A theoretical framework for ESG reporting to investors

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Abstract: We provide a theoretical framework for reporting of firms' environmental, social, and governance (ESG) activities to investors. In our model, investors receive an ESG report and use it to price the firm. Because the manager is interested in the firm's price, disclosing an ESG report provides effort and greenwashing incentives. We analyze the impact of different reporting characteristics on the firm's price, cash flows, and ESG performance. In particular, we investigate the consequences of whether the report captures ESG inputs or outcomes, how the report aggregates different ESG dimensions, and the manager's tradeoffs regarding ESG efforts and reporting bias. We find that, for example, an ESG report that weights efforts by their impact on the firm's cash flows tends to have a stronger price reaction than an ESG report that focuses on the ESG impact per se. ESG reports aligned with investors' aggregate preferences provide stronger incentives and lead to higher cash flows and ESG than reports that focus on either ESG or cash flow effects individually. Additionally, in the presence of informative financial reporting, ESG reports that focus on ESG impacts lead to the same cash flow and better ESG results than reports focusing on cash flow impacts alone.

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1 Introduction

As concerns about climate change and social inequity become more widespread, consumers, investors, and regulators are increasingly interested in firms' environmental, social, and governance (ESG) activities. Information is a necessary input to the decisions that stakeholders make. Investor and stakeholder pressure can motivate firms to shift activities towards those more desirable from an ESG or cash-flow perspective, even when direct regulation of underlying activities is difficult or infeasible. As a result, governments, industry bodies, and even firms are developing standards for corporate ESG reporting. In many jurisdictions (e.g., the U.S.), the reporting is targeted at investors, either because reporting to investors falls under the purview of securities regulators, or because investors, as suppliers of capital, can demand such disclosures.[1] We contribute to the ongoing development of ESG reporting research and practice by providing a theoretical framework, highlighting economic forces that affect the impact of ESG reports used by investors.

Our framework is developed around a model of a firm that reports its ESG to investors. Some investors incorporate ESG into their demand for shares, due to, for example, altruism, a concern over ESG outcomes, or positive affect (i.e., warm glow) from allocating wealth in line with ESG. The firm's ESG and cash flows depend on different corporate actions, and we allow these actions to have heterogeneous effects on ESG and cash flows. We assume a manager chooses the corporate actions (the manager's effort, which can be alternatively interpreted as investments in projects) to increase the firm's price and that investors are uncertain about the manager's effort choice because the manager's preference has a stochastic component unobservable to outside investors.

ESG reporting entails a noisy public report of the firm's ESG activities, aggregating

[1] The European Union's Nonfinancial Reporting Directive places some weight on information demands of non-investor stakeholders, with reporting requirements targeted at both publicly listed firms as well as large private firms.

across efforts and aspects of ESG performance outside of the managers' control. The reporting weights on the efforts (and other variables) allow us to nest scenarios in which a report measures the firm's ESG directly and those where a report measures the effects of the firms' ESG choices on its expected cash flows. The report is also subject to costly biasing by the manager, which captures an aspect of greenwashing whereby firms manipulate their ESG reports to appear "green." Random variation in the reporting cost function prevents investors from unraveling the biasing (as in Dye and Sridhar, 2004), which allows greenwashing to have a material effect on price that does not depend on the manager passing the cost of greenwashing on to investors (as in pure window-dressing models). Underpinning the potential for harmful greenwashing are two stylized facts. First, investors are increasingly concerned about firms' ESG impacts (e.g., **?**). Second, much of the data on ESG is difficult to measure (e.g., quality of jobs provided), or if easy to measure (e.g., smokestack emissions), difficult to translate into dollar equivalents (e.g., effect of smokestack emissions on local and global welfare or cost of avoiding emissions).

Our framework suggests three alternative mechanisms through with ESG reporting can affect prices. First, because investors update their beliefs when information arrives, there is an immediate price response. Second, ESG reporting has real effects by changing managers' choices. Finally, when investors learn about the firm's activities from the ESG report, they face less uncertainty about cash flows and, as a result, demand a lower risk premium. All three effects are simultaneously determined in equilibrium. In particular, when investors learn more from the ESG report, the price response will be stronger, managers will have greater incentives to change their actions, and there will be a larger reduction in the risk premium. At the same time, a stronger price response also provides more incentives to bias the report. Additionally, as we discuss later, different investors may be interested in learning different things from the same report, so a report that is more sensitive to, for instance, ESG outcomes unrelated to cash flows, may be viewed as more informative to ESG-minded investors but noisier to investors focused on learning about cash flows.

