The Impact of Discretionary Disclosure on Financial Reporting Systems: An Extension of Bayesian Persuasion

Henry Friedman John Hughes Beatrice Michaeli*

Anderson School of Management
University of California, Los Angeles

January 2016

* We thank Anil Arya, Tim Baldenius, Anne Beyer, Judson Caskey, Pingyang Gao (discussant), Robert Gox, Thomas Hemmer, Navin Kartik, Christian Leuz, Ivan Marinovic, Brian Mittendorf, Stefan Reichelstein, Richard Sansing, Naomi Soderstrom, Philip Stocken, Alfred Wagenhoffer (discussant), Yun Zhang (discussant) and seminar participants at Dartmouth, Ohio State, Stanford, Baruch, Melbourne, UCLA Anderson's 2015 Accounting Mini-Conference, the 9th ARW Conference in Zurich, the 2015 HKUST Accounting Research Symposium, and the 2016 FARS meeting for helpful comments and suggestions.

The Impact of Discretionary Disclosure on Financial Reporting Systems: An Extension of Bayesian Persuasion

Abstract: We use a Bayesian persuasion model to consider how a firm's design of its financial reporting system may be impacted by subsequent receipt and discretionary disclosure of private information. The firm's objective is to induce posterior expectations that meet a threshold. In the absence of private information, the firm prefers an imperfectly informative reporting system, notwithstanding that a perfectly informative reporting system is costless. Anticipating private information may cause the firm either to increase or decrease the reporting system's informativeness, depending on prior beliefs and the informativeness of private signals. However, whichever direction the impact of private information on the financial reporting system may take, the introduction of private information makes the firm unambiguously worse off. Setting aside feasibility issues, mandatory disclosure of private information may either increase or decrease the firm's welfare. Interestingly, there may also exist an equilibrium in which the firm chooses to never disclose private information.

Key Words: financial reporting, Bayesian persuasion, discretionary disclosure

I. INTRODUCTION

Firms make choices concerning the properties of financial reporting systems through the accounting policies that they adopt and the information they gather in arriving at estimates. Policy choices may reflect conservative or liberal biases, and information gathered for estimates may be biased in the sense that more attention is given to acquiring data that aligns financial statement users' actions with managers' preferences. Such biases are likely to arise in settings in which the firm avoids a substantial loss if and only if an outside party's beliefs meet or exceed a critical threshold level. Common examples of such settings include firms seeking to meet criteria for strong credit ratings, tight debt covenants, exchange listing, index inclusion, unqualified audit opinions, and asset impairment tests. In each of these settings, the firm is interested in influencing, through information produced by the reporting system, the beliefs of the outside party be it a rating agency, bank, regulator, investor, or auditor. Our study considers a complicating factor, focusing on how the subsequent receipt and discretionary disclosure of private information that might also influence an outsider affects the firm's design of its financial reporting system. In a parsimonious model, we show how the addition of a stage at which the firm may receive private information and exercise discretion over its public release impacts on the *ex ante* design of its reporting system, affecting its bias, informativeness, and the firm's expected benefits. We also characterize situations in which the firm would prefer to commit to disclose its private information, rather than maintaining the option not to disclose.

There are a number of situations that illustrate accounting choices that fall within generally accepted accounting principles and serve the preferences of firms facing important thresholds. A manufacturing firm offering product warranties and facing a threshold based on expected earnings might choose to recognize revenue at the time of sale rather than defer revenue until claims are submitted (a policy choice). It could then limit the information it gathers for estimating claims by conducting tests less likely to reveal product defects, resulting in lower warranty accruals. Similarly, a construction firm might choose the percentage of completion method of accounting for a project without gathering information about causes of project failures or delays that raise estimates of future costs. A merchandising firm may choose liberal credit policies (technically an operating rather than reporting choice) without fully investigating the risks of non-collection. In these cases, further information from warranty claims, cost realizations, or customer defaults might arise after financial reports and, if damaging, might not be disclosed.

Models like ours, examining the design of information systems, where the sender can commit to a design that is observable to the intended receivers, are referred to as Bayesian persuasion models by Kamenica and Gentzkow (2011).[1] Financial reporting systems fit the Bayesian persuasion framework reasonably well in the sense that firms have flexibility in choosing accounting policies that may advance their interests. The firm's overriding interest in our setting is to meet a crucial threshold. We embellish this setting by recognizing that subsequent to issuing financial reports, firms are likely to receive private information, the disclosure of which is discretionary. Press releases, public announcements, management forecasts, and supplemental SEC filings are among the more familiar conduits through which private information may be disclosed. Since disclosure or non-disclosure of this information may also contribute to the formation of posterior expectations affecting whether thresholds are met, anticipation of receiving private information may factor into the design of financial reporting systems. Our study explores this interaction. We also compare discretionary and mandatory disclosure of private information in assessing the value to the firm of the option to not disclose. Stepping back, we consider the value of private information *per se*, i.e., whether the firm is better or worse off when it may become privately informed.[2]

In a pure Bayesian persuasion context, firms facing threshold concerns may seek to dampen the informativeness of financial reports.[3] The basic idea is that the firm may increase the relative frequency of reports that are just good enough to meet the threshold by allowing imperfection in reporting of states that would exceed the threshold. For instance, suppose that a perfectly informative, but relatively infrequent, good report implied a state that strictly exceeded the threshold. By allowing a good report to also sometimes be generated in a state that (if perfectly observed) would not meet the threshold, the firm may be able to increase the relative

[1] These types of models are relatively new in the literature and have also been studied, for example, in Duggan and Martinelli (2011), Gentzkow and Kamenica (2013), Michaeli (2014), Taneva (2014), Alonso and Camara (2014), and Wang (2013). Without being cast as persuasion, Goex and Wagenhoffer (2009) and Arya, Glover and Sivaramakrishnan (1997) also consider ex ante commitment to information gathering.

[2] Hedlund (2015) considers a setting in which the sender has private information at the time of choosing the signal structure. There, the mere choice of reporting system conveys information about the sender's private information.

[3] As shown in Kamenica and Gentzkow (2011), the driving force behind the interior solution is that the firm's payoff as a function the outsider's beliefs has both strictly convex and concave regions. If the payoff was globally convex (concave), then a perfectly informative (uninformative) reporting system would be optimal.

frequency of a good report, thereby still meeting the threshold but with higher *ex ante* probability. In a vernacular familiar to accountants, threshold concerns create an incentive for introducing a liberal bias into financial reporting systems. Biases motivated in response to threshold concerns can therefore manifest in liberal accounting policy choices.[4]

Many studies in accounting employ models of financial reporting systems with state-dependent asymmetric informativeness, or bias, similar to our model. Gigler and Hemmer (2001) show how a conservative bias may reduce pre-emptive voluntary disclosure, thereby mitigating the value of communication between managers and shareholders. While they seek to address the question of how reporting quality affects discretionary disclosure, we seek to address how the prospect of discretionary disclosure influences properties of public reporting systems. Kwon, Newman, and Suh (2001) consider optimal compensation arrangements in a moral hazard context with limited liability for which bad reports are less informative and good reports more informative of underlying bad and good states, respectively. Gigler et al. (2009) show how bias in a reporting system may make it more or less likely that a favorable or unfavorable signal accurately reports the underlying state in a setting where investment continuation decisions are at stake. Beyer (2012) considers an aggregate reporting system for a multi-segment firm that only reports losses and not gains in asset values. Such a system is less informative about gains in values, but is more informative about losses by comparison with a system that reports both since losses are not offset by gains. Friedman, Hughes, and Saouma (2015) portray effects of reporting biases on competition. Of particular interest is the distinction they draw between bias and precision in showing how bias may increase overall reporting system informativeness holding symmetric precision constant.

The addition of a subsequent stage at which firms may or may not disclose private information not encompassed by its financial reports influences the optimal design of financial

[4] There is considerable empirical support regarding the importance of meeting thresholds in avoiding losses. Kausar, Taffler, and Tan (2006) find that institutional investors tend to divest after going concern qualifications. Menon and Williams (2010) find negative market reactions to going concern qualifications in audit reports likely driven by the dependencies of exchange listings, debt terms, and financing on obtaining unqualified reports. Graham and Harvey (2001) find that credit ratings are a major concern for CFOs in capital structure decisions, while Kisgen (2006) notes that an inability to maintain high ratings may exclude institutions from holding bonds, trigger higher interest rates, etc., thereby affecting capital structure decisions. Beneish and Press (1993) find that violations of debt covenants lead to increases in interest rates, and in a later study Beneish and Press (1995) detect negative market reactions associated with such violations. Li et al. (2011) find indirect evidence that failing to pass goodwill impairment tests was a principal concern of firms given the negative impact of impairments on analysts' and market expectations.

reporting systems in a surprising way. We identify conditions on the informativeness of private signals and on the prior beliefs such that, in anticipation of the effects of discretionary disclosure on posterior expectations, firms choose more informative financial reporting systems that reduce the probability of meeting the thresholds in comparison to the case where firms do not expect to receive private information. Also possible is the somewhat more intuitive case for discretionary disclosure of private information to induce firms to choose less informative financial reporting systems to offset the anticipated effects of information contained in such disclosures. As we elaborate below, a design that provides more informative financial reports of states that exceed the threshold may be necessary to overcome the reduction in posterior expectations from non-disclosure of a private signal. Increasing the informativeness of the reports comes at a cost: a reduced frequency of reports that cause beliefs to meet the threshold. We further find that the option to not disclose may or may not be valuable to the firm in comparison to mandatory disclosure (presuming that such mandatory disclosure could be enforced). Stepping back to consider the impact of private information on the firm's welfare, we find that the firm is better off in meeting a crucial threshold without the potential to receive private information. Last, we consider the efficacy of commitments not to disclose private information and find that in the absence of some benefit to disclosure beyond meeting the threshold, such commitments are sustainable.

As is typical in models of discretionary disclosure (e.g., Verrecchia 1983, Dye 1985 and Jung and Kwon 1988), in equilibrium, a low-end pool is formed and private information that would lower posterior expectations is suppressed. Only signals that would raise posterior expectations above the prior expectations are disclosed. Of course, rational receivers would lower their expectations upon not observing a disclosure to take into account that the sender may have realized a low signal. Holding constant the design of the financial reporting system with no private information, raising expectations from those induced by financial reports may be excessive to maintaining the highest probability of just meeting the threshold. While the firm may adjust for this effect by making the financial reporting system less informative, this response is mitigated by the need to offset the effect of non-disclosure in lowering expectations. Hence, as we alluded to in summarizing our results, interesting questions are whether the option to not disclose private information is valuable to the firm and whether either discretionary or mandatory disclosure of private information is valuable to the firm in meeting a crucial posterior expectations threshold.

Among the issues we have technically suppressed in our model is the prospect of *ex post* manipulation of financial reports. In the absence of some added friction or noise, we can ignore such biasing since rational receivers of those reports will undo their effects, making them irrelevant. We could allow for biases that could not be undone as long as there are limitations on a firm's flexibility in distorting reports before they are disseminated. The important feature of the financial reporting system structure in our model is that one cannot completely undo the effects of *ex ante* design choices *ex post*. We also ignore any out-of-pocket costs to increasing the informativeness of the financial reporting system; a perfectly informative system is feasible at no such cost. From a modeling perspective, such costs are often introduced as a means to obtain interior rather than corner solutions. In our model, out-of-pocket costs that prevent corner solutions are unnecessary and would merely obscure the following insight: that a less than perfectly informative system may be desirable as a means of inducing beliefs that meet a threshold with greater probability.

The most closely related study to ours is Stocken and Verrecchia (2004). In their model, an *ex ante* choice of financial reporting system precision is followed by the sender's manipulation of a report based on the realization of the signal generated by that system and a further private signal realization. The sender's ability to manipulate the report *ex post* may induce a less precise *ex ante* reporting system choice. In contrast, our paper focuses on *ex ante* choices that affect precision and bias and the sender's ability to exercise discretion over disclosure of a private signal. In our paper, the potential for discretionary disclosure can have a negative or *positive* effect on the informativeness of the reporting system chosen *ex ante*. Similar to our study, Kamenica and Gentzkow (2011) consider how an optimal information system will be set when the sender (the firm in our case) is uncertain about the beliefs of a receiver (the outside party in our case). In our model, given that the firm does not know if and what private information it will observe, there is also uncertainty about the beliefs of the outside party at the stage in which the reporting system is set. However, the firm has a partial control, because it can choose to disclose or withhold this private information. In this context, we show that the firm cannot, by discretionary disclosure of subsequently acquired private information, improve the likelihood of meeting the threshold beyond that achievable from the public reporting system alone; i.e., given a choice, the firm strictly prefers not to obtain private information.

Our study also relates closely to two streams of empirical literature. First, several studies document associations between properties of financial reports (e.g., earnings qaulity or complexity) and discretionary disclosure as represented by management forecasts or guidance (e.g., Ball et al., 2012; Francis et al., 2008; Guay, Samuels, and Taylor, 2015; Gong et al., 2009; Lennox and Park, 2006). Overall, the average sign of the association between financial reporting quality and the frequency and accuracy of management forecasts varies across these studies. Our study provides a theoretical foundation for observing mixed empirical evidence.

The second stream of related empirical work provides substantial support for firms utilizing accounting practices as devices for boosting the likelihood that thresholds will be met. Bartov, Gul, and Tsui (2001) find an association between discretionary accruals and audit report qualifications.[5] Press and Weintrop (1990) find that firms use accounting flexibility to meet debt covenants. Healy and Palepu (1990) find the opposite; however, Begley (1990) suggests that this could be an identification issue. Sweeney (1994) finds that firms approaching covenant violation early-adopt mandatory income-increasing changes and that firm's discretionary changes are increasing in default costs. Dichev and Skinner (2002) find that a large number of firms meet or beat covenants suggesting manipulation of reports upon which covenants are based. Kim and Kross (1998) find evidence of manipulation of loan loss provisions coincident with a change in bank capital standards. Ramanna and Watts (2012) find firms tend to use discretion in applying tests of goodwill impairment. Chen, Lethmathe and Soderstrom (2015), study the firm's reporting behavior when their objective is to meet a return level required to be accepted into a UN carbon emission program. Bonachi, Mara and Shalev (2015) find evidence consistent with parent firms accounting for business combinations under common control at fair value when their leverage is high and they have net covenants.

Our study makes several contributions to the accounting literature. To the best of our knowledge, we are the first to model the impact of *ex post* discretionary disclosure of private information on the *ex ante* design of public reporting systems. Notwithstanding a high level of abstraction, our model captures an incentive for biased financial reporting distinct from other incentives characterized in the literature. Especially noteworthy is our result that a more

[5] Signed accruals have been a common workhorse for detecting earnings manipulation. We suggest that biased accruals could be an artifact of accounting policies chosen *ex ante* as well as a consequence of *ex post* manipulations. The former would appear to be more likely in a context where thresholds apply over multiple reporting periods.

informative financial reporting system induced by discretionary disclosure of private information may weaken the effect of the reporting system in raising the probability of meeting a crucial threshold.[6] The results we obtain on the value of discretionary disclosure of private information or the value of private information under either discretionary or mandatory disclosure offer further insights on the influence of meeting crucial thresholds that firms may face.

