis seem, at least to me, to fare a lot better in this gut-check contest. First of all, unlike in the case of Holmström’s (1982) Theorem 9 *both* own and peer performance matter. Second, *relative* performance matters too, because a simple linear aggregate as in Banker and Datar (1989) is not sufficient in general to communicate the implications of own and peer performance. More importantly perhaps, the notion that it is the *change* in the covariance that is key appears intuitively consistent with how we go about *RPE* in all kinds of standard situations. Think, for example, about a parent trying to encourage his/her child to excel in High School. How to best do this may of course differ by the nature of the school, culture, etc., but I don’t believe the typical approach is that the parent compares the kid with the average student in the school or deducting some fraction of average test scores from their own child’s scores as part of metering out a reward (or punishment). Rather, it is to identify peers that the student should try to be more like (think honor roll) and/or peers that the student should try and be less like (I’ll leave identification of this group to the reader’s own experience and imagination).

This approach to identify not one but multiple groups of peers seems so much more in line with how I believe we think about *RPE* in general. For a cell phone maker, for example, my guess is that the benchmark used for *RPE* is not based on a weighted average of Apple and RIM. Rather, *RPE* is used to encourage managers at this point in time to make their firms *more* like the Apples of the world and *less* like the RIMs. Politicians argue that we should be less like Greece and more like whatever country they view as having desirable

properties to be emulated. People have heroes and role models they aspire and strive to be like and similarly reference groups they actively try and disassociate themselves from. The examples are endless and I'd therefore argue that this kind of "good peer, bad peer" *RPE* suggested by my very basic analysis is the norm while the use of an "average peer" is at best a marginal and, as the evidence suggests, clearly an empirically irrelevant exception.

Furthermore, the task of identifying the relevant peer groups I think is actually quite straight forward. Becoming more like an ideal group implies as far as I am concerned not simply boosting ones performance relative to this group, but more importantly increasing ones covariance with it. Becoming less like another group of peers have the same relative consequences but also implies reducing the covariance with that group. And as I have shown in the previous section, reward structures that provide incentives to become more like some group differ fundamentally from reward structures that provide incentives to distance oneself from another group. Moreover, in neither case does the optimal reward structure disintegrate into simply placing a positive weight on own performance and a negative weight on average peer performance. Instead it adjust own performance with relative performance as captured by the realized correlation between own and peer performance.

5 Conclusion

In this paper I have developed a set of agency theory based predictions about *RPE* using the basic framework of Holmström and Milgrom (1987). As I show, generally the covariance between jointly normal own and peer performance is not going to be independent of the agent's effort. Accordingly, as pointed out by Banker and Datar (1989), linearly aggregated performance measures are therefore not sufficient and a contract written on own minus scaled peer performance is not optimal. What is optimal depends on how the agent's effort impacts the covariance between own and peer performance. If it increases the covariance the optimal contract, while increasing in own performance, rewards "similar" performance with peers. If

the covariance is decreasing, the optimal contract instead rewards "dissimilar" performance. As a result, the optimal contract varies in the *correlation* between own and peer performance but, consistent with the empirical evidence, optimal *RPE* is independent of *aggregate* peer performance.