An important feature of ESG reporting is greenwashing, whereby firms present positively-biased portrayals of their ESG activities or outcomes. Our framework illustrates how greenwashing is reflected in multiple aspects of the report, beyond the simple biasing of reporting low emissions when emissions were in fact high. We elaborate on various ways firms might engage in greenwashing, and illustrate how parameters affecting different types of greenwashing additionally affect cash flows, ESG efforts and outcomes, stock prices, and stock price reactions to ESG reports. In particular, we examine ex-ante greenwashing related to how the report is set up, as well as ex-post greenwashing driven by the manager's reporting and effort choices. Ex-ante greenwashing in our model relates to how ESG efforts and stochastic outcomes outside of the managers' control are aggregated. Aggregation policies are important because different sensitivities to efforts or stochastic components can affect market responses and managers' incentives (i.e., what ESG scores measure and how they are aggregated matters). Ex-post greenwashing in our model comes from the manager's ability to directly exaggerate (or dampen) reported ESG. Additionally, managers can increase efforts that are captured relatively well in the ESG report, and at the same time reduce efforts that have relatively small impacts on the report. Note that while reporting greenwashing happens after the ESG efforts are chosen, it affects the market response and therefore, in equilibrium, can have an effect on the manager's effort choice. Through this channel, ESG reporting features have real effects on ESG activities and outcomes.

Our results provide guidance on how the characteristics of ESG reporting can affect the firm's ESG efforts and performance as well as the firm's cash flows and stock price risk premia. For example, when we focus on a single-effort setting (to abstract away from issues involving aggregation and effort-report congruence) we show that many changes in report characteristics affect investors' price response to the ESG report, the manager's effort level, and reporting greenwashing in the same direction. Capital market incentives encourage both ESG-related efforts and greenwashing. Similarly, changes to investor and firm characteristics, e.g., investors' average ESG concerns and ex-ante uncertainty about the manager's efforts, tend to move price responses, efforts, and greenwashing in the same direction. The multiple paths for effects on prices, reporting, and managerial efforts suggest caution in interpreting empirical results (e.g., on inferring cash flow effects of ESG efforts from returns around ESG reports or attributing heterogeneity in responses to ESG reports to the informativeness of those reports without controlling for heterogeneity in investors' ESG concerns).

Two particular characteristics of interest are the degree to which the report captures ESG efforts/inputs versus performance/outcomes and the sensitivity of the report to different types of efforts. Reporting ESG efforts rather than outcomes has a direct effect of providing stronger managerial motivation, since the report becomes more sensitive to controllable efforts rather than uncontrollable outcomes. However, the effort incentives are also driven by the price responses to the report, which can be larger with outcome reporting. Effort incentives are larger with outcome reporting if many investors are concerned about ESG performance and a significant portion of ESG performance is outside of the manager's control.

Focusing on how the report aggregates the manager's efforts, we investigate reports that are congruent to cash flows, to ESG, and to the average investor's values, while abstracting away from reporting noise and uncontrollable ESG outcomes. Because investors use the report to learn about the manager's efforts, a report that aggregates multiple efforts according to their effect on ESG does not perfectly reveal the cash-flow consequences of the manager's efforts. This ESG-congruent report thus leaves investors with residual cash flow uncertainty. In contrast, a cash-flow congruent report reveals the effects of efforts on cash flows, which are valued by all investors. This causes prices to react more strongly to the cash-flow congruent report relative to the ESG-congruent report.[2] We show that, while a cash-flow-congruent report leads to higher expected cash flows, an ESG-congruent report leads to higher expected ESG. Different regulators or standard-setters with different objective functions (e.g., cash flow versus ESG maximization) would thus naturally prefer reports with different sensitivities

[2] A necessary condition for the ESG-congruent report to have a higher price reaction is that the efforts have a stronger effect on ESG than on cash flows. In this situation, cash-flow congruency perfectly reveals cash flow effects that, in sum, are smaller than the imperfectly revealed ESG effects.

to managerial efforts. Interestingly, a values-congruent report (reflecting cash flow impact plus ESG impact weighted by the fraction of investors who value ESG) yields the same expected cash flows as the cash-flow-congruent report and the same expected ESG as the ESG-congruent report. Unfortunately, the weights of the efforts in the values-congruent report depend on the fraction of ESG investors, a parameter that likely varies over time and across firms and is difficult to measure for regulators, standard-setters, or firms interested in designing ESG reports.