II. THE MODEL

There is a stochastic state that indirectly affects the payoff for a risk-neutral firm. The firm experiences a significant benefit (equivalently avoids a significant loss) whenever the beliefs of a Bayesian outside party meet or exceed a threshold representing an unmodeled decision made by the outsider. The firm, through the design of a financial reporting system, seeks to maximize the probability that the outside party's posterior expectation of the state (i.e., after receiving reports and messages from the firm) at least meets the threshold. The players have common prior beliefs, and the accounting policies that comprise the firm's *ex ante* choice of its financial reporting system are publicly observable. We assume that the firm receives a private signal with a probability strictly less than one as in Dye (1985) and Jung and Kwon (1988) after the financial reporting system has been implemented. This probability and the distribution generating private signals are also common knowledge. Disclosed signals are credible and it is not possible to credibly communicate not having received a signal.

For analytic tractability, we adopt a binary state and reporting structure similar to Gigler and Hemmer (2001), Kwon et al. (2001), Bagnoli and Watts (2005), Smith (2007), Chen and Jorgensen (2012), Guo (2012), and Friedman et al. (2015); albeit in a different context. While parsimonious, the structure is adequate for depicting persuasive behavior on the part of the firm in choosing its reporting system. A similar binary structure for the firm's private signal, if received, is sufficient for depicting the impact of discretion over disclosure on the reporting system design choice. In order to focus on the impact of discretionary disclosure on the design of the financial reporting system, we assume a parameterization that preserves pooling of a low signal realization with non-receipt of a signal as a rational strategy.

We allow the firm to choose the properties of the reporting system but take the properties

[6] The opposite case of discretionary disclosure of private information inducing a less informative financial reporting system is also possible.

of the private signals as exogenous. This captures the idea that the firm has flexability in designing its financial reporting system, but often cannot control the arrival and nature of private information. In contrast, Gigler and Hemmer (2001) explore a setting in which the reporting system is fixed and the private information system is endogenously chosen. In Section IV, we discuss an extension in which the firm has control over both the financial reporting and the private information systems.

Formally, the firm's random state is represented by $\theta \in \{H, L\}$ where H and L represent high and low values, respectively. We normalize values by setting $H = 1$ and $L = 0$. The outsider's threshold against which he compares posterior expectations is represented by $k \in (0,1)$. Common prior beliefs are defined by $\alpha = \Pr(\theta = H)$.[7] We assume $\alpha < k$ to avoid the trivial case where the threshold is met even in the absence of additional information provided through reports and messages. The financial reporting system generates a report with the structure:

$$\Pr(r = g | \theta = H) = \beta_H \in [0,1]$$

$$\Pr(r = g | \theta = L) = \beta_L \in [0,1]$$

where $\beta_H \geq \beta_L$. The manager chooses $\boldsymbol{\beta} \equiv (\beta_H, \beta_L)$ prior to potentially receiving a private signal, s. With probability $q \in (0,1)$, the firm receives a non-empty private signal $s \in \{h, l\}$ with the following structure:

$$\Pr(s = h | \theta = H, s \neq \emptyset) = \gamma_H \in [0,1]$$

$$\Pr(s = h | \theta = L,, s \neq \emptyset) = \gamma_L \in [0,1]$$

where $\gamma_H > \gamma_L$. The firm cannot credibly communicate not having received a signal, $s = \emptyset$, which happens with probability $1 - q$. Upon receiving a non-empty signal, s, the firm can either truthfully disclose that signal by sending a message $m = s$ or not disclose, in which case the message $m = \emptyset$ is the same as when a signal is not received.

We assume that the firm's payoff is increasing in the posterior expectation of the outside party about the firm's state. Of principal interest, the firm receives an additional benefit if the expected state meets or exceeds a threshold. Formally, we define the firm's *ex post* payoff as:

$$\pi \equiv \sigma E[\theta | m, r] + 1_{E[\theta|m,r] \geq k} S$$

where S is the discrete benefit (or loss avoided) from meeting the threshold, k, normalized to 1,

[7] In our model, beliefs are equivalent to the probability that the state is high, which, given our assumption that H = 1 is also the expected value. As such, we tend to use expectation and beliefs interchangeably.

and $\sigma > 0$ is the sensitivity of the *ex post* payoff to an increase in the posterior expectation.[8]

The expression we use for the firm's *ex post* payoff is meant to capture typical concerns while introducing necessary elements for a theoretically interesting interior solution in a minimally-parameterized function. There are two components: a linear component, $\sigma E[\theta|m,r]$; and a step-function component, $1_{E[\theta|m,r]\geq k}S$. As we show below, the step function gives the firm an incentive to set a reporting system that is informative but does not fully reveal the underlying state (i.e., an interior solution to the problem of setting the reporting system). The linear component provides important incentives in the discretionary disclosure stage of the game. As in Kamenica and Gentzkow (2011), obtaining an interior solution requires that the firm's *ex post* payoff should have both strictly convex and concave regions. This could be achieved by more general functions or by more densely-parameterized functions (e.g., piecewise linear with $n > 3$ pieces). However, our simple parameterization provides the necessary components while maintaining parsimony, tractability, and foundations in the institutional environment. For example, our two-component model concisely captures a firm whose stock price increases linearly in the market's average belief about θ, and experiences a jump when the price justifies inclusion in an index such as the Russel 3000.[9]

Figure 1 depicts the timeline of events. At date 1, the firm chooses the parameters $\boldsymbol{\beta} \equiv \{\beta_H, \beta_L\}$ governing the financial reporting system. The state, θ, is drawn by nature, but observed by neither the firm nor the outsider. At date 2, the financial report is realized and observed by both players, and either a private signal s is realized and privately observed only by the firm, or no signal is received. At date 3, the firm sends either a message $m = s$, or $m = \emptyset$ to the outside party. At date 4, the outside party forms a posterior expectation of the state and assesses whether the threshold has been met. The firm receives $\sigma E[\theta|m,r]$ and an additional benefit S normalized to 1 if the outsider's posterior expectations meet or beat the threshold. In

[8] Figures 3, 4, and 5 present plots of the firm's ex post payoff as a function of the outside party's beliefs. We discuss the case of $\sigma = 0$ in Section IV.

[9] Formally, we take the outsider as a passive Bayesian, but note that the firm's payoff function could be derived from an outsider who chooses two actions, $a_1 \in \{0,1\}$ and $a_2 \in [0,1]$, to maximize a utility function given by $u = -\left(a_1 - 1_{\mathrm{E}[\theta|m,r]\geq k}\right)^2 - (a_2 - \theta)^2$. In the context of the Russel 3000 example, the outsider stands in for two types of investors, where a_1 represents mechanical investment choices made by Russell 3000 index funds, and a_2 captures portfolio allocation choices made by other investors. For a given report, r, and message, m, the outsider's optimal actions are $\{\hat{a}_1, \hat{a}_2\} \in \arg\max_{a_1,a_2} E[u|r,m]$ and the firm's payoff is given by $\pi \equiv \sigma\hat{a}_2 + S\hat{a}_1 = \sigma E[\theta|m,r] + 1_{E[\theta|m,r]\geq k}S$, where σ and S represent the sensitivities of the firm's stock price to the different actions.

our analysis, $\boldsymbol{\beta}$ is chosen before the firm potentially observes the private signal. Otherwise, the choice of $\boldsymbol{\beta}$ would signal the firm's private information, as in Hedlund (2015). The timing of the public report, *r*, relative to the private signal, *s*, and the message, *m*, is inconsequential, as is the timing of the firm's choice of $\boldsymbol{\beta}$ relative to nature's unobserved draw of θ.

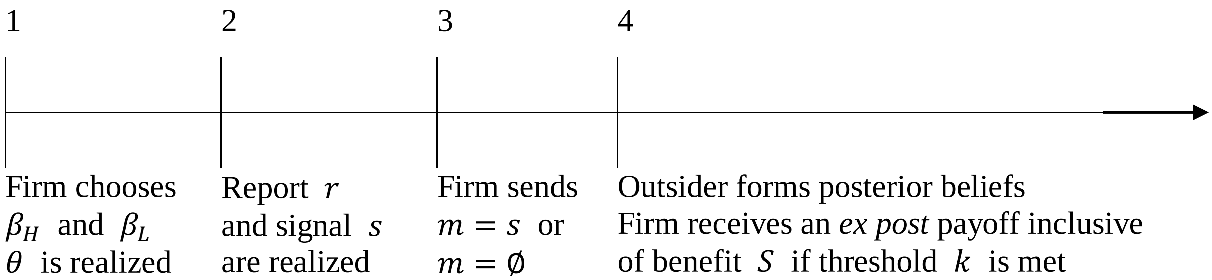

Figure 1
Timeline of events

III. ANALYSIS

Optimization Objective

Using the Law of Iterated Expectations, the firm's *ex ante* expected payoff simplifies to

$$E[\pi] = \sigma\alpha + B,$$

where

$$B \equiv E[1_{E[\theta|m,r]\geq k}]S = Pr(E[\theta|m,r] \geq k) \cdot 1 = Pr(E[\theta|m,r] \geq k)$$

is the expected benefit to the firm of meeting (or exceeding) the threshold which, under our maintained assumptions, is equal to the probability that the threshold is met. It is straightforward to show that maximizing the firm's expected payoff of is equivalent to maximizing the expected benefit B:

$$\underset{\boldsymbol{\beta}\in X}{\operatorname{argmax}} E[\pi] = \underset{\boldsymbol{\beta}\in X}{\operatorname{argmax}} B$$

where X is the set of plausible values of $\boldsymbol{\beta}$, since $\sigma\alpha$ is independent of the firm's reporting system choices.[10] When choosing across regimes and reporting systems we therefore focus only on the variable portion, B.

[10] Even though the first term of the firm's payoff, $\sigma\alpha$, does not matter from an *ex ante* perspective, $\sigma > 0$ *is* important for obtaining a unique disclosure strategy in the voluntary disclosure stage.

Financial Reporting With No Private Information

We first consider as a benchmark the special case in which the firm never receives private information. This case is a pure persuasion game in which posterior expectations are based only on the firm's financial report. Consider the extreme choices of $\boldsymbol{\beta}$. Setting $\beta_H = \beta_L$ implies an uninformative reporting system with no updating of beliefs. Hence, the outside party's posterior beliefs are equal to its prior beliefs $\alpha < k$, the threshold is not met, and $B = 0$. At the other extreme, $\beta_H = 1$ and $\beta_L = 0$ implies a perfectly informative system. In this case, the outside party's posterior belief equals 1 if $r = g$, implying the threshold is exceeded, and 0 if $r = b$, implying the threshold is not met. It follows that $B = \alpha S = \alpha$. Notably, assurance of a high state given a good report is a stronger condition than is necessary to meet the threshold. The firm can increase the probability of meeting the threshold by allowing some good reports to be generated in a low state. While this diminishes the posterior expectation given a good report, the expectation may still be sufficient to meet the threshold. Accordingly, in the absence of private information the firm maximizes the probability of a good report, subject to meeting the threshold. This is accomplished by setting $\beta_H = 1$ and solving for β_L in the following expression:

$$\Pr(\theta = H | r = g) = \frac{\alpha \Pr(r = g | \theta = H)}{\alpha \Pr(r = g | \theta = H) + (1 - \alpha) \Pr(r = g | \theta = L)} = \frac{\alpha \beta_H}{\alpha \beta_H + (1 - \alpha) \beta_L} = k.$$

The optimal choice of $\boldsymbol{\beta}$, with superscript "P" to indicate the "pure persuasion" benchmark, is

$$\beta_H^P = 1 \quad \text{and} \quad \beta_L^P = \frac{\alpha(1-k)}{k(1-\alpha)} \in (0,1),$$

implying an expected benefit $B^P = \frac{\alpha}{k} > \alpha$. While with perfect information the firm only meets the threshold with probability α, an optimal reporting system improves the odds to $\frac{\alpha}{k}$. Both parties are rational and update consistent with Bayes' Rule, notwithstanding that the information provided by the firm's reporting system is slanted in a manner that serves the firm's interests.

The distribution over posterior beliefs (i.e., the outsider's expectation that the underlying state is high) generated by reports is as follows. The outsider has a posterior belief equal to k with probability $\frac{\alpha}{k}$ and a posterior belief of 0 with probability $\frac{k-\alpha}{k}$. We note that these posterior beliefs satisfy the law of iterated expectations; i.e., $\frac{k-\alpha}{k} \times 0 + \frac{\alpha}{k} \times k = \alpha$.[11]

Financial Reporting With Private Information

The possible receipt and discretionary disclosure of a private signal adds a second stage at which the firm makes a decision and the outside party updates beliefs. Accordingly, we solve the model by backward induction. Recall that the firm receives a benefit, S, if and only if $E[\theta|r,m] \geq k$. Having normalized the states at $H = 1$ and $L = 0$, the above expectation is simply the posterior probability of $\theta = H$ given a report r and message m, i.e.,

$$E[\theta|r,m] = \Pr(\theta = H|r,m).$$

Suppose the firm receives a private signal s. Since $\gamma_H > \gamma_L$, the posterior probability of a high state is greater conditional on message $m = h$ than on $m = \emptyset$ and also greater conditional on message $m = \emptyset$ than on $m = l$. The lemma below follows immediately:

Lemma 1: The firm always discloses when $s = h$ and never discloses when $s = l$.

Moving back to the choice of parameters governing the financial reporting system $\boldsymbol{\beta}$, as is the case without private information, the firm wants to maximize the expected benefit B which equates to maximizing the probability that posterior beliefs meet or exceed the threshold. There are four combinations of reports and messages that might maximize the joint probability of meeting the threshold, shown below.[12] Each combination gives rise to a constrained optimization program for which some $\boldsymbol{\beta}$ is optimal. We solve the firm's problem by first solving for the optimal $\boldsymbol{\beta}$ that maximizes the probability of meeting or exceeding the threshold with a specific set of report-message combinations, and then determining which set, at its optimum, is best for the firm. The combinations and related programs are as follows, where "D" denotes discretionary disclosure:

[11] This is equivalent to the "Bayesian plausibility" requirement in Kamenica and Gentzkow (2011). In our case, we incorporate this requirement in our calculations of posterior beliefs using Bayes' Rule rather than explicitly including the requirement as an additional constraint in the optimization programs.