References

- [1] Albuquerque, A., 2009, Peer Firms in Relative Performance Evaluation. *Journal of Accounting and Economics* 48, 69-89.
- [2] Albuquerque, A., 2011, Do Growth Options Firms Use Less Relative Performance Evaluation?. *Working paper*.
- [3] Antle, R., and A. Smith, 1986, An Empirical Investigation of the Relative Performance Evaluation of Corporate Executives. *Journal of Accounting Research* 24, pp. 1-39.
- [4] Banker, R., and S. Datar, 1989, Sensitivity, Precision, and Linear Aggregation of Signals for Performance Evaluation, *Journal of Accounting Research* 27, 21-39.
- [5] Celentani, C., and R. Loveira, 2006, A simple explanation of the relative performance evaluation puzzle. *Review of Economic Dynamics* 9, 525–540.
- [6] Core, J., W. Guay and D. Larcker, 2003, Executive Equity Compensation and Incentives: A Survey. *Economic Policy Review* 9, 26-45.
- [7] Core, J., W. Guay and R. Verrecchia, 2003, Price versus Non-Price Performance Measures in Optimal CEO Compensation Contracts, *The Accounting Review* 78, 957-981.
- [8] Dikoli, S., C. Hofmann and T. Pfeiffer, 2011a, Relative Performance Evaluation and Peer-Performance Summarization Errors. *Review of Accounting Studies*. *Forthcoming*.
- [9] Dikoli, S., C. Hofmann and T. Pfeiffer, 2011b, Accounting for Net Performance in Managerial Compensation Contracts. *Working paper*.
- [10] Dye, R., 1992. Relative Performance Evaluation and Project Selection. *Journal of Accounting Research* 30, 27-52.
- [11] Hellwig and Schmidt, 2002, Discrete-time Approximation of the Holmström-Milgrom Brownian-Motion Model of Intertemporal Incentive Problems. *Econometrica* 70, 2225-2264.

- [12] Hemmer, T., 2004, Lessons Lost in Linearity: A Critical Assessment of the General Usefulness of LEN Models in Compensation Research. *Journal of Management Accounting Research* 16, 149-162.
- [13] Holmström, B., 1979, Moral Hazard and Observability. *The Bell Journal of Economics* 10, 74-91.
- [14] Holmström, B., 1982, Moral Hazard in Teams, *The Bell Journal of Economics* 13, 324-340.
- [15] Holmström, B., and P. Milgrom, 1987, Aggregation and Linearity in the Provision of Intertemporal Incentives. *Econometrica* 55, 303-28.
- [16] Janakiraman, R. Lambert and D. Larcker, 1992, An Empirical Investigation of the Relative Performance Evaluation Hypothesis." *Journal of Accounting Research* 30, 53-69.
- [17] Mirrlees, J., 1974, Notes on Welfare Economics, Information and Uncertainty. In Balch, McFadden, and Wu, editors, *Essays on Economic Behavior under Uncertainty*, Amsterdam: North Holland.