In an extension, we examine the addition of the ESG report to a setting with a noisy financial report already in place. The financial report is congruent to cash flows, but its noise allows the ESG report to provide incremental information about the manager's efforts as well as ESG outcomes outside of the manager's control. The general tenor of the equilibrium is preserved, in that both reports affect prices and managerial efforts, and greenwashing persists. The addition of the ESG report increases the expected return around information releases, which is due to the greater resolution of cash flow uncertainty and thus larger reduction in the risk premium at disclosures of the ESG and financial reports relative to the financial report alone. Finally, the existence of a high-quality financial report provides sufficient cash-flow motivation to the manager such that an ESG-congruent ESG report leads to the same cash flows than a cash-flow-congruent ESG report. However, the ESG-congruent report leads to a higher expected ESG performance. In other words, in the presence of high quality financial reporting, an ESG report does not distort cash flow incentives.

2 Literature Review

We address regulators' and researchers' call to improve our understanding of trade-offs related to characteristics of managers' reports about internal and external effects of their ESG actions (Christensen et al., 2019; Grewal and Serafeim, 2020). Our model broadly combines three strands of literature. First, similar to the literature on multitask effort allocation with moral hazard (e.g., Holmstrom and Milgrom, 1991; Datar et al., 2001), our model features a manager who privately takes multiple actions that affect the firm's outcomes. Second, as in the literature on earnings management (e.g., Dye and Sridhar, 2004; Fischer and Verrecchia, 2000), our focal firm discloses a report that need not be truthful. Finally, some investors who receive the report incorporate their beliefs about the firm's ESG when forming their demand, similar to Pástor et al. (2020), Friedman and Heinle (2016), and additional studies discussed in greater detail below.

We follow Paul (1992) and Feltham and Xie (1994) on their notion of the incongruity of performance measures. We depart from these works in that we introduce an additional (non-financial) dimension of firm performance that is valued by some investors, which, in turn, creates a second type of incongruity: between the social impact's and measure's sensitivities to agent's actions.

Our model extends the literature on earnings management (e.g., Dye and Sridhar, 2008) by allowing a manager to manipulate the report of firms' ESG, i.e., to engage in greenwashing. We analyze the manager's reporting strategy as a function of parameters capturing the manager's incentives, information asymmetry between the manager and investors, and the congruity of the report to the social and financial impacts it may be designed to capture.

Several studies provide evidence that individuals value social impact of their investments. For example, the survey in Krueger et al. (2020) suggests that institutional investors recognize the importance of climate risks for their portfolios' cash flows. Similarly, Bauer et al. (2021) survey members of pension funds and find that two-thirds of respondents are willing to sacrifice some financial benefits to invest in companies whose goals are aligned with sustainable development goals (SDG). Barber et al. (2021) and Bolton and Kacperczyk (2021) provide further evidence of tradeoffs between ESG and market performance.

The pricing of companies' non-financial performance has received much recent academic interest. Closely related is Pástor et al. (2020), who show that agents' tastes for green holdings affect asset prices in equilibrium and derive predictions about the returns on a green factor. Zerbib (2020) develops an asset-pricing model where ESG performance is priced due to the impact of two investor groups: those that exclude certain assets from their investment options and those that internalize private costs of externalities in their expected returns. These investors cause two types of premia to occur: taste premia and exclusion premia. Pedersen et al. (2020) analyze an economy where the ESG score contains information related to firm fundamentals and some investors have preferences about firms' non-financial performance. They show that in equilibrium, prices of assets satisfy a four-fund separation theorem: each asset is a portfolio of a risk-free asset, a tangent portfolio, a minimal-variance portfolio, and an ESG-tangent portfolio. Chowdhry et al. (2018), Oehmke and Opp (2019), and Friedman and Heinle (2021) derive conditions for impact investment to improve social outcomes when some investors value impact as well as cash flows.