[12] Recall that as per our discussion early on the firm sets the reporting system to maximize the expected benefit, which under our assumptions equates to maximizing the probability of meeting the threshold.

$$\mathcal{P}1(D): \quad \max_{\boldsymbol{\beta}} \ \Pr(r = g, m = h) + \Pr(r = b, m = h) + \Pr(r = g, m = \emptyset)$$

$$s.t. \quad E[\theta | r = g, m = h] \geq k, \ E[\theta | r = b, m = h] \geq k,$$

$$E[\theta | r = g, m = \emptyset] \geq k, \ 1 \geq \beta_H \geq \beta_L \geq 0;$$

$$\mathcal{P}2(D): \quad \max_{\boldsymbol{\beta}} \ \Pr(r = g, m = h) + \Pr(r = b, m = h)$$

$$s.t. \quad E[\theta | r = g, m = h] \geq k, \ E[\theta | r = b, m = h] \geq k,$$

$$1 \geq \beta_H \geq \beta_L \geq 0;$$

$$\mathcal{P}3(D): \quad \max_{\boldsymbol{\beta}} \ \Pr(r = g, m = h) + \Pr(r = g, m = \emptyset)$$

$$s.t. \quad E[\theta | r = g, m = h] \geq k, \ E[\theta | r = g, m = \emptyset] \geq k,$$

$$1 \geq \beta_H \geq \beta_L \geq 0;$$

$$\mathcal{P}4(D): \quad \max_{\boldsymbol{\beta}} \ \Pr(r = g, m = h)$$

$$s.t. \quad E[\theta | r = g, m = h] \geq k, \ 1 \geq \beta_H \geq \beta_L \geq 0.$$

Elaborating on $\mathcal{P}1(D)$, the objective function is composed of the unconditional joint probability of report-message combinations including a good report and disclosure of a high signal, bad report and disclosure of a high signal, and good report and non-disclosure of a signal. The constraints ensure that the threshold is met for each combination and assumed properties of $\boldsymbol{\beta}$. The first constraint (good report-high signal) will be slack, while at least one of the next two constraints (good report-no message or bad report-high signal) will bind, as both imply a lower probability of meeting the threshold. Each of the next two programs, $\mathcal{P}2(D)$ and $\mathcal{P}3(D)$, considers two combinations of reports and messagess while eliminating one of the potentially binding constraints in $\mathcal{P}1(D)$. $\mathcal{P}4(D)$ considers only one combination while eliminating both of the potentially binding constraints in $\mathcal{P}1(D)$, which allows the remaining constraint to bind. Eliminating constraints enlarges the feasible regions, but reduces the set of report-message combinations that result in posterior beliefs at or above the threshold. Hence, *a priori* we cannot say which program solution will provide the highest probability and related expected benefit of meeting the threshold for a given set of exogenous parameters. Solutions to the programs are provided in the Appendix.

Characteristics of Optimal Financial Reporting Systems

We begin this section by identifying a set of conditions on model parameters that have a bearing on which of the solutions to the above programs dominates. These conditions lead to characterizations of optimal financial reporting systems. We further assess the impact of discretionary disclosure and, separately, the potential availability of private information by comparing the optimal financial reporting system with the solution to the pure persuasion game benchmark.

Condition 1: $\gamma_H \geq \overline{g} \equiv \frac{1-\gamma_L kq}{q(2-\gamma_L q-k)}$

We refer to the above condition as capturing private signal informativeness. Note that the lower bound, $\overline{g}$, on the probability of a high signal given a high state in Condition 1 is increasing in the probability of a high signal given a low state, γ_L. Either an increase in γ_H or a decrease in γ_L widens the spread between those probabilities, which naturally captures private signal informativeness. Accordingly, we classify private signals as more informative if Condition 1 is satisfied and as less informative otherwise.

Condition 2: $\alpha > \frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$

Prior beliefs are said to be optimistic if Condition 2 is satisfied and pessimistic otherwise.

Proposition 1: If Condition 1 is not satisfied (less informative private signals), then the firm's optimal financial reporting system is defined by $1 = \beta^*_{\mathrm{H}} > \beta^*_{\mathrm{L}} > 0$ and the threshold is met or exceeded with a report of $r = g$ independent of the message.

A less informative private signal implies a smaller shift in the outside party's posterior beliefs and, therefore, meeting the threshold requires a good public report. In this case, the solution of $\mathcal{P}3(D)$ is globally optimal, i.e. the threshold is met following a good financial report irrespective of the message sent by the firm. In comparison with the solution to the benchmark pure persuasion game (equivalent to a completely uninformative private signal or $q = 0$), it is optimal for the firm to choose a financial reporting system that generates a somewhat more informative good report. This is accomplished by reducing the probability of a good report in a low state, $\beta_L^P > \beta_L^*$, while holding constant the probability of a good report in a high state $\beta_H^* =$

$\beta_H^P = 1$. Although decreasing β_L implies a more informative good report, it also reduces the frequency of a good report, thereby lessening the unconditional probability of a report that induces a posterior expectation that meets the threshold. The former effect is necessary to allow the firm to meet the threshold with the combination of a good report and non-disclosure of a private signal. Although meeting the threshold with only a good report and a high signal as in $\mathcal{P}4(D)$ would allow the firm to increase the frequency of a good report, the joint unconditional probability of just this combination is lower, implying that the threshold would not be met as often.

As the next proposition establishes, increasing private signal informativeness to the point where Condition 1 is satisfied changes the way that the firm's private information affects its financial reporting system:

Proposition 2: When Condition 1 is satisfied (more informative private signals):

(i) If Condition 2 is not satisfied (pessimistic priors), then the firm's optimal financial reporting system is defined by $1 = \beta_{\mathrm{H}}^{**} > \beta_{\mathrm{L}}^{**} > 0$ and the threshold is met for report $\mathrm{r} = \mathrm{g}$ and message $\mathrm{m} = \mathrm{h}$.

(ii) If Condition 2 is satisfied (optimistic priors), then $1 > \beta_{\mathrm{H}}^{***} > \beta_{\mathrm{L}}^{***} > 0$ and the threshold is met for either report $\mathrm{r} = \mathrm{g}$ or message $\mathrm{m} = \mathrm{h}$.

Recall that Condition 1 is satisfied when private signals are more informative and Condition 2 is satisfied when prior beliefs are optimistic. It is useful to compare the solution in part (i) of Proposition 2 to the solution in Proposition 1 in assessing the effect of satisfying Condition 1. A more informative private signal under program $\mathcal{P}3(D)$ makes it more difficult to meet the threshold with the combination of a good report and non-disclosure of a signal. In other words, this combination implies a lower posterior belief, tightening the constraint on meeting the threshold for that combination due to a more informative low signal. As a consequence, the firm must choose a more informative but less frequent good report, which is accomplished by reducing the probability of a good report in a low state. However, the firm can do better in program $\mathcal{P}4(D)$, where a good report and a more informative high signal imply a higher posterior belief. This combination allows the firm to relax the constraint on meeting the threshold by choosing a less informative but more frequent good report, achievable by increasing the probability of a good report in a low state in comparison to the benchmark pure persuasion game, i.e., $\beta_L^P < \beta_L^{**}$.

	Optimistic beliefs	Pessimistic beliefs
Less informative private signal	Threshold k met IFF $r = g$ Chooses $\boldsymbol{\beta}^*$ such that $1 = \beta_H^* = \beta_H^P$ and $0 < \beta_L^* < \beta_L^P$	
More informative private signal	Threshold k met IFF $r = g$ OR $m = h$ Chooses $\boldsymbol{\beta}^{***}$ such that $1 = \beta_H^P > \beta_H^{***}$ and $0 < \beta_L^{***} < \beta_L^P$	Threshold k met IFF $r = g$ AND $m = h$ Chooses $\boldsymbol{\beta}^{**}$ such that $1 = \beta_H^P = \beta_H^{**}$ and $0 < \beta_L^P < \beta_L^{**}$

Figure 2

Firm's choice of financial reporting system defined by β_H and β_L. Stars indicate optima as described in Propositions 1 (*), 2.i (**) and 2.ii (***); $r \in \{g, b\}$ is the public report; $m \in \{s, \emptyset\}$ is the discretionary message based on the private signal $s \in \{h, l\}$; and β_H (β_L) is the probability of $r = g$ conditional on the state being high (low).

When Conditions 1 and 2 are satisfied, having both optimistic prior beliefs and a more informative private signal makes it possible for the firm to meet the threshold with a combination of a bad report and high private signal. This is achieved by reducing the probability of a good report in a high state such that a bad report no longer implies a low state with certainty. Accordingly, under part (ii), the firm does best in program $\mathcal{P}1(D)$ where the threshold is met by a combination of good report and any message or a bad report and a high message. In this case, the firm also reduces the probability of a good report in a low state in comparison to the benchmark pure persuasion game; i.e., $\beta_H^{***} < \beta_H^P = 1$ and $\beta_L^{***} < \beta_L^P$. While the firm can benefit from having a high message, in order to benefit when a low message is sent, it is crucial to make the bad report less than fully informative, which implies setting $\beta_H < 1$.

Figure 2 summarizes the results of Propositions 1 and 2. As we would anticipate, public financial reports and messages of private information are partial substitutes. Under pessimistic prior beliefs, less (more) informative private signals imply the choice of a more (less) informative financial reporting system i.e., $\beta_L^{**} > \beta_L^*$. The implication of more informative private signals for the informativeness of the financial reporting system in the remaining case of optimistic prior beliefs requires a measure of informativeness that encompasses both parameters

β_H and β_L. For this case, we resort to the variance of conditional expectations (VCE) to show that less (more) informative private signals again imply the choice of a more (less) informative financial reporting system.[13]

Some Welfare Implications

Our analysis focuses on the expected utility of the firm, but there are several settings in which the firm's expected utility is a sufficient statistic for certain broader welfare orderings. In some instances, we might view the outside party as an intermediary who assesses compliance with a threshold based on pre-established rules; e.g., an auditor for whom criteria for an unqualified opinion are set by generally accepted auditing standards. For example, a sufficiently low posterior expectation of firm value could lead to a going concern qualification, which might then trigger a loss. While the auditor may have little, if any, flexibility in applying such rules without incurring penalties for non-compliance, he might prefer that the client achieves the threshold under the assumption that the likelihood of a continuing engagement is advanced by an unqualified opinion. Similarly, a bond rating organization obliged to follow a fixed protocol might stand to benefit from a firm meeting the criteria for a high rating because higher ratings generate greater interest from traders who subscribe to the service. In other instances, the outside party might be less invested in whether a threshold is met. An example here could be an unbiased bank examiner who assesses the adequacy of loan loss reserves. In all of these cases, the firm's expected utility is a sufficient statistic for the welfare of the firm-outside party pair. Missing from these orderings is the welfare of those who rely on assessments made by intermediaries and for whom rules may be set.

Another class of outside parties includes a competitive lender or an investor for whom the threshold takes the form of a required expected return. If the outside party represents a set of *competitive* lenders or investors, then the firm's expected utility would also serve as a sufficient statistic for the joint utility of the firm and the outsider(s). For example, consider an entrepreneur who must raise capital to implement a project that would otherwise be lost at some cost or impose a loss in expected utility on the entrepreneur. Given that the entrepreneur can exploit the competition amongst investors to extract surplus in excess of the outside party's required return,

[13] See the appendix for the proof. Note that VCE and equivalent measures have been used in prior studies as measures of information content (e.g., Friedman et al. 2015).

the welfare ordering reduces to the ordering implied by the expected benefits to the firm as depicted by our propositions.[14] Note that in this example, the entrepreneur derives a further benefit beyond that of meeting the threshold in the form of expected returns in excess of the outside party's required return.

Implied Liberal Bias in Financial Reporting

We relate our results expressed in terms of $\boldsymbol{\beta}$ to a bias proxy, denoted, χ, as in Friedman et al. (2015) through the following transformation of variables:

$$\chi \equiv \frac{1-\beta_L-\beta_H}{2}.$$

A positive value of χ connotes a conservative bias while a negative value connotes a liberal bias. The implied biases corresponding to the solutions in the benchmark pure persuasion case and Propositions 1 and 2 are liberal consistent with the tendency in all cases to increase the frequency of good reports while reducing their informativeness in order to produce the highest joint unconditional probability of meeting the threshold. Only the solution to Proposition 2 (i) includes a liberal bias greater than that in the pure persuasion game; i.e., $\chi^{**} < \chi^P$. This is because with pessimistic prior beliefs, the firm relies on both a good financial report and a more informative high private signal to meet the threshold. The latter allows the firm to further increase the unconditional probability of a good report by more liberally biasing the reporting system than in the pure persuasion game. In the other two cases, discretionary disclosure of private signals leads to less liberal biasing of the financial reporting system; i.e., $\chi^* > \chi^P$ and $\chi^{***} > \chi^P$. Supposing that regulators such as the SEC and FASB may seek on general principles to induce more informative financial reporting, then this is advanced by less liberal (equivalently, more conservative) reporting in the sense of reducing the probability of a good report in a low state; i.e., decreasing β_L.

IV. EXTENSIONS

Are Firms Better Off with Private Information?

Comparing the expected benefit (probability of meeting the threshold multiplied by the

---

[14] There is a subtlety here in that the deadweight costs or loss in utility that the entrepreneur avoids by meeting the threshold dominates the inefficiency implied by overinvestment in comparison to a perfectly informative reporting system.

firm's benefit of 1 when meeting the threshold) corresponding to the solution in Propositions 1, B^*, with that in the pure persuasion game, B^P, we see that the addition of a less informative private signal to financial reports lowers the expected benefit. The firm still expects to do better, though, than it would if it provided a perfectly informative financial reporting system:

> **Corollary 1:** Suppose Condition 1 (less informative private signals) is not satisfied. Then, $B^P > B^* > \alpha$.

The proof is omitted as it follows directly from the comparison of the expected benefits at $\boldsymbol{\beta}^P$ and $\boldsymbol{\beta}^*$.

To explain the driving forces behind this result let

$$\mu \equiv E[\theta|r] \quad \text{and} \quad \Pi(\mu) = E_m[\pi(\mu, m)].$$

As shown in Kamenica and Gentzkow (2011) the expected benefit from the optimal reporting system depends on the concave closure of $\Pi(\mu)$ when the firm might have access to private information, and depends on the concave closure of $\pi(\mu)$ when the firm surely lacks such access. Kamenica and Gentzkow (2011) provide this result in terms of a receiver with beliefs that the sender does not know when designing the reporting system. In our setting, the firm's potential receipt and disclosure of private information causes it to be uncertain of the outsider's message-dependent beliefs at the time when the firm chooses $\boldsymbol{\beta}$. As defined above, $\pi(\mu)$ has a jump of S at the point where the posterior expectation based on the firm's report, $\mu(r)$, equals the threshold, k. $\Pi(\mu)$ has a jump of $S * \Pr(m = h)$ at the point where the posterior expectation based on the firm's report, $\mu(r)$, combined with a high message, $m = h$, equals k, and a further jump of $S * \Pr(m = \emptyset)$ at the point where the posterior, $\mu(r)$, combined with a null message, $m = \emptyset$, equals k.[15] The total vertical distance of the two jumps in $\Pi(\mu)$ is equal to the vertical jump in $\pi(\mu)$, since the first step is $S \cdot \Pr(m = h)$, the second step is $S \cdot \Pr(m = \emptyset)$, and $\Pr(m = \emptyset) + \Pr(m = h) = 1$.

[15] Both $\Pi(\mu)$ and $\pi(\mu)$ include a linear component as well, but it is irrelevant for this discussion.