6 Figures

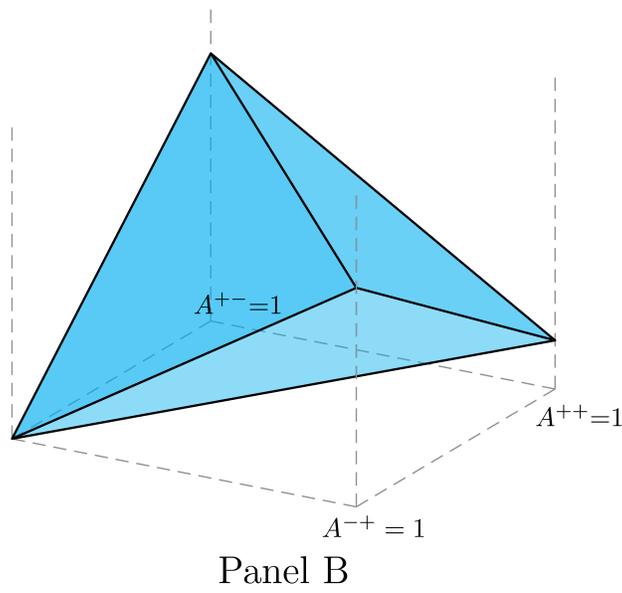
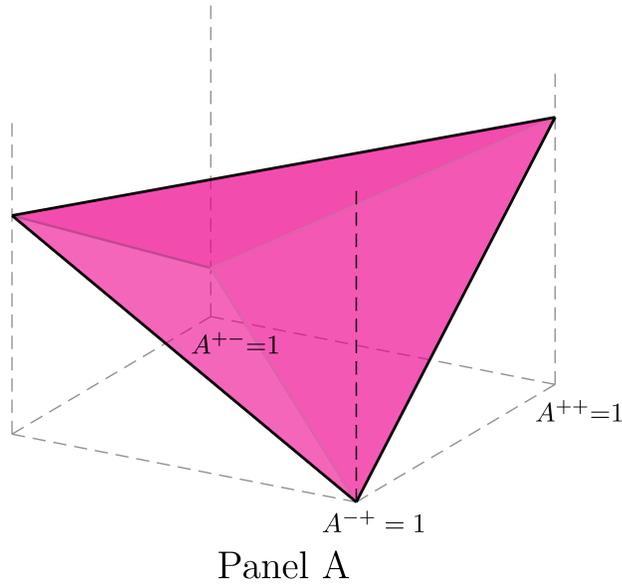
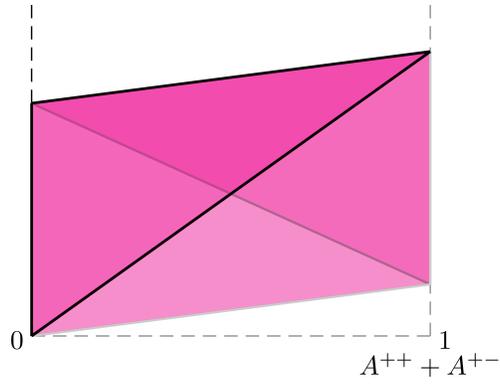
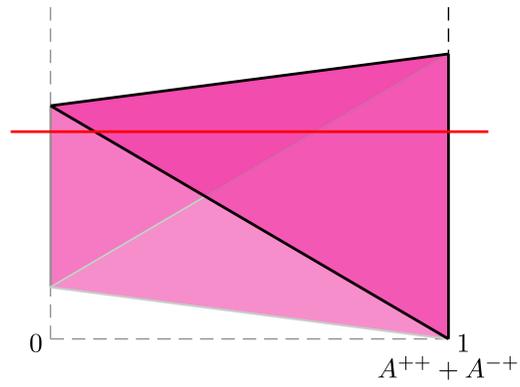


Figure 1. $\tilde{S}(\vec{\omega}, \vec{\pi})$ for $\frac{d\gamma}{dp} > 0$ (Panel A) and $\frac{d\gamma}{dp} < 0$ (Panel B).

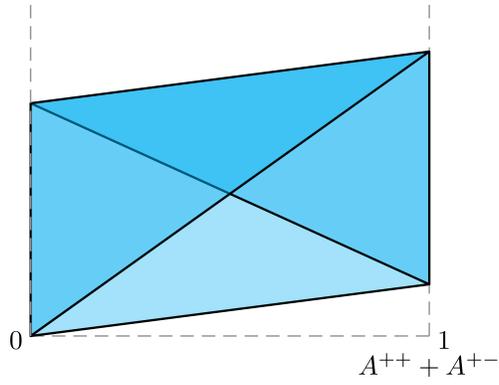


Panel A

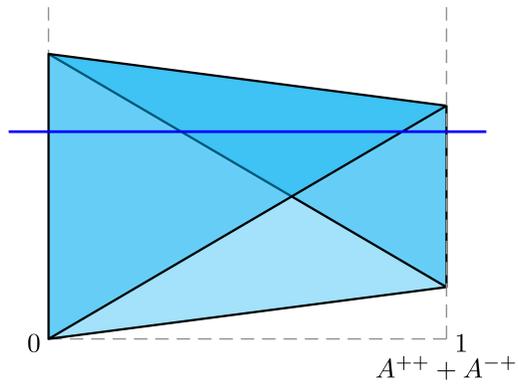


Panel B

Figure 2. Compensation from Figure 1, Panel A levels plotted against own (Panel A) and peer (Panel B) performance.



Panel A



Panel B

Figure 3. Compensation from Figure 1, Panel B levels plottet against own (Panel A) and peer (Panel B) performance.