Most of the literature assumes symmetric information and/or is silent on the source of the information that investors have about firms' non-financial performance. Lyon and Maxwell (2011) provide a model of greenwashing based on discretionary disclosure of favorable signals (e.g., Jung and Kwon, 1988), in contrast to our model of reporting bias with uncertain costs. Despite the relative paucity of theoretical research, there exists rich empirical evidence for firms' greenwashing or providing inappropriate information on their ESG activities (e.g., Bingler et al., 2021; Basu et al., 2021; Delmas and Burbano, 2011; Marquis et al., 2016), as well as numerous examples from the popular and business press (e.g., Brogger and Marsh, 2021; Kowsmann and Brown, 2021).

A separate literature has focused on the materiality of ESG disclosures (e.g., Khan et al., 2016; Jebe, 2019). Amel-Zadeh and Serafeim (2018) report survey evidence that mainstream investment organizations primarily use ESG information because of its relevance to investment performance, ahead of client demand and ethical considerations, though Moss et al. (2020) find no evidence of retail investors reacting to ESG press releases. Materiality implies "relevant to investor decision-making," and can be evaluated based either on relevance to fundamentals, i.e., future cash flows or discount rates, or based on investor responses to ESG information releases. The Sustainability Accounting Standards Board (SASB) has promulgated industry-specific sustainability standards that focus on materiality, while the SEC encourages disclosure of material ESG information under existing disclosure rules. Our model, by clearly delineating between cash flow relevance, investor response, and effects on ESG allows us to show how focusing on different definitions of materiality in designing ESG reports can affect prices, greenwashing, and corporate ESG efforts.

A related debate to investor-focused materiality is on how trading activity and investor engagement affect firms' social impact. Landier and Lovo (2020) show how the policy of an ESG fund forces companies to internalize (at least partially) their externalities. An ESG fund's optimal strategy is to invest in firms with the strongest capital search frictions and most inefficient externalities. Green and Roth (2021) derive optimal strategies for social investors to maximize social welfare in an environment of competition between commercial and social investors. De Angelis et al. (2020) show how companies' greenhouse gas emissions can be reduced through the increase in the cost of capital for those companies, wherein the cost of capital becomes more sensitive to emissions as the share of green investors and environmental stringency increase.

Our contribution to these streams of literature is through explicitly modeling firms' reporting of ESG activities as well as potential greenwashing. We show that in equilibrium, a firm's price is sensitive to its ESG report, which is an aggregate signal of a manager's effort and a part of the firm's ESG performance that is out of the manager's control. We analyze how price and its sensitivity to the report varies with report characteristics and show how this sensitivity in turn affects managers' real and reporting choices.

3 Model and equilibrium

The manager of a firm chooses two efforts, $\mathbf{e} = (e_1, e_2)$, each of which can affect both the firm's cash flow and the firm's ESG output.[3] In particular, we assume that the firm has per-share cash flows of $\tilde{x} = \bar{x} - \boldsymbol{\theta}^T \mathbf{e} + \tilde{\varepsilon}_x$ and per-share ESG of $\tilde{y} = \boldsymbol{\eta}^T \mathbf{e} + \tilde{\varepsilon}_y$, where $\tilde{\varepsilon}_x$ and $\tilde{\varepsilon}_y$ are independent, normally distributed random variables with means of 0 and variances of σ_x^2 and σ_y^2, respectively, and where $\boldsymbol{\theta} = (\theta_1, \theta_2) \in \Re^2$ and $\boldsymbol{\eta} = (\eta_1, \eta_2) \in \Re^2$ are constant vectors known by all actors. With $\eta_i > 0$, effort e_i that increases the expected ESG output decreases cash flows when $\theta_i > 0$ and increases cash flows when $\theta_i < 0$.