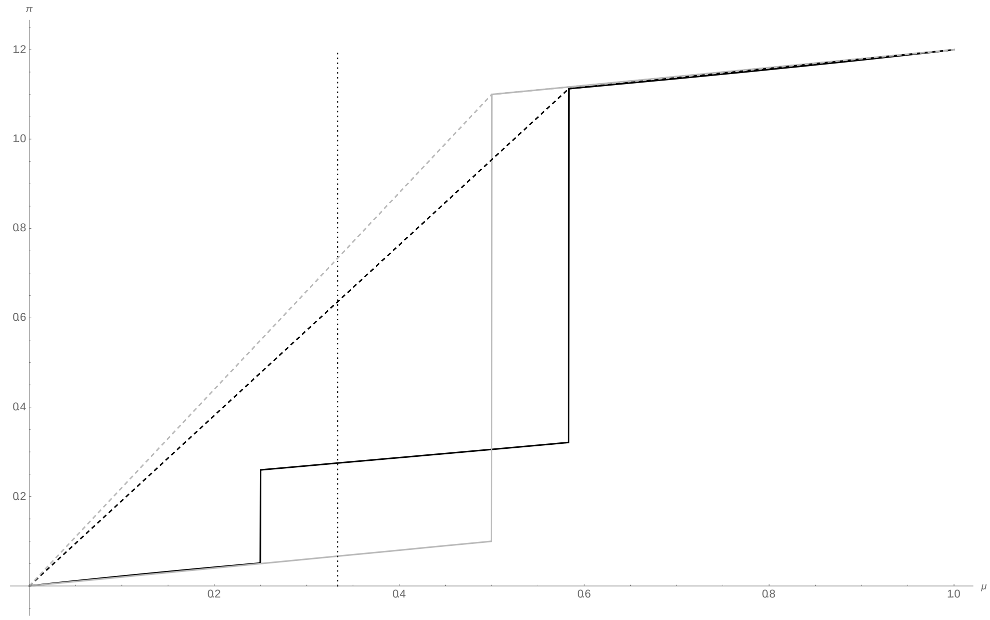

Figure 3

Comparison of expected benefits in the pure persuasion benchmark (gray) and when the firm may have private information (black) when Condition 1 is not satisfied (Corollary 1). Solid lines represent firm benefits. Dashed lines are concave closures, and the vertical dotted line marks $\mu = \alpha$. Parameters are set as $S = 1,\ \sigma = \frac{1}{5},\ k = \frac{1}{2},\ \alpha = \frac{1}{3},\ \gamma_L = \frac{1}{4},\ \gamma_H = \frac{3}{4}$, and $q = \frac{1}{2}$.

Similar to Kamenica and Gentzkow (2011) the maximum expected payoff, which is achieved with the optimal reporting system, is the concave closure of $\Pi(\mu)$ or $\pi(\mu)$ (depending on whether the firm might have private information) evaluated at the prior belief α. That is, our primary concern from an expected benefit standpoint is the value of the concave closure of the payoff function, evaluated at α. As illustrated by the numerical example in Figure 3, the concave closure of $\pi(\mu)$ is above the concave closure of $\Pi(\mu)$ evaluated at the prior belief, α, when Condition 1 is not satisfied. This implies that the firm's expected payoff is always lower when it has potential access to private information, compared to the case when it is known to be uninformed.

Similar to the ordering of expected benefits in Corollary 1 for the case described in Proposition 1, expected benefits in both cases considered in Proposition 2 are lower than in the benchmark pure persuasion game:

Corollary 2: Suppose Condition 1 is satisfied (more informative private signals).

(i) If Condition 2 is not satisfied (pessimistic priors), then $B^P > B^{**} > \alpha$.

(ii) If Condition 2 is satisfied (optimistic priors), then $B^P > B^{***} > \alpha$.

The proofs are omitted and their intuition follows a similar logic to that of Corollary 1.

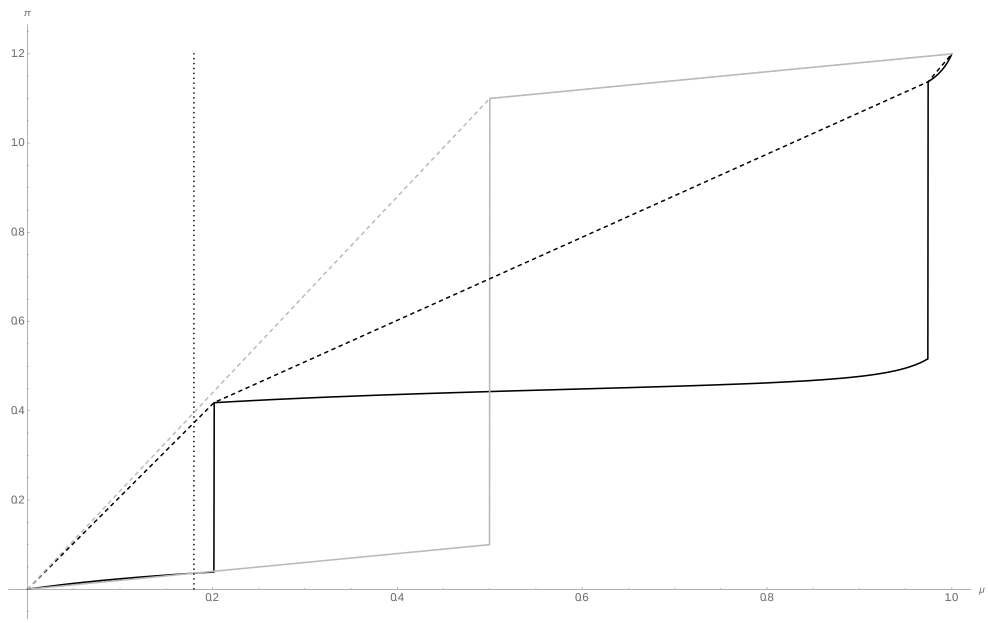

Figure 4

Comparison of expected benefits in the pure persuasion benchmark (gray) and when the firm may have private information (black) when Condition 1 is satisfied but Condition 2 is not (Corollary 2(i)). Solid lines represent firm benefits. Dashed lines are concave closures, and the vertical dotted line marks $\mu = \alpha$. Parameters are set as $S = 1$, $\sigma = \frac{1}{5}$, $k = \frac{1}{2}$, $\alpha = \frac{18}{100}$, $\gamma_L = \frac{1}{4}$, $\gamma_H = \frac{99}{100}$, and $q = \frac{99}{100}$.

Numerical examples in Figures 4 and 5 illustrate parts (i) and (ii) of Corollary 2. It is evident from Corollaries 1 and 2 that, in the context of our model, discretionary disclosure of private information does not enhance the firm's ability to meet the threshold over what the firm could achieve with the financial reporting system alone, absent potential receipt of private information.

These results further imply that if the firm had control over the private information, it would choose never to receive such information (i.e., set $q = 0$) or choose a completely uninformative private information system so that the outsiders ignore the message (i.e., set $\gamma_H = \gamma_L$). However, an ability to forestall the receipt of private information would seem to be impossible given all of the ways in which information may arrive. It would appear to be similarly impossible to design commitments not to disclose information when there are potential benefits from influencing outsiders' beliefs.

An important feature underlying Corollaries 1 and 2 is that the firm has unrestricted control over the properties of the reporting system. Through these properties, the firm essentially chooses the distribution of posterior beliefs over the states of nature induced by the report, constrained only by the fact that the Bayesian outsider's expected beliefs must equal his prior beliefs. In the benchmark case, the firm has complete control over this distribution. In contrast, when the firm has access to private information, it loses a degree of control because, for any report that it sends, multiple beliefs might be induced over which we take expectations. Intuitively, it is possible for the firm in the benchmark case to choose signal properties that would induce the same distribution of posterior beliefs as would be induced by the combination of the reporting system and the potentially-disclosed message. However, since $\boldsymbol{\beta^P}$ is unique, adding private information and discretionary disclosure that does not replicate the distribution in the benchmark case reduces the firm's expected payoff.

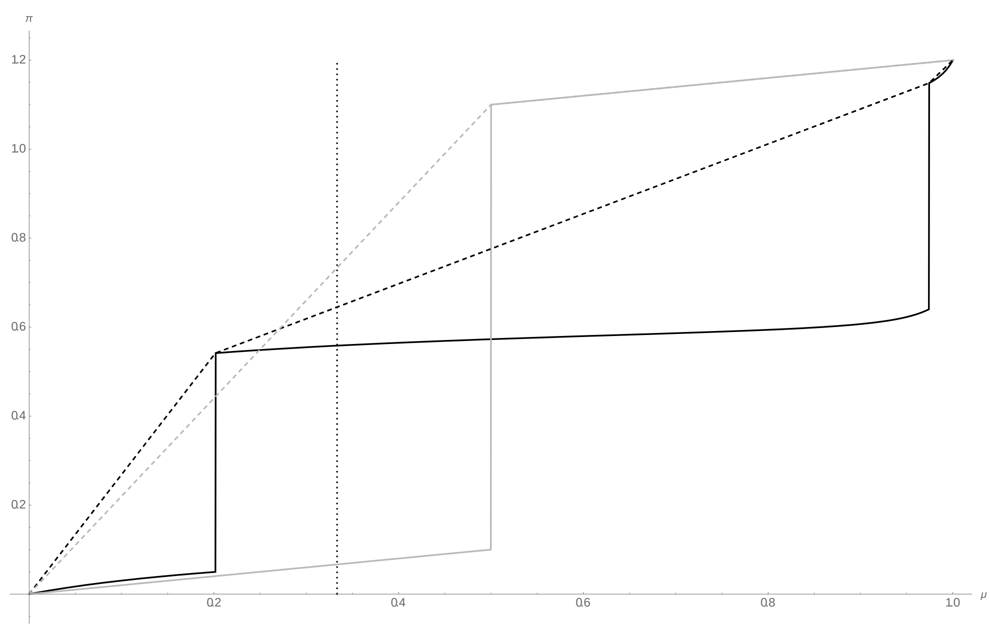

Figure 5

Comparison of expected benefits in the pure persuasion benchmark (gray) and when the firm may have private information (black) when Conditions 1 and 2 are satisfied (Corollary 2(ii)). Solid lines represent firm benefits. Dashed lines are concave closures, and the vertical dotted line marks $\mu = \alpha$. Parameters are set as $S = 1$, $\sigma = \frac{1}{5}$, $k = \frac{1}{2}$, $\alpha = \frac{1}{3}$, $\gamma_L = \frac{1}{4}$, $\gamma_H = \frac{99}{100}$, and $q = \frac{99}{100}$.

Non-disclosure Equilibrium

In the preceding section, we showed that in the absence of control over the arrival of private information the firm is stuck in a less desirable equilibrium. In this subsection, we discuss the implications of assuming no further benefit to reporting beyond meeting the threshold (i.e., $\sigma = 0$). Interestingly, when $\sigma = 0$, there exists an equilibrium in which the firm chooses never to disclose private information. When $\sigma = 0$ the threshold decision rule of our model leads to regions of indifference with respect to the disclosure of a high signal.

Suppose that the outside party believed the firm would never disclose a private signal. Now assume that the firm receives a high signal. Would the firm disclose? If $\sigma = 0$, then the firm might have no benefit to disclosing. In order to sustain the outside party's belief that it would not disclose a high signal if received, the firm chooses the same financial reporting system as in the pure persuasion game, $\boldsymbol{\beta^P}$. Following a good report, the outsider party's beliefs are high enough to just meet the threshold, i.e., $E[\theta|r = g] = k$, and disclosing the high signal would induce a posterior expectation by the outside party in excess of the threshold which provides no explicit further benefit to the firm. For a bad report, disclosing a high signal is moot since a bad report implies a low state with certainty, i.e., $E[\theta|r = b, m = g] = E[\theta|r = b, m = \emptyset] = 0$. Since *ex ante* the solution to the pure persuasion game at least weakly dominates the solution under discretionary disclosure, the firm has no incentive to defect at either stage.

A natural question is whether these equilibria can be ordered from the firm's point of view. As we showed, in each of the cases represented in Propositions 1 and 2, the expected benefit to the firm is greater in the benchmark pure persuasion game than in any case with discretionary disclosure of informative private signals. It follows that, in the absence of an additional marginal benefit from disclosure of a high signal *per se* (i.e., $\sigma = 0$), a non-disclosure equilibrium exists and dominates from the firm's point of view. While we find this result interesting, the assumption of no further benefit to disclosure of a high signal is not descriptive of situations that firms actually face given the many other roles that have been ascribed to discretionary disclosure. Any marginal benefit to disclosure of a high private signal beyond that of meeting a crucial threshold, in and of itself, no matter how negligible, suffices to eliminate this equilibrium, notwithstanding that the firm may be better off with no disclosure. Of course, if a more informative reporting system involves a (higher) cost, then this could be sufficient to offset an added benefit beyond meeting the threshold thereby recovering a non-disclosure equilibrium.

The Value of Discretion to Disclose Private Information

We next consider whether the option to disclose or not disclose is beneficial to the firm. To do so, we solve for the optimal financial reporting system design through a series of programs similar to those in the previous section except that both low and high signals are disclosed. We refer to this as a *mandatory* disclosure setting, as the firm is assumed to disclose its private information. While we abstract from costs of mandatory disclosure, our results in this setting can be interepreted as informative about when a firm would seek additional certification of whether it posseses potentially private information and the nature of any such information. The set of programs under mandatory disclosure are as follows, with solutions and programs dentoed by "M":

$$\mathcal{P}1(M): \quad \max_{\boldsymbol{\beta}} \Pr(r = g, m = h)$$
$$s.t. \quad E[\theta | r = g, m = h] \geq k, \; 1 \geq \beta_H \geq \beta_L \geq 0.$$

$$\mathcal{P}2(M): \quad \max_{\boldsymbol{\beta}} \Pr(r = g, m = h) + \Pr(r = g, m = \emptyset)$$
$$s.t. \quad E[\theta | r = g, m = h] \geq k, \; E[\theta | r = g, m = \emptyset] \geq k,$$
$$1 \geq \beta_H \geq \beta_L \geq 0;$$

$$\mathcal{P}3(M): \quad \max_{\boldsymbol{\beta}} \Pr(r = g, m = h) + \Pr(r = b, m = h)$$
$$s.t. \quad E[\theta | r = g, m = h] \geq k, \; E[\theta | r = b, m = h] \geq k,$$
$$1 \geq \beta_H \geq \beta_L \geq 0;$$

$$\mathcal{P}4(M): \quad \max_{\boldsymbol{\beta}} \Pr(r = g, m = h) + \Pr(r = b, m = h) + \Pr(r = g, m = \emptyset)$$
$$s.t. \quad E[\theta | r = g, m = h] \geq k, E[\theta | r = b, m = h] \geq k,$$
$$E[\theta | r = g, m = \emptyset] \geq k, \; 1 \geq \beta_H \geq \beta_L \geq 0;$$

$$\mathcal{P}5(M): \quad \max_{\boldsymbol{\beta}} \Pr(r = g, m = h) + \Pr(r = g, m = \emptyset) + \Pr(r = g, m = l)$$

$$s.t. \quad E[\theta | r = g, m = h] \geq k,\ E[\theta | r = g, m = \emptyset] \geq k,$$

$$E[\theta | r = g, m = l] \geq k, 1 \geq \beta_H \geq \beta_L \geq 0$$

$$\mathcal{P}6(M): \quad \max_{\boldsymbol{\beta}} \Pr(r = g, m = h) + \Pr(r = g, m = \emptyset)$$

$$+\Pr(r = g, m = l) + \Pr(r = b, m = h)$$

$$s.t. \quad E[\theta | r = g, m = h] \geq k,\ E[\theta | r = g, m = \emptyset] \geq k,$$

$$E[\theta | r = g, m = l] \geq k,\ E[\theta | r = b, m = h] \geq k,$$

$$1 \geq \beta_H \geq \beta_L \geq 0.$$

Proposition 3: Suppose Condition 1 is not satisfied (less informative private signals), with γ_H sufficiently low, i.e., $\gamma_H < \min\{\overline{g}, g_o, g_{oo}\}$. Then, the firm strictly prefers discretion over mandatory disclosure.