We assume that there is a continuum of investors with unit mass, and the supply of shares is fixed at 1. There is also a risk-free asset (money) with perfectly-elastic supply in which investors can borrow or lend.[4]

All investors value cash flows and are risk averse with respect to their cash holdings. A λ-fraction of investors also value corporate ESG. We assume that the ESG-concerned investors are risk-neutral with respect to ESG. Let q_i and l_i denote the amount of shares and risk-free money held by investor i. Denote type-1 investors as those who care only about cash flows. Their utility is $u_1 = -\exp\left[-\rho\left(q_1 \tilde{x} + l_1\right)\right]$. Type-2 investors, who also value the firm's ESG performance (in risk-neutral expectation), have utility defined by $u_2 = -\exp\left[-\rho\left(q_2 \tilde{x} + l_2\right) - \rho q_2 E\left[\tilde{y}|\Omega\right]\right] = -\exp\left[-\rho\left(q_2 \tilde{x} + l_2\right)\right] \exp\left[-\rho q_2 E\left[\tilde{y}|\Omega\right]\right]$. Ω is the information on which investors condition their expectations.

The firm's manager is interested in maximizing the firm's stock price and has a preference over her efforts. In particular, we model the manager's preference for the ESG efforts with the cost $\sum_{i \in \{1,2\}} \frac{c_e}{2} \left(e_i - \phi_i\right)^2$, where ϕ_1 and ϕ_2 are realizations of the random vector, $\tilde{\boldsymbol{\phi}} \sim N\left(\bar{\boldsymbol{\phi}}, \Sigma_\phi\right)$, privately observed by the manager, with $\bar{\boldsymbol{\phi}} = \left(\bar{\phi}_1, \bar{\phi}_2\right)$. Investors do not observe the manager's preferences, but $\bar{\phi}_i$, the manager's expected bliss action on effort e_i, and

[3] We choose two efforts as the minimal number needed to capture issues related to report congruence and effort allocation. The analysis extends straightforwardly to higher-dimension effort vectors.

[4] Note that our setup eliminates wealth effects, consistent with much of the extant literature on ESG in accounting and finance.

Σ_ϕ, the positive definite covariance matrix, are common knowledge. We assume all random variables, ε_x, ε_y, and $\boldsymbol{\phi}$, are independent.

Absent the ESG report, introduced below, the timeline is as follows: at $t = 0$, the manager privately observes $\boldsymbol{\phi} = (\phi_1, \phi_2)^T$ and chooses $\mathbf{e} = (e_1, e_2)^T$ to maximize $u_{m'} = p - \sum_{i\in\{1,2\}} \frac{c_e}{2} (e_i - \phi_i)^2$. At $t = 1$ investors trade in the shares and establish the stock price, p. At $t = 2$ cash flows are paid out and ESG performance is revealed. Proposition 1 summarizes the equilibrium efforts and price without the ESG report, which will be useful in our subsequent discussion of the effects of the ESG report.

Proposition 1 *In the equilibrium without the ESG report, the manager's efforts and the stock price are given by*

$$\mathbf{e}^\dagger = \boldsymbol{\phi} \text{ and} \tag{1}$$
$$p^\dagger = \bar{x} + \left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\bar{\boldsymbol{\phi}} - \rho\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta} + \sigma_x^2\right). \tag{2}$$

As Proposition 1 shows, the manager chooses efforts equal to her bliss points, $e_i = \phi_i$, $\forall i$. As a result, the expected efforts are given by their expected bliss points, $\bar{\phi}_i$. Because investors receive no additional information, the firm's price is given by expected cash flows, $E[x] = \bar{x} - \boldsymbol{\theta}^T\bar{\boldsymbol{\phi}}$, minus a risk premium for the uncertainty in cash flows, $Var[x] = \rho\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta} + \sigma_x^2\right)$, plus the preference-weighted expected ESG output, $\lambda E[y] = \lambda\boldsymbol{\eta}^T\bar{\boldsymbol{\phi}}$. These results follow prior literature where ESG output is random and a λ-fraction of investors has a risk-neutral warm glow from holding shares in a firm that provides this ESG output (e.g., Chowdhry et al., 2018; Pástor et al., 2020; Pedersen et al., 2020).

Note that we allow for a correlation in cash flows and ESG: an incremental unit of effort e_i increases ESG output by η_i and decreases cash flows by θ_i. As a result, when all investors know that the manager has taken an additional unit of effort e_i, price changes by $\lambda\eta_i - \theta_i$. In addition, because investors are risk averse with respect to cash flows, the uncertainty about the effort effect on cash flows increases the risk premium. That is, the risk premium arises in response to both the fundamental risk in cash flows, σ_x^2, as well as the variation in cash flows that arises from the manager's effort choice, $\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta}$.