The applicable discretionary disclosure case for this parameterization is depicted in Proposition 1. Under mandatory disclosure, a low private signal is no longer pooled with non-receipt of a signal. As a consequence, a good report need not be as informative as under discretionary disclosure in order for the posterior beliefs following a combination of a good report and non-disclosure to meet the threshold. However, a good report that is only sufficiently informative to meet the threshold for that combination, when combined with a low signal, will not meet the threshold. If under mandatory disclosure the firm sought to meet the threshold for both non-receipt of a signal and a low signal, then the effect of having to compensate for a low signal in choosing a reporting system implies a worse solution than for $\mathcal{P}3(D)$. In the proof, we show that the solution to $\mathcal{P}3(D)$ exceeds to solution to all programs $\mathcal{P}1(M) - \mathcal{P}6(M)$. Hence, discretion in this case is valuable to the firm.

Proposition 4: Suppose Condition 1 is satisfied (more informative private signals), with $\gamma_H > \max\{\overline{g}, g^o, g^{oo}\}$. Then,

(i) if Condition 2 is not satisfied (pessimistic priors), then the firm is indifferent between discretion and mandatory disclosure.

(ii) if Condition 2 is satisfied (optimistic priors), then the firm strictly prefers mandatory disclosure to discretion.

The applicable discretionary disclosure cases for these parameterizations are $\mathcal{P}4(D)$ and $\mathcal{P}1(D)$, respectively. In part (i), we show that $\mathcal{P}1(M)$ is globally optimal. Since it corresponds to $\mathcal{P}4(D)$ by considering only the combination of a good report and high signal in meeting the threshold, then the solutions are identical implying indifference by the firm between discretionary and mandatory disclosure. As for part (ii), we show that $\mathcal{P}4(M)$ brings a higher expected benefit to the firm than the globally optimal program with discretion, $\mathcal{P}1(D)$. While they both consider combinations of a good report and high signal, bad report and high signal, and high report and non-disclosure, the former does not pool non-receipt of a signal with a low signal, thereby making it possible to meet the threshold with a less informative but more frequent good report. Hence, the firm strictly prefers mandatory disclosure to discretion. Figure 6 depicts the regions in which the firm prefers either of the two regimes.

Less informative private signal	The firm strictly prefers discretion	
More informative private signal	The firm strictly prefers commitment	The firm is indifferent
	Optimistic prior beliefs	Pessimistic prior beliefs

Figure 6
Firm's preference over regimes

Although interesting as a benchmark in appreciating the consequences of discretion, mandatory disclosure may not be a realistic option given that the costs of monitoring compliance and enforcing penalties for non-compliance when a firm's receipt of information is uncertain are likely to be prohibitively high. Our results do suggest, though, that firms with optimistic priors who expect to receive informative private signals might be willing to pay for certification services that provide a mechanism for *ex ante* commitment to disclosure of potentially private information.

V. CONCLUSION

We consider the effects of discretionary disclosure of private information on financial reporting system design choices. Our model is an extension of Bayesian persuasion games in which a sender makes an *ex ante* choice of a reporting system, with the objective of maximizing the expectation of meeting a posterior beliefs threshold set by a receiver and upon which the sender's welfare depends. The sender in the context of our model is a firm and the receiver is an outside party such as an auditor, credit rater, lender, investor, or certifying agency, whose beliefs influence the firm's payoffs through, for example, audit opinions, debt ratings, debt key covenant waivers or renegotiations, price-setting, or any of myriad certification requirements. While a perfectly informative reporting system is assumed to be feasible at no cost, the firm can do better with a less informative system that enhances the firm's odds of generating posterior beliefs that just meet the threshold. The firm's optimal design in such a setting can be viewed as a liberal or aggressive set of accounting policies. Although our model is highly stylized to focus on but one tension the firm faces in choosing the properties of its reporting system, we believe that meeting crucial thresholds could be an overriding concern for some firms during time spans long enough to influence financial accounting policy decisions. The flexibility afforded firms by accounting standards in choosing accounting policies constitutes a natural device for firms to employ in seeking to meet thresholds or otherwise influence the beliefs held by financial statement users.

The prospect of receiving private information, over which disclosure by the firm is discretionary, induces the firm to change the properties of its financial reporting system. When private signals are less informative, the firm directs its financial reporting system toward providing more informative favorable reports. This is because such reports have to raise the posterior beliefs sufficiently to offset the negative influence of the potential non-disclosure of a private signal, given that such non-disclosure may be due to an unfavorable private signal or no private signal having been received. When private signals are more informative and prior beliefs are pessimistic, then the firm would choose less informative favorable reports, anticipating that disclosure of a sufficiently favorable private signal would compensate for the effect of an unfavorable report on posterior expectations. The firm's finacial reporting system choices in the remaining case of more informative private signals and optimistic prior beliefs are more complex involving both less informative favorable reports and more informative unfavorable reports. Constructively, financial reports and private signals are partial substitutes. Less informative private signals in general imply a choice of more informative financial reporting systems.

Broadly speaking, in settings where outside parties employ only a threshold decision rule, discretionary disclosure of private information provides no benefit to the firm beyond that achievable through a judicious choice of a public financial reporting system. While likely to be less descriptive of situations that firms may face, absent a marginal further benefit to disclosing favorable private signals *per se*, an alternative equilibrium exists in which the firm does not disclose even those signals which would increase the likelihood of meeting or exceeding the threshold. Comparing regimes with discretionary and mandatory disclosure of private information, there are conditions under which the firm may prefer one or the other. In particular, a combination of optimistic prior beliefs and highly informative private signals implies a preference for mandatory disclosure. Although useful as a theoretical benchmark, implementing mandatory disclosure would require monitoring of the receipt of private information and penalties for non-compliance, which may be infeasible or, at best, very costly.

While we have focused on the application of our model to financial reporting by firms, the structure we employ may also be suitable for characterizing reporting choices for intermediaries that gather information for distribution to other parties. Financial analysts may curry favor with firms and brokerage firms by seeking to primarily acquire good news in arriving at their estimates and ratings provided to investors that may or may not be reinforced by information subsequently received as they update. Information gathering is one of the tasks that sell-side analysts perform (Michaely and Womack 2005). To the extent that this task may be biased is consistent with the *ex ante* concept of information system choices in our model. Bond rating agencies have likewise been thought to limit the extent to which they search for news that might result in lower ratings or downgrades. Jiang, Stanford and Xie (2012) find evidence of upwardly biased ratings for issuer-pay firms. Whether bias manifests in information gathering or later in the process is an open question. Extending the jurisprudence context of Kamenica and Gentzkow (2011), a prosecutor who slants information gathered for a fair-minded judge may subsequently be faced with a decision regarding whether to suppress or reveal information subsequently obtained. More broadly speaking, the advocacy nature of the legal system suggests a high likelihood of misaligned preferences, and rules of discovery and penalties for evidence tampering suggest incentives for biases to enter at the information gathering stage. While withholding subsequently obtained evidence is unlawful, recent high-profile cases suggest it still occurs (Patrice 2015; Simmerman 2012).

Giving some thought to empirical applications, we note that in 2005 the S.E.C. liberalized its “quiet period” policies to allow more information to be communicated for certain organizations following the filing of a registration statement. For IPOs this period is often referred to as a “cooling-off period.” In the context of our study, such a period, if enforced, may serve as a commitment device that benefits the firm, notwithstanding that its effect may be to diminish the informativeness of prospectuses. Relaxing these policies may have the opposite effect suggesting a natural experiment to test our predictions may be feasible. There is some prospect that these policies may be further liberalized or even eliminated given the commonly held view echoed by Fortune magazine’s 2011 feature article, “It’s time to kill the IPO quiet period.” Given that the ability and motivation to meet a crucial threshold may only be present and substantial for some firms, there is scope for cross-sectional differences that could contribute to the power of one’s tests.

APPENDIX

Proof of Lemma 1: The firm discloses $s = h$ whenever:

$$\begin{aligned}\Delta_h \equiv\, & E[\theta|r,m=h] - E[\theta|r,m=\emptyset] \\ &= \Pr(\theta=H|r,m=h) - \Pr(\theta=H|r,m=\emptyset) \\ &\geq 0, \quad \forall r = g,b.\end{aligned}$$

It is straightforward to verify that, because $\gamma_H > \gamma_L$ by assumption, $\Pr(\theta=H|r,m=h) \geq \Pr(\theta=H|r,m=\emptyset)$, $\forall r = g,b$. Hence, the firm discloses $s = h$. Next, we show that the firm withholds $s = l$:

$$\begin{aligned}\Delta_l \equiv\, & E[\theta|r,m=l] - E[\theta|r,m=\emptyset] \\ &= \Pr(\theta=H|r,m=l) - \Pr(\theta=H|r,m=\emptyset) \\ &\leq 0, \quad \forall r = g,b.\end{aligned}$$

It is straightforward to verify that, because $\gamma_H > \gamma_L$ by assumption, $\Pr(\theta=H|r,m=l) \leq \Pr(\theta=H|r,m=\emptyset)$, $\forall r = g,b$. Hence, the firm withholds $s = l$.

Proof of Proposition 1: $\mathcal{P}1(D)$ can be rewritten as:

$$\begin{aligned}\max_{\beta_H,\beta_L}\ & \alpha\beta_H + (1-\alpha)\beta_L + (1-p)(\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L) \\ s.t.\quad & E[\theta|r=g,m=h] \geq k, \\ & E[\theta|r=g,m=\emptyset] \geq k, \\ & E[\theta|r=b,m=h] \geq k, \\ & 1 \geq \beta_H \geq \beta_L \geq 0.\end{aligned}$$

The first condition is slack whenever either the second, or the third condition are satisfied (because $E[\theta|r=g,m=h] \geq E[\theta|r=g,m=\emptyset]$ and $E[\theta|r=g,m=h] \geq E[\theta|r=b,m=h]$). Therefore, the Lagrangian is

$$\begin{aligned}\mathcal{L}_1 = & \alpha\beta_H + (1-\alpha)\beta_L + q(\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L) \\ & +\mu_1(E[\theta|r=g,m=\emptyset] - k) + \mu_2(E[\theta|r=b,m=h] - k) \\ & +\mu_3(1-\beta_H) + \mu_4(\beta_H - \beta_L) + \mu_5\beta_L.\end{aligned}$$

The Karush-Kuhn-Tucker stationarity conditions are:

$$\begin{aligned}\frac{\partial\mathcal{L}_1}{\partial\beta_H} = & \alpha(1-q\gamma_H) \\ & +\mu_1\frac{\partial E[\theta|r=g,m=\emptyset]}{\partial\beta_H} + \mu_2\frac{\partial E[\theta|r=b,m=h]}{\partial\beta_H} \\ & -\mu_3 + \mu_4 = 0 \end{aligned} \tag{1}$$

$$\begin{aligned}\frac{\partial\mathcal{L}_1}{\partial\beta_L} = & (1-\alpha)(1-q\gamma_L) \\ & +\mu_1\frac{\partial E[\theta|r=g,m=\emptyset]}{\partial\beta_L} + \mu_2\frac{\partial E[\theta|r=b,m=h]}{\partial\beta_L} \\ & -\mu_4 + \mu_5 = 0,\end{aligned} \tag{2}$$

the Karush-Kuhn-Tucker feasibility conditions are:

$$E[\theta|r=g,m=\emptyset] - k \geq 0 \tag{3}$$

$$E[\theta|r=b,m=h] - k \geq 0 \tag{4}$$

$$1 - \beta_H \geq 0 \tag{5}$$

$$\beta_H - \beta_L \geq 0, \tag{6}$$

$$\beta_L \geq 0 \tag{7}$$

$$\mu_i \geq 0, \quad i = 1,2,3,4,5. \tag{8}$$

and the Karush-Kuhn-Tucker complementarity slackness conditions are:

$$\mu_1(E[\theta|r=g,m=\emptyset] - k) = 0 \tag{9}$$

$$\mu_2(E[\theta|r=b,m=h]-k)=0 \tag{10}$$
$$\mu_3(1-\beta_H)=0 \tag{11}$$
$$\mu_4(\beta_H-\beta_L)=0 \tag{12}$$
$$\mu_5\beta_L=0. \tag{13}$$

With five complementarity slackness conditions there are $2^5=32$ cases. We can immediately rule out:

- All cases with $\mu_3>0$ (so $\beta_H=1$) because if $\beta_H=1$, then $E[\theta|r=b,m=\emptyset]=0<k$ which is a contradiction;
- All cases with $\mu_1>0$, $\mu_5>0$ (so $E[\theta|r=g,m=\emptyset]=k$, $\beta_L=0$) because if $\beta_L=0$, then $E[\theta|r=g,m=\emptyset]=1>k$ which is a contradiction;
- All cases with $\mu_4>0$ ($\beta_H=\beta_L$), because if $\beta_H=\beta_L$, then $E[\theta|r=g,m=\emptyset]=E[\theta|r=b,m=\emptyset]<k$ which is a contradiction.
- All cases with $\mu_2=0$, $\mu_3=0$ (so $E[\theta|r=b,m=h]>k$ and $1>\beta_H$), because then $\frac{\partial\mathcal{L}_1}{\partial\beta_H}=\alpha(1-q\gamma_H)+\mu_1\frac{\partial E[\theta|r=g,m=\emptyset]}{\partial\beta_H}+\mu_4>\mu_1\frac{\partial E[\theta|r=g,m=\emptyset]}{\partial\beta_H}\geq 0$, implying $\beta_H=1$ which is a contradiction;
- All cases with $\mu_1=0$, $\mu_4=0$ (so $E[\theta|r=g,m=\emptyset]>k$ and $\beta_H>\beta_L$), because then $\frac{\partial\mathcal{L}_1}{\partial\beta_L}=(1-\alpha)(1-q\gamma_L)+\mu_2\frac{\partial E[\theta|r=b,m=h]}{\partial\beta_L}+\mu_5>\mu_2\frac{\partial E[\theta|r=b,m=h]}{\partial\beta_L}\geq 0$, implying $\beta_L=1\geq\beta_H$ which is a contradiction;

We are left with only one case to consider:

- $\mu_1>0$, $\mu_2>0$, $\mu_3=0$, $\mu_4=0$, $\mu_5=0$, $E[\theta|r=g,m=\emptyset]=k$, $E[\theta|r=b,m=h]=k$, $1>\beta_H>\beta_L>0$

We solve the four equations below

$$E[\theta|r=g,m=\emptyset]-k=0$$
$$E[\theta|r=b,m=h]-k=0$$
$$\alpha(1-q\gamma_H)+\mu_1\frac{\partial E[\theta|r=g,m=\emptyset]}{\partial\beta_H}+\mu_2\frac{\partial E[\theta|r=b,m=h]}{\partial\beta_H}=0$$
$$(1-\alpha)(1-q\gamma_L)+\mu_1\frac{\partial E[\theta|r=g,m=\emptyset]}{\partial\beta_L}+\mu_2\frac{\partial E[\theta|r=b,m=h]}{\partial\beta_L}=0$$

and get

$$\beta_H=\frac{(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha))(1-q\gamma_L)}{\alpha(\gamma_H-\gamma_L)(1-k)}$$
$$\beta_L=\frac{(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha))(1-q\gamma_H)}{k(\gamma_H-\gamma_L)(1-\alpha)}$$
$$\mu_1=\frac{\Gamma(\gamma_H(1-k)\alpha-\gamma_L k(1-\alpha))(\gamma_L k+\gamma_H(1-k-\gamma_L q))}{(\gamma_H-\gamma_L)^2(1-k)^2k^2}$$
$$\mu_2=\frac{\Gamma\gamma_H\gamma_L(k-\alpha(1-q(\gamma_H k+\gamma_L(1-k))-\gamma_L kq))}{(\gamma_H-\gamma_L)^2(1-k)^2k^2}$$

where $\Gamma\equiv(1-\gamma_H q)(1-\gamma_L q)$. It is straightforward to verify that this case is feasible in a sense that $1>\beta_H>\beta_L>0$ and $\mu_1>0$, $\mu_2>0$ whenever $\alpha>\frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$. For future reference,

- if $\alpha>\frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$, then

$$\begin{aligned}1&>\frac{(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha))(1-q\gamma_L)}{\alpha(\gamma_H-\gamma_L)(1-k)}\\&=\beta_H\\&>\beta_L\end{aligned}$$

$$= \frac{(\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha))(1-q\gamma_H)}{k(\gamma_H - \gamma_L)(1-\alpha)}$$
$$> 0$$

and the value of $\mathcal{P}1(D)$ is:

$$\frac{M_A \cdot M_B + M_C}{k(1-k)(\gamma_H - \gamma_L)};$$

where $M_A \equiv \alpha(1-(\gamma_H(1-k)+\gamma_L k)q)$, $M_B \equiv -(\gamma_L k + \gamma_H(1-k-\gamma_L q))$ and $M_C \equiv \gamma_L k(1-q(\gamma_L k + \gamma_H(2-k-\gamma_L q)))$.