3.1 ESG reporting

In the no-report setting above, investors receive no information about the manager's effort or the other random components in cash flows or ESG performance. In what follows, we extend the model to include a firm's report about its aggregate ESG performance. In particular, After observing $\boldsymbol{\phi}$ and choosing $\mathbf{e}$, the manager releases a report about the firm's ESG performance, y.

To analyze the effect of different reporting regimes and choices, we allow efforts and ε_y to have differential effects on the ESG report. Furthermore, to incorporate ESG-report biasing, an important facet of greenwashing, we assume that the report, r, need not be the manager's truthful expectation of y but can be biased. Specifically, we assume that the manager's cost of providing report r is $\frac{c_r}{2}\left(r - \boldsymbol{\zeta}^T\mathbf{e} - \nu\varepsilon_y - \varepsilon_r\right)^2$. Here ε_r is normally distributed and independent of the other random variables, with $\tilde{\varepsilon}_r \sim N\left(0, \sigma_r^2\right)$. As in Dye and Sridhar (2004, p. 56), ε_r "reflects idiosyncratic circumstances that influence the [manager's] misreporting costs" and prevents unraveling of the reporting bias effect in pricing, and could alternatively be incorporated via a mechanism as in Fischer and Verrecchia (2000) with uncertain incentives. The non-random reporting cost term, c_r, is a positive constant.

The $\boldsymbol{\zeta} = \left(\zeta_1, \zeta_2\right)^T$ term captures sensitivities of the report (and therefore reporting costs) to efforts, while ν captures the sensitivity of the report to the uncontrolled component of ESG, ε_y. For example, when $\varepsilon_r = 0$, the manager can avoid any reporting costs by choosing $r = \boldsymbol{\zeta}^T\mathbf{e} + \nu\varepsilon_y$. As such we can interpret $(\boldsymbol{\zeta}, \nu)$ as the weights on different elements defined in ESG reporting regulation or in a firm's ESG reporting policies. These reporting parameters allow us to capture whether the report reflects ESG inputs (i.e., efforts) or ESG outcomes (i.e., efforts and the uncontrollable ε_y) as well as two types of reporting incongruity. The first, which we might call cash flow incongruity, is the degree to which $\frac{\zeta_1}{\zeta_2} \neq \frac{\theta_1}{\theta_2}$. With cash flow incongruity, which relates to materiality considerations of fundamental investors, the ESG report fails to proportionately capture the relative impacts of e_1 and e_2 on expected cash flows, $E\left[x\right]$. The second, which we call ESG incongruity, is the degree to which $\frac{\zeta_1}{\zeta_2} \neq \frac{\eta_1}{\eta_2}$.

ESG incongruity implies that the ESG report, r, fails to proportionately capture the relative impacts of e_1 and e_2 on expected ESG, $E[y]$.

The manager's objective is $E[u_m]$, where u_m is stock price net of effort and reporting costs:

$$u_m = p - \frac{c_e}{2}(\mathbf{e} - \boldsymbol{\phi})^T(\mathbf{e} - \boldsymbol{\phi}) - \frac{c_r}{2}\left(r - \boldsymbol{\zeta}^T\mathbf{e} - \nu\varepsilon_y - \varepsilon_r\right)^2.$$

The timeline incorporating the ESG report is as follows: at $t = 0$ the manager privately observes $\boldsymbol{\phi} = (\phi_1, \phi_2)^T$ and chooses $\mathbf{e} = (e_1, e_2)^T$ to maximize $E[u_m|\boldsymbol{\phi}]$. At $t = 1$ the manager privately observes $\boldsymbol{\zeta}^T\mathbf{e} + \nu\varepsilon_y + \varepsilon_r$ and provides a report r to the market, after which investors trade in the shares and establish price p. The manager's choice of report is chosen to maximize $E\left[u_m|\boldsymbol{\zeta}^T\mathbf{e} + \nu\varepsilon_y + \varepsilon_r\right]$.[5] At $t = 2$ cash flows are paid out and ESG performance is revealed.

3.2 Equilibrium with ESG reporting

The following Proposition summarizes the equilibrium efforts, report, and price, as well as the effect of the ESG disclosure on price by comparing with $p^\dagger$ derived in Proposition 1.