- if $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$, then the value of $\mathcal{P}1(D)$ is zero.

$\mathcal{P}2(D)$ can be rewritten as:

$$\max_{\beta_H, \beta_L} q(\alpha\gamma_H + (1-\alpha)\gamma_L)$$
$$\text{s.t. } E[\theta | r = g, m = h] \geq k,$$
$$E[\theta | r = b, m = h] \geq k,$$
$$1 \geq \beta_H \geq \beta_L \geq 0.$$

The maximand is independent of β_H and β_L so we just need to ensure that the conditions are satisfied. The first condition is slack if the second condition holds (because $E[\theta | r = g, m = h] > E[\theta | r = b, m = h]$) so we only need to verify that the second and third condition are feasible. Substituting for $E[\theta | r = b, m = h]$ in the second condition and rearranging we get

$$\frac{1-\beta_H}{1-\beta_L} \geq \frac{\gamma_L}{\gamma_H} \frac{(1-\alpha)}{(1-k)} \frac{k}{\alpha}; \tag{14}$$
$$1 \geq \beta_H \geq \beta_L \geq 0. \tag{15}$$

We note that if (15) is satisfied, then $\frac{1-\beta_H}{1-\beta_L} \in [0,1]$. Therefore:

- if the RHS of (14), is bigger than one, i.e., when

$$\frac{\gamma_L}{\gamma_H} \frac{(1-\alpha)}{(1-k)} \frac{k}{\alpha} \geq 1 \quad \Leftrightarrow \quad \alpha \leq \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$$

then (14) cannot be satisfied for any β_H and β_L that satisfy (15).

- if the RHS of (14), is smaller than one, i.e., when

$$\frac{\gamma_L}{\gamma_H} \frac{(1-\alpha)}{(1-k)} \frac{k}{\alpha} < 1 \quad \Leftrightarrow \quad \alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$$

then the firm sets β_H and β_L that satisfy (14) and (15) simultaneously.

For future reference,

- if $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$, then the value of $\mathcal{P}2(D)$ is $q(\alpha\gamma_H + (1-\alpha)\gamma_L)$
- if $\alpha \leq \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$ then the value of $\mathcal{P}2(D)$ is zero.

$\mathcal{P}3(D)$ can be rewritten as:

$$\max_{\beta_H, \beta_L} \alpha\beta_H + (1-\alpha)\beta_L$$
$$s.t. \quad E[\theta | r = g, m = h] \geq k,$$
$$E[\theta | r = g, m = \emptyset] \geq k,$$
$$1 \geq \beta_H \geq \beta_L \geq 0.$$

Setting the second condition binding ensures that the first condition is satisfied (because $E[\theta | r = g, m = h] \geq E[\theta | r = g, m = \emptyset]$) and allows us to express β_L:

$$E[\theta | r = g, m = \emptyset] = k \quad \Rightarrow \quad \beta_L = \beta_H \frac{\alpha(1-k)(1-\gamma_H q)}{k(1-\alpha)(1-\gamma_L q)}$$

Substituting and simplifying, we can rewrite the optimization program as:

$$\max_{\beta_H,\beta_L} \alpha\beta_H\left(1+\frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)}\right)$$
$$s.t. \quad 1 \geq \beta_H \geq \beta_L \geq 0$$

Taking derivative with respect to β_H yields

$$\alpha\left(1+\frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)}\right) > 0$$

and therefore $\beta_H = 1$ and $\beta_L = \frac{\alpha(1-k)(1-\gamma_H q)}{k(1-\alpha)(1-\gamma_L q)}$ (note that $1 \geq \beta_H \geq \beta_L \geq 0$ is satisfied because $0 < \alpha < k < 1$ and $0 \leq \gamma_L \leq \gamma_H \leq 1$ by assumption). For future reference, the value of $\mathcal{P}3(D)$ is $\alpha\left(1+\frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)}\right)$.

$\mathcal{P}4(D)$ can be rewritten as:

$$\max_{\beta_H,\beta_L} q(\alpha\beta_H\gamma_H + (1-\alpha)\beta_L\gamma_L)$$
$$s.t. \quad E[\theta|r=g,m=h] \geq k,$$
$$1 \geq \beta_H \geq \beta_L \geq 0.$$

The Lagrangian is:

$$\mathcal{L}_4 = q(\alpha\beta_H\gamma_H + (1-\alpha)\beta_L\gamma_L)$$
$$+\mu_1(E[\theta|r=g,m=h]-k) + \mu_2(1-\beta_H) + \mu_3(\beta_H-\beta_L) + \mu_4\beta_L.$$

The Karush-Kuhn-Tucker stationarity conditions are

$$\frac{\partial\mathcal{L}_4}{\partial\beta_H} = q\alpha\gamma_H + \mu_1\frac{\partial E[\theta|r=g,m=h]}{\partial\beta_H} - \mu_2 + \mu_3 = 0 \quad (16)$$
$$\frac{\partial\mathcal{L}_4}{\partial\beta_L} = q(1-\alpha)\gamma_L + \mu_1\frac{\partial E[\theta|r=g,m=h]}{\partial\beta_L} - \mu_3 + \mu_4 = 0, \quad (17)$$

the Karush-Kuhn-Tucker feasibility conditions are:

$$E[\theta|r=g,m=h]-k \geq 0 \quad (18)$$
$$1-\beta_H \geq 0 \quad (19)$$
$$\beta_H-\beta_L \geq 0 \quad (20)$$
$$\beta_L \geq 0, \quad (21)$$
$$\mu_i \geq 0, \quad i=1,2,3,4. \quad (22)$$

and the Karush-Kuhn-Tucker complementarity slackness conditions are:

$$\mu_1(E[\theta|r=g,m=h]-k) = 0 \quad (23)$$
$$\mu_2(1-\beta_H) = 0 \quad (24)$$
$$\mu_3(\beta_H-\beta_L) = 0 \quad (25)$$
$$\mu_4\beta_L = 0. \quad (26)$$

With four complementarity slackness conditions there are $2^4 = 16$ cases. We can immediately rule out:

- All cases with $\mu_3 > 0$, $\mu_4 > 0$ (so $\beta_H = \beta_L = 0$), because then $E[\theta|r=g,m=h] = 0 < k$, which is a contradiction;
- All cases with $\mu_1 > 0$, $\mu_4 > 0$ (so $E[\theta|r=g,m=h] = k$, $\beta_L = 0$) because if $\beta_L = 0$, then $E[\theta|r=g,m=h] = 1 > k$ which is a contradiction;
- All cases with $\mu_2 = 0$ (so $1 > \beta_H$), because then $\frac{\partial\mathcal{L}_4}{\partial\beta_H} = q\alpha\gamma_H + \mu_1\frac{\partial E[\theta|r=g,m=h]}{\partial\beta_H} + \mu_3 \geq \mu_1\frac{\partial E[\theta|r=g,m=h]}{\partial\beta_H} \geq 0$, implying $\beta_H = 1$ which is a contradiction;
- All cases with $\mu_1 = 0$ and $\mu_3 = 0$ (so $E[\theta|r=g,m=h] > k$ and $1 \geq \beta_H > \beta_L$) because $\frac{\partial\mathcal{L}_4}{\partial\beta_L} = q(1-\alpha)\gamma_L + \mu_4 \geq 0$, implying $\beta_L = 1$ which is a contradiction;

We are left with only three cases to consider:

- $\mu_1 > 0,\ \mu_2 > 0,\ \mu_3 = 0,\ \mu_4 = 0,\ E[\theta|r = g, m = h] = k,\ 1 = \beta_H > \beta_L > 0$

$$E[\theta|r = g, m = h] = k \quad \Rightarrow \quad \beta_L = \beta_H \frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L}$$

Substituting $\beta_H = 1$ and $\beta_L = \frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L}$ into (17) and solving yields $\mu_1 = \frac{\alpha\gamma_H q}{k^2} > 0$. Substituting β_H, β_L and μ_1 into (17) yields $\mu_2 = \frac{\alpha\gamma_H q}{k} > 0$. This case is feasible only if $\beta_L = \frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L} < \beta_H = 1$, which is equivalent to the requirement $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$.

- $\mu_1 > 0,\ \mu_2 > 0,\ \mu_3 > 0,\ \mu_4 = 0,\ E[\theta|r = g, m = h] = k,\ 1 = \beta_H = \beta_L > 0$

If $1 = \beta_H = \beta_L$, then $E[\theta|r = g, m = h] = \frac{\alpha\gamma_H}{\alpha\gamma_H + (1-\alpha)\gamma_H}$, i.e., the investors rationally ignore the report because it is uninformative. By $E[\theta|r = g, m = h] = k$ it follows that this case can only be feasible when

$$\frac{\gamma_L}{\gamma_H} = \frac{\alpha(1-k)}{k(1-\alpha)} \tag{27}$$

Substituting $1 = \beta_H = \beta_L$ and (27) into (17) yields $\mu_1 = \frac{\alpha\gamma_H q(1-k) - k\mu_3}{(1-k)k^2}$. Substituting $1 = \beta_H = \beta_L$, (27) and μ_1 into (16) yields $\mu_2 = \frac{\alpha\gamma_H q}{k} > 0$. Substituting $1 = \beta_H = \beta_L$, μ_1 and μ_2 into (16) yields $\mu_3 = \frac{\alpha\gamma_H(1-k)q}{k} > 0$. Then, $\mu_1 = 0$ which is a contradiction.

- $\mu_1 = 0,\ \mu_2 > 0,\ \mu_3 > 0,\ \mu_4 = 0,\ E[\theta|r = g, m = h] > k,\ 1 = \beta_H = \beta_L > 0$

Substituting $\beta_H = \beta_L = 1$ into (17) implies that $\mu_3 = q(1-\alpha)\gamma_L > 0$. Substituting $\beta_H = \beta_L = 1$ and μ_3 into (16) implies $\mu_2 = q(\alpha\gamma_H + (1-\alpha)\gamma_L) > 0$. This case is feasible if $E[\theta|r = g, m = h] = \frac{\alpha\beta_H\gamma_H}{\alpha\beta_H\gamma_H + (1-\alpha)\beta_L\gamma_H} = \frac{\alpha\gamma_H}{\alpha\gamma_H + (1-\alpha)\gamma_H} > k$ which is equivalent to the requirement $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$. However, we note that if $\beta_H = \beta_L = 1$ the investors rationally ignore the report because it is uninformative (this case is considered under a separate optimization program). Hence, this solution is not feasible.

For future reference,

- if $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$, the value of $\mathcal{P}4(D)$ is zero.
- if $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$, then $1 = \beta_H > \beta_L = \frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L} > 0$ and the value of $\mathcal{P}4(D)$ is $\frac{q\alpha\gamma_H}{k}$.

Below is a summary of the values of the programs:

- If $\alpha < \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$, then
  - The value of $\mathcal{P}1(D)$ is zero;
  - The value of $\mathcal{P}2(D)$ is zero;
  - The value of $\mathcal{P}3(D)$ is $\alpha\left(1 + \frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)}\right)$;
  - The value of $\mathcal{P}4(D)$ is $\frac{(1-p)\alpha\gamma_H}{k}$.
- If $\alpha > \frac{\gamma_L k}{\gamma_L k + \gamma_H(1-k)}$, then
  - The value of $\mathcal{P}1(D)$ is: $\frac{M_A \cdot M_B + M_C}{k(1-k)(\gamma_H - \gamma_L)}$

where $M_A \equiv \alpha(1 - (\gamma_H(1-k) + \gamma_L k)q)$, $M_B \equiv \gamma_L k + \gamma_H(1-k) - \gamma_H\gamma_L q$ and $M_C \equiv$

$\gamma_L k(1-q(\gamma_L k+\gamma_H(2-k)-\gamma_H\gamma_L q))$;

- The value of $\mathcal{P}2(D)$ is $q(\alpha\gamma_H+(1-\alpha)\gamma_L)$;
- The value of $\mathcal{P}3(D)$ is $\alpha\left(1+\frac{(1-k)(1-\gamma_H q)}{k(1-\gamma_L q)}\right)$;
- The value of $\mathcal{P}4(D)$ is zero.

As a last step we compare the values of the programs. The comparison reveals that if either $\gamma_H<\overline{g}$, then $\mathcal{P}3(D)$ has the highest value (for any $\alpha\in(0,1)$). Hence, if Condition 1 is not satisfied the firm sets $\beta_H^*=1$ and $\beta_L^*=\frac{\alpha(1-k)(1-\gamma_H q)}{k(1-\alpha)(1-\gamma_L q)}$ and the threshold is met or exceeded whenever the public report is favorable. Note that $\beta_H^*=1>\beta_L^*>0$.

Proof of Proposition 2 :

Item (i): Using the proof of Proposition 1, we note that if $\alpha<\frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$ and $\gamma_H>\overline{g}$, then $\mathcal{P}4(D)$ has the highest value. The firm sets $\beta_H^{**}=1$ and $\beta_L^{**}=\frac{\alpha(1-k)\gamma_H}{k(1-\alpha)\gamma_L}\in(0,1)$ and the threshold k is met whenever both the public report and the disclosure are favorable. As a last step we verify that this case is feasible, i.e., that $\overline{g}<1$. This is true when $q>\overline{q}\equiv\frac{2-k(1-\gamma_L)-\sqrt{(1-\gamma_L)(4-k(4-k(1-\gamma_L))}}{2\gamma_L}$ (note that $\overline{q}<1$).

Item (ii): Using the proof of Proposition 1, we note that if $\alpha>\frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$ and $\gamma_H>\overline{g}$, then $\mathcal{P}1(D)$ has the highest value. The firm sets

$$\begin{aligned}1&>\frac{(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha))(1-q\gamma_L)}{\alpha(\gamma_H-\gamma_L)(1-k)}\\&=\beta_H\\&>\beta_L\\&=\frac{(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha))(1-q\gamma_H)}{k(\gamma_H-\gamma_L)(1-\alpha)}\\&>0\end{aligned}$$

and the threshold is met or exceeded whenever either the public report or the disclosure are favorable. Using the proof of item (i), we note that this case is feasible.

Proof of Footnote 13 claim: The variance of conditional expectations (VCE), conditoining on the report, r, is defined as a function of the $\boldsymbol{\beta}$ vector as $VCE(\boldsymbol{\beta})=Var\big[E[\theta|r]\big]=E[\big(E[\theta|r]-E\big[E[\theta|r]\big]\big)^2]$, which is equal to $E[(Pr[\theta=1|r]-\alpha)^2]$ and can be expressed as

$$VCE(\boldsymbol{\beta})=\frac{(\alpha-1)^2\,\alpha^2(\beta_H-\beta_L)^2}{(1-\alpha\beta_H-(1-\alpha)\beta_L)\,(\alpha\beta_H+(1-\alpha)\beta_L)}.$$

Plugging in the values for $\boldsymbol{\beta}^*$ and $\boldsymbol{\beta}^{***}$ yields

$$VCE(\boldsymbol{\beta}^*)=\frac{\alpha\,(\alpha\ +k(\alpha\gamma_H q+\gamma_L(1-\alpha)q-1)-\alpha\gamma_H q)}{q(\gamma_H(1-k)+\gamma_L k)-1},\text{ and}$$

$$VCE(\boldsymbol{\beta}^{***})=\frac{\alpha\gamma_H(1-k)-k\gamma_L(1-\alpha)}{\alpha(\gamma_H(1-k)+\gamma_L k)}*VCE(\boldsymbol{\beta}^*).$$

For feasible values of the exogenous parameters, i.e., $0<\alpha<k<1$ and $0<\gamma_L\le\gamma_H<1$, we have $VCE(\boldsymbol{\beta}^*)>VCE(\boldsymbol{\beta}^{***})$. If $\gamma_L=0$, then $VCE(\boldsymbol{\beta}^*)=VCE(\boldsymbol{\beta}^{***})$.

Proof of Proposition 3:

$\mathcal{P}1(M)$ is identical to $\mathcal{P}4(D)$ from the proof of Proposition 1.

$\mathcal{P}2(M)$ can be rewritten as:

$$\max_{\boldsymbol{\beta}} q(\alpha\beta_H\gamma_H + (1-\alpha)\beta_L\gamma_L) + (1-q)(\alpha\beta_H + (1-\alpha)\beta_L)$$
$$s.t. \quad E[\theta|r=g, m=h] \geq k,$$
$$E[\theta|r=g, m=\emptyset] \geq k,$$
$$1 \geq \beta_H \geq \beta_L \geq 0;$$

The first constraint is slack if the second constraint holds. The second constraint binds:

$$\frac{\beta_H\alpha(1-k)}{k(1-\alpha)} = \beta_L$$

and so the optimization program can be rewritten as

$$\max_{\boldsymbol{\beta}} \beta_H \left(q\left(\alpha\gamma_H + (1-\alpha)\frac{\alpha(1-k)}{k(1-\alpha)}\gamma_L\right) + (1-q)\left(\alpha + (1-\alpha)\frac{\alpha(1-k)}{k(1-\alpha)}\right)\right)$$

We note that the expected payoff is increasing in β_H and therefore $\beta_H = 1$. Substituting, we find that $\beta_L = \frac{\alpha(1-k)}{(1-\alpha)k} < 1$. For future reference the value of the optimization program is $\frac{\alpha}{k}(q(k\gamma_H + (1-k)\gamma_L) + (1-q))$.

$\mathcal{P}3(M)$ is identical to $\mathcal{P}2(D)$ from the proof of Proposition 1.

$\mathcal{P}4(M)$ can be rewritten as:

$$\max_{\boldsymbol{\beta}} p(\beta_L(1-\alpha) + \alpha\beta_H)$$
$$s.t. \quad E[\theta|r=g, m=h] \geq k,$$
$$E[\theta|r=b, m=h] \geq k,$$
$$E[\theta|r=g, m=\emptyset] \geq k,$$
$$1 \geq \beta_H \geq \beta_L \geq 0;$$

The first constraint is slack if the third constraint holds. The Lagrangean is:

$$\mathcal{L}_4 = (1-q)(\beta_L(1-\alpha) + \alpha\beta_H) + \mu_1(E[\theta|r=b, m=h] - k)$$
$$+\mu_2(E[\theta|r=g, m=\emptyset] - k) + \mu_3(1-\beta_H) + \mu_4(\beta_H - \beta_L) + \mu_5(\beta_L)$$

The Karush-Kuhn-Tucker stationarity conditions are

$$\frac{\partial\mathcal{L}_4}{\partial\beta_H} = (1-q)\alpha + \mu_1\frac{\partial E[\theta|r=b,m=h]}{\partial\beta_H} + \mu_2\frac{\partial E[\theta|r=g,m=\emptyset]}{\partial\beta_H} - \mu_3 + \mu_4 = 0$$
$$\frac{\partial\mathcal{L}_4}{\partial\beta_L} = (1-q)(1-\alpha) + \mu_1\frac{\partial E[\theta|r=b,m=h]}{\partial\beta_L} + \mu_2\frac{\partial E[\theta|r=g,m=\emptyset]}{\partial\beta_L} - \mu_4 + \mu_5 = 0,$$

the Karush-Kuhn-Tucker feasibility conditions are

$$\frac{\alpha(1-\beta_H)\gamma_H}{\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L} - k \geq 0$$
$$\frac{\alpha\beta_H}{\alpha\beta_H + (1-\alpha)\beta_L} - k \geq 0$$
$$1 - \beta_H \geq 0$$
$$\beta_H - \beta_L \geq 0$$
$$\beta_L \geq 0$$

and the Karush-Kuhn-Tucker complementarity slackness conditions are

$$\left(\frac{\alpha(1-\beta_H)\gamma_H}{\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L} - k\right)\mu_1 = 0$$
$$\left(\frac{\alpha\beta_H}{\alpha\beta_H + (1-\alpha)\beta_L} - k\right)\mu_2 = 0$$
$$(1-\beta_H)\mu_3 = 0$$
$$(\beta_H - \beta_L)\mu_4 = 0$$

$\beta_L \mu_5 = 0$

We know:

1. $\beta_H > \beta_L$ because, otherwise, if $\Pr(H|r = g, m = \emptyset) - k \geq 0$ then it has to be that $\Pr(H|r = b, m = \emptyset) - k \geq 0$. But we know that $\Pr(H|r = b, m = \emptyset) < \alpha \Rightarrow \Pr(H|r = b, m = \emptyset) - k < \alpha - k < 0$. It follows that $\mu_4 = 0$.
2. $\beta_H < 1$, because otherwise $\Pr(H|r = b, m = h) = 0 \Rightarrow \Pr(H|r = b, m = h) - k < 0$, so it follows that $\mu_3 = 0$.
3. Since $\mu_4 = 0$, it must be true that $\mu_2 > 0$ because otherwise $\frac{\partial \mathcal{L}_4}{\partial \beta_L} > 0$ implying $\beta_L = 1$ which contradicts $\beta_L < \beta_H < 1$.
4. Since $\mu_3 = 0$ ($\beta_H < 1$), it must be true that $\mu_1 > 0$ because otherwise $\frac{\partial \mathcal{L}_4}{\partial \beta_H} > 0$ implying $\beta_H = 1$ which is a contradiction.

It follows that β_H and β_L are defined by the binding constraints:

$$\beta_H = \frac{\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha)}{\alpha(1-k)(\gamma_H - \gamma_L)} < 1$$

$$\beta_L = \frac{\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha)}{k(1-\alpha)(\gamma_H - \gamma_L)} < \beta_H$$

If $\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha) < 0$, then $\beta_L = 0$ and the first constraint gives us

$$\frac{\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha)}{\gamma_H \alpha(1-k)} = \beta_H$$

but because we assumed $\gamma_H \alpha(1-k) - \gamma_L k(1-\alpha) < 0$, this implies $\beta_H < 0$, which is not feasible. The second constraint gives us $1 - k = 0$, contradicts our assumption of $1 > k$. So $\mathcal{P}4(M)$ has a solution only for $\gamma_H > \frac{k(1-\alpha)}{\alpha(1-k)} \gamma_L$. For future reference, the value of the optimization program is $q\big(\alpha \gamma_H + \gamma_L(1-\alpha)\big) + (1-q)\frac{\alpha(1-k)\gamma_H - k(1-\alpha)\gamma_L}{(1-k)(\gamma_H - \gamma_L)k}$.

$\mathcal{P}5(M)$ can be rewritten as:

$$\begin{aligned} &\max_{\boldsymbol{\beta}} \alpha \beta_H + (1-\alpha)\beta_L \\ s.t.\quad & E[\theta | r = g, m = h] \geq k, \\ & E[\theta | r = g, m = \emptyset] \geq k, \\ & E[\theta | r = g, m = l] \geq k, \\ & 1 \geq \beta_H \geq \beta_L \geq 0 \end{aligned}$$

The first and second constraints are slack if the third constraint is satisfied. The expected payoff is increasing in both β_H and β_L. We examine the third constraint and note that:

$$\frac{\partial}{\partial \beta_H}\left(\frac{\alpha \beta_H (1-\gamma_H)}{\alpha \beta_H (1-\gamma_H) + (1-\alpha)\beta_L(1-\gamma_L)}\right) \propto (1-\gamma_L)(1-\gamma_H)(1-\alpha)\alpha\beta_L > 0$$

This suggests $\beta_H = 1$. β_L will be defined by

$$0 = \frac{\alpha(1-\gamma_H)}{\alpha(1-\gamma_H) + (1-\alpha)\beta_L(1-\gamma_L)} - k$$

$$\beta_L = \frac{\alpha(1-k)(1-\gamma_H)}{k(1-\alpha)(1-\gamma_L)}$$

For future reference the value of the optimization program is $\alpha\left(\frac{k(1-\gamma_L) + (1-k)(1-\gamma_H)}{k(1-\gamma_L)}\right)$.

$\mathcal{P}6(M)$ can be rewritten as:

$$\begin{aligned} &\max_{\boldsymbol{\beta}}\ \alpha \beta_H + (1-\alpha)\beta_L + q(\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L) \\ s.t.\quad & E[\theta | r = g, m = h] \geq k, \\ & E[\theta | r = g, m = \emptyset] \geq k, \\ & E[\theta | r = b, m = h] \geq k, \end{aligned}$$

$$E[\theta|r = g, m = l] \geq k,$$
$$1 \geq \beta_H \geq \beta_L \geq 0.$$

The first and second constraints are slack if the third and fourth are satisfied. Hence, the Lagrangean is

$$\mathcal{L}_6 = \alpha\beta_H + (1-\alpha)\beta_L + q(\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L)$$
$$+\mu_1(E[\theta|r = b, m = h] - k) + \mu_2(E[\theta|r = g, m = l] - k)$$
$$+\mu_3(1-\beta_H) + \mu_4(\beta_H - \beta_L) + \mu_5(\beta_L)$$

The Karush-Kuhn-Tucker stationarity conditions are

$$\frac{\partial \mathcal{L}_6}{\partial \beta_H} = \alpha(1-\gamma_H q)$$
$$+\mu_1 \frac{\partial E[\theta|r=b,m=h]}{\partial \beta_H} + \mu_2 \frac{\partial E[\theta|r=g,m=l]}{\partial \beta_H} - \mu_3 + \mu_4 = 0$$
$$\frac{\partial \mathcal{L}_6}{\partial \beta_L} = (1-\alpha)(1-\gamma_L q)$$
$$+\mu_1 \frac{\partial E[\theta|r=b,m=h]}{\partial \beta_L} + \mu_2 \frac{\partial E[\theta|r=g,m=l]}{\partial \beta_L} - \mu_4 + \mu_5 = 0,$$

the Karush-Kuhn-Tucker feasibility conditions are

$$\frac{\alpha(1-\beta_H)\gamma_H}{\alpha(1-\beta_H)\gamma_H + (1-\alpha)(1-\beta_L)\gamma_L} - k \geq 0$$
$$\frac{\alpha\beta_H(1-\gamma_H)}{\alpha\beta_H(1-\gamma_H) + (1-\alpha)\beta_L(1-\gamma_L)} - k \geq 0$$
$$1 - \beta_H \geq 0$$
$$\beta_H - \beta_L \geq 0$$
$$\beta_L \geq 0$$

and the Karush-Kuhn-Tucker complementarity slackness conditions are

$$\mu_1(E[\theta|r = b, m = h] - k) = 0$$
$$\mu_2(E[\theta|r = g, m = l] - k) = 0$$
$$\mu_3(1-\beta_H) = 0$$
$$\mu_4(\beta_H - \beta_L) = 0$$
$$\mu_5(\beta_L) = 0$$

We know:

1. $\mu_4 = 0$ (and so $\beta_H > \beta_L$) because otherwise, if $\Pr(H|r = g, m = \emptyset) - k \geq 0$ then it has to be that $\Pr(H|r = b, m = \emptyset) - k \geq 0$. But we know that $\Pr(H|r = b, m = \emptyset) < \alpha \Rightarrow \Pr(H|r = b, m = \emptyset) - k < \alpha - k < 0$ which is a contradiction.

2. $\mu_3 = 0$ (and so $\beta_H < 1$), because otherwise $\Pr(H|r = b, m = h) = 0$ which implies $\Pr(H|r = b, m = h) - k < 0$ (and contradicts the constraint).

3. Since $\mu_3 = 0$ ($\beta_H < 1$), then it must be true that $\mu_1 > 0$ because otherwise $\frac{\partial \mathcal{L}_6}{\partial \beta_H} > 0$ implying $\beta_H = 1$ which is a contradiction.
4. Since $\mu_4 = 0$, then it must be true that $\mu_2 > 0$ because otherwise $\frac{\partial \mathcal{L}_6}{\partial \beta_L} > 0$ implying $\beta_L = 1$ which contradicts $\beta_L < \beta_H < 1$.

We note that β_H and β_L are defined by the first and second constraints binding:

$$\beta_H = \frac{(\gamma_H\alpha(1-k) - \gamma_L k(1-\alpha))}{(\gamma_H - \gamma_L)} \frac{(1-\gamma_L)}{\alpha(1-k)}$$

$$\beta_L = \frac{\left(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha)\right)}{(\gamma_H-\gamma_L)}\frac{(1-\gamma_H)}{k(1-\alpha)}$$

We need $\left(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha)\right) > 0$ for β_H and β_L to be non-negative. If this condition does not hold then $\beta_L = 0$ and

$$\frac{\alpha(1-\beta_H)\gamma_H}{\alpha(1-\beta_H)\gamma_H+(1-\alpha)\gamma_L} - k = 0$$
$$\frac{\alpha(1-k)\gamma_H-k(1-\alpha)\gamma_L}{\alpha(1-k)\gamma_H} = \beta_H$$

but because we assumed $\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha) < 0$, this implies $\beta_H < 0$, which is not feasible. So $\mathcal{P}6(M)$ has a solution only for $\gamma_H > \frac{k(1-\alpha)}{\alpha(1-k)}\gamma_L$. Lastly, we note that $\beta_H < 1$ because

$$\begin{aligned} 1-\beta_H &= 1-\frac{\left(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha)\right)}{(\gamma_H-\gamma_L)}\frac{(1-\gamma_L)}{\alpha(1-k)} \\ &\propto \alpha(1-k)(\gamma_H-\gamma_L)-(1-\gamma_L)\left(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha)\right) \\ &= \gamma_L\left(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha)\right)+\gamma_L(k-\alpha) \\ &> 0, \end{aligned}$$

by assumption. For future reference, the value of the optimization program is:

$$M_D\cdot(k(1-\gamma_L)+(1-k)(1-\gamma_H)+\gamma_H\gamma_L q)+(k-\alpha)\gamma_H\gamma_L q$$

where $M_D \equiv \frac{\left(\gamma_H\alpha(1-k)-\gamma_L k(1-\alpha)\right)}{(\gamma_H-\gamma_L)(1-k)k}$.

Below is a summary of the values of the programs:

- If $\alpha < \frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$, then
  - The value of $\mathcal{P}1(M)$ is $\frac{q\alpha\gamma_H}{k}$;
  - The value of $\mathcal{P}2(M)$ is $\frac{\alpha}{k}(q(k\gamma_H+(1-k)\gamma_L)+(1-q))$;
  - The value of $\mathcal{P}3(M)$ is zero;
  - The value of $\mathcal{P}4(M)$ is zero;
  - The value of $\mathcal{P}5(M)$ is $\alpha\left(\frac{k(1-\gamma_L)+(1-k)(1-\gamma_H)}{k(1-\gamma_L)}\right)$;
  - The value of $\mathcal{P}6(M)$ is zero;

- If $\alpha > \frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$, then
  - The value of $\mathcal{P}1(M)$ is zero;
  - The value of $\mathcal{P}2(M)$ is $\frac{\alpha}{k}(q(k\gamma_H+(1-k)\gamma_L)+(1-q))$;
  - The value of $\mathcal{P}3(M)$ is $q(\alpha\gamma_H+(1-\alpha)\gamma_L)$;
  - The value of $\mathcal{P}4(M)$ is $q\left(\alpha\gamma_H+\gamma_L(1-\alpha)\right)+(1-q)\frac{\alpha(1-k)\gamma_H-k(1-\alpha)\gamma_L}{(1-k)(\gamma_H-\gamma_L)k}$;
  - The value of $\mathcal{P}5(M)$ is $\alpha\left(\frac{k(1-\gamma_L)+(1-k)(1-\gamma_H)}{k(1-\gamma_L)}\right)$;
  - The value of $\mathcal{P}6(M)$ is $\frac{\alpha(1-\gamma_H(1-k)-\gamma_L k)(\gamma_L k+\gamma_H(1-\gamma_L q-k))-M_D}{(\gamma_H-\gamma_L)(1-k)k}$ where $M_D \equiv \gamma_L k(1-\gamma_L k-\gamma_H(1+(1-\gamma_L)q-k+q))$.

It is immediate that in case (B) the value of program $\mathcal{P}3(M)$ is lower than the value of program $\mathcal{P}4(M)$. So we only need to consider:

- If $\alpha < \frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$, the values of $\mathcal{P}1(M)$, $\mathcal{P}2(M)$ and $\mathcal{P}5(M)$.

- If $\alpha > \frac{\gamma_L k}{\gamma_L k+\gamma_H(1-k)}$, the values of $\mathcal{P}2(M)$, $\mathcal{P}4(M)$, $\mathcal{P}5(M)$ and $\mathcal{P}6(M)$.

As a last step we consider the case when Condition 1 is not satisfied and compare the values of the programs above with the value of program $\mathcal{P}3(D)$ (the highest value program under the discretion regime as shown in Proposition 1). The comparison reveals that the value of program $\mathcal{P}3(D)$ is strictly larger than:

- the values of programs $\mathcal{P}1(M)$ and $\mathcal{P}5(M)$;
- the value of program $\mathcal{P}2(M)$ if $\gamma_H < g_o$, where $g_o \equiv \frac{1-\gamma_L(1-\gamma_L(1-k))q}{1-\gamma_L kq} > 0$;
- the value of program $\mathcal{P}6(M)$ because the value of program $\mathcal{P}6(M)$ is lower than the value of program $\mathcal{P}1(D)$ which is lower than the value of program program $\mathcal{P}3(D)$;
- the value of program $\mathcal{P}4(M)$ because the value of program $\mathcal{P}4(M)$ is lower than the value of program $\mathcal{P}6(D)$ if $\gamma_H < g_{oo}$, where $g_{oo} \equiv \frac{q-\gamma_L(1-(1-q)(1-k))}{1-\gamma_L q-k(1-q)}$. Feasibility requires that $g_{oo} > 0$ which holds when $p < \frac{1-\gamma_L}{1-\gamma_L(1-k)}$. Further, (as we show above) the value of program $\mathcal{P}6(M)$ is lower than the value of program program $\mathcal{P}3(D)$;

It follows that a sufficient condition for discretion to be strictly valuable is that $\gamma_H < \min\{\overline{g}, g_o, g_{oo}\}$ and $p < \frac{1-\gamma_L}{1-\gamma_L(1-k)}$.

Proof of Proposition 4 :

Item (i): Using the proof of Proposition 3, we consider the case when Conditions 1 and 2 are not satisfied and compare the values of programs $\mathcal{P}1(M)$, $\mathcal{P}2(M)$ and $\mathcal{P}5(M)$ (the relevant programs when Condition 2 is not satisfied) with the value of program $\mathcal{P}4(D)$ (the highest value program under the discretion regime as shown in Proposition 2, item (i)). The comparison reveals that the value of program $\mathcal{P}4(D)$:

- is equal to the value of program $\mathcal{P}1(M)$;
- is strictly larger than the value of program $\mathcal{P}2(M)$ if $\gamma_H > \frac{\gamma_L(1-k)q+1-q}{(1-k)q} \equiv g^o$. Feasibility requires that $g^o < 1$ which holds if $q > \frac{1}{2-\gamma_L(1-k)-k}$;
- is strictly larger than the value of program $\mathcal{P}5(M)$

It follows that the firm is indifferent between discretion and mandatory disclosure if $\gamma_H > \max\{\overline{g}, g^o\}$ and $q > \frac{1}{2-\gamma_L(1-k)-k}$.

Item (ii): Using the proof of Proposition 3, we consider the case when Conditions 1 and 2 are satisfied and compare the values of programs $\mathcal{P}2(M)$, $\mathcal{P}4(M)$, $\mathcal{P}5(M)$ and $\mathcal{P}6(M)$ (the relevant programs when Condition 2 is satisfied) with the value of program $\mathcal{P}1(D)$ (the highest value program under the discretion regime as shown in Proposition 2, item (ii)). The comparison reveals that the value of program $\mathcal{P}1(D)$ is strictly lower than the value of program $\mathcal{P}4(M)$ if $\gamma_H > \frac{1-\gamma_L}{1-\gamma_L(1-p)} \equiv g^{oo}$. We note that $g^{oo} < 1$ (so $\gamma_H > g^{oo}$ is feasible). It immediately follows that the firm strictly prefers mandatory disclosure if $\gamma_H > \max\{\overline{g}, g^{oo}\}$.

REFERENCES

Alonso, R. and O. Camara. 2014. Persuading Voters. *Working paper*.

Arya, A., Glover, J., Sivaramakrishnan, K. 1997. Commitment issues in Budgeting. *Journal of Accounting Research* 35 (2): 273-278.

Bagnoli, M. and S. Watts. 2005. Conservative accounting choices. *Management Scienc* 51 (5): 786-801.

Ball, R., S. Jayaraman and L. Shivakumar. 2012. Audited Financial Reporting and Voluntary Disclosure as Complements: A Test of the Confirmation Hypothesis, *Journal of Accounting and Economics* 53: 136-166

Bartov, E., Gul, F. and J. Tsui. 2001. Discretionary-accruals models and audit qualifications. *Journal of Accounting and Economics* 30 (3): 421-452.

Begley, J. 1990. Debt covenants and accounting choice. *Journal of Accounting and Economics* 12 (1): 125-139.

Beneish, M and E. Press.1993. Costs of technical violation of accounting-based debt covenants. *The Accounting Review* 68 (2): 233-257.

Beneish, M and E. Press. 1995. The resolution of technical default. *The Accounting Review*, 70 (2): 337-353.

Beyer, A. 2013. Conservatism and aggregation: the effect on cost of equity capital and the efficiency of debt contracts. *Rock Center for Corporate Governance at Stanford University Working Paper*.

Bonachi, M., A. Mara and R. Shalev. 2015. Fair Value Accounting and Firm Indebtedness - Evidence from Business Combinations under Common Control. *Working paper*.

Chen, H. and B. N. Jorgensen. 2012. Firm exit through bankruptcy-the effect of accounting bias on product market competition. *Working paper*.

Chen, H., Lethmathe, P. and N. Soderstrom. 2015. Estimation bias and Monitoring in clean development Mechanism Projects. *Working paper*.

Dichev, I. and D. Skinner. 2002. Large-sample evidence on the debt covenant hypothesis. *Journal of Accounting Research* 40 (4): 1091-1123.

Dye, R. A. 1985. Disclosure of nonproprietary information. *Journal of Accounting Research*, 23 (1): 123-145.

Duggan, J., and C. Martinelli. 2011. A Spatial Theory of Media Slant and Voter Choice. *Review of Economic Studies* 78 (2): 640-666.

Francis, J., D. Nanda and P. Olsson. 2008. Voluntary Disclosure, Earnings Quality, and Cost of Capital. *Journal of Accounting Research* 46: 53–99

Friedman, H. L., J. S. Hughes, and R. Saouma. 2015. Implications of biased reporting: Conservative and liberal accounting policies in oligopolies. *Review of Accounting Studies*, forthcoming.

Gentzkow, M and E. Kamenica. 2014. Costly Persuasion. *American Economic Review: Papers & Proceedings* 104 (5): 457-462.

Gigler, F. and T. Hemmer. 2001. Conservatism, optimal disclosure policy, and the timeliness of financial reports. *The Accounting Review* 76 (4): 471-493.

Gigler, F. Kanodia, C., Sapra, H., and R. Venugopalan. 2009. Accounting conservatism and the efficiency of debt contracts. *Journal of Accounting Research* 47 (3): 767-797.

Healy, P. and K. Palepu.1990. Effectiveness of accounting-based dividend covenants. *Journal of Accounting and Economics* 12 (1): 97-123.

Göx, R. and A. Wagenhofer. 2009. Optimal Impairment Rules. *Journal of Accounting and Economics* 48 (1): 2-16.
Gong, G., Y. Li, and H. Xie. 2009. The Association between Management Earnings Forecast Errors and Accruals. *The Accounting Review* 84 (2): 497–530.
Guay, W., D. Samuels and D. Taylor. 2015. Guiding Through the Fog: Financial Statement Complexity and Voluntary Disclosure. *Working paper*.
Guo, Q. 2012. Conservative reporting and product market competition. *Working paper*.
Hedlund, J. 2015. Bayesian persuasion by a privately informed sender. *Working paper*.
Jiang, J., Stanford, M. H. and Xie,Y. 2012. Does it matters who pays for bond ratings? Historical evidence. *Journal of Financial Economics.*
Jung, W. and Y. K. Kwon. 1988. Disclosure when the market is unsure of information endowment of managers. *Journal of Accounting Research* 26 (1), 146-153.
Kamenica, E. and M. Gentzkow. 2011. Bayesian persuasion. *American Economic Review* 101 (6): 2590-2615.
Kausar, A., Taffler, R. and C. Tan. 2006. Anomalous price reaction and differential stockholder response to going-concern audit opinions and withdrawals. *Working paper*.
Kim, M. and W. Kross. 1998. The impact of the 1989 change in bank capital standards on loan loss provisions and loan write-offs. *Journal of Accounting and Economics* 25 (1): 69-99.
Kisgen, D. J. 2006. Credit ratings and capital structure. *The Journal of Finance* 61 (3): 1035-1072.
Kwon, Y., D. Newman, and Y. Suh. 2001. The demand for accounting conservatism for management control. *Review of Accounting Studies* 6 (1): 29-52.
Lennox, C., and C. Park. 2006. The informativeness of earnings and management's issuance of earnings forecasts. *Journal of Accounting and Economics* 42: 439–458.
Li, Z., Shroff, P., Venkataraman, R. and I. Zhang. 2011. Causes and consequences of goodwill impairment losses. *Review of Accounting Studies* 16 (4): 745-778.
Menon, K. and D. Williams. 2010. Investor reaction to going concern audit reports. *The Accounting Review* 85 (6): 2075-2105.
Michaeli, B. 2014. Divide and Inform: Rationing Information to Facilitate Persuasion. *Working paper*.
Michaely, R. and Womack, K. 2005. Brokerage Recommendations: Stylized Characteristics, Market Responses, and Biases. *Advances in Behavioral Finance II*, Richard Thaler Princeton University Press.
Patrice, J. 2015. New Law Cracking Down on Prosecutorial Misconduct But Does It Do Enough? *Above the Law* (online). URL: http://abovethelaw.com/2015/10/new-law-cracking-down-on-prosecutorial-misconduct-but-does-it-do-enough/.
Press, E. and J. Weintrop. 1990. Accounting-based constraints in public and private debt agreements: Their association with leverage and impact on accounting choice. *Journal of Accounting and Economics* 12 (1): 65-95.
Ramanna, K., and R. Watts. 2012. Evidence on the Use of Unverifiable Estimates in Required Goodwill Impairment." *Review of Accounting Studies* 17 (4): 749-780.
Simerman, J. 2012. Prosecutors' tactics amounted to civil rights violations, exonerated convicts maintain. *NOLA.com | The Times-Picayune*. URL: http://www.nola.com/crime/index.ssf/2012/07/prosecutors_tactics_amounted_t.html.

Smith, M. 2007. Accounting conservatism and real options. *Journal of Accounting, Auditing & Finance* 22 (3): 449-467.

Stocken, P. and R. Verrecchia. 2004. Financial Reporting System Choice and Disclosure Management. *The Accounting Review* 79 (4): 1181-1203.

Sweeney, A. P. 1994. Debt-covenant violations and managers' accounting responses. *Journal of Accounting and Economics* 17 (3): 281-308.

Verrecchia, R. E. 1983. Discretionary disclosure. *Journal of Accounting and Economics* 5: 179-194.

Taneva, I. 2014. Information Design. *Working paper*.

Wang, Y. 2013. Bayesian Persuasion with Multiple Receivers. *Working paper*.