Proposition 2 *In the equilibrium with disclosure, the manager's efforts, the disclosed report, the stock market price and the price difference relative to the benchmark with no ESG*

[5] We assume that the manager cannot credibly provide additional information to investors.

report are given by

$$\mathbf{e}^* = \boldsymbol{\phi}+\frac{\psi}{c_e}\boldsymbol{\zeta}, \tag{3}$$

$$r^* = \frac{\psi}{c_r}+\boldsymbol{\zeta}^T\mathbf{e}^*+\nu\varepsilon_y+\varepsilon_r, \text{ and} \tag{4}$$

$$\begin{aligned} p^* &= \bar{x}+\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)\left(\frac{\psi}{c_e}\boldsymbol{\zeta}+\bar{\boldsymbol{\phi}}\right)+\psi\left(r^*-\left(\frac{\psi}{c_r}+\frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta}+\boldsymbol{\zeta}^T\bar{\boldsymbol{\phi}}\right)\right) \\ &\quad -\rho\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta}+\sigma_x^2-\frac{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2}\right), \end{aligned} \tag{5}$$

$$\begin{aligned} p^*-p^\dagger &= \left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)\frac{\psi}{c_e}\boldsymbol{\zeta}+\psi\left(\boldsymbol{\zeta}^T\left(\boldsymbol{\phi}-\bar{\boldsymbol{\phi}}\right)+\nu\varepsilon_y+\varepsilon_r\right) \\ &\quad +\rho\frac{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2} \end{aligned} \tag{6}$$

where $\psi=\frac{dp^*}{dr}=\frac{(\lambda\boldsymbol{\eta}-\boldsymbol{\theta})^T\Sigma_\phi\boldsymbol{\zeta}+\lambda\nu\sigma_y^2}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2}$.

Proposition 2 summarizes the equilibrium in the game with an ESG disclosure and compares the price to the setting without an ESG disclosure. The equilibrium price in (5) and the price difference in (6) show that the disclosure of the ESG report to the market has three effects on the price of the firm. First, there is a real effect through the manager's effort incentives. This effect is given by the first term in (6), $\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)\frac{\psi}{c_e}\boldsymbol{\zeta}$. Second, investors revise their beliefs about the firm's cash flows and ESG outcomes, which manifests in a price reaction to the report. This effect is given by the second term in (6), $\psi\left(r^*-\left(\frac{\psi}{c_r}+\frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta}+\boldsymbol{\zeta}^T\bar{\boldsymbol{\phi}}\right)\right)=\psi\left(r^*-E\left[r^*\right]\right)=\psi\left(\boldsymbol{\zeta}^T\left(\boldsymbol{\phi}-\bar{\boldsymbol{\phi}}\right)+\nu\varepsilon_y+\varepsilon_r\right)$. Note that the market only observes a measure that aggregates three types of random variables: the effects of the manager's efforts, $\boldsymbol{\phi}$; the firm's ESG performance outside the manager's control, ε_y; and the reporting incentives, ε_r. While the first two are relevant to investors, the third uniformly reduces the amount of information that investors can glean about cash flows and ESG performance. Finally, investors learn about the manager's effort choices, which reduces their uncertainty about the firm's cash flows and lowers the risk premium impounded into stock price. This effect can be seen in the last term in (6), $\rho\frac{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2}$. Note that investors are risk averse with respect to cash flows only, so the reduction in the risk premium comes from learning about the manager's efforts and their effects on cash flows, rather than from the ESG performance beyond the manager's control, ε_y.[6]

3.3 Greenwashing

A fourth implication of Proposition 2 is that when investors receive and respond to an ESG report, the manager has incentives to alter the report in order to change investors' perceptions about the firm. We refer to this behavior as greenwashing. Prior literature has provided multiple definitions for greenwashing. Delmas and Burbano (2011, p. 65) define greenwashing as "the intersection of two firm behaviors: poor environmental performance and positive communication about environmental performance." Implicit in their definition is a divergence between performance and communication. Lyon and Maxwell (2011, p. 9) capture greenwashing in a selective disclosure model, and define greenwashing as "selective disclosure of positive information about a company's environmental or social performance, without full disclosure of negative information on these dimensions, so as to create an overly positive corporate image." In our model, selective disclosure could be related to the properties of the report $(\boldsymbol{\zeta}, \nu)$ when some actions or some random outcomes are not measured. However, our model does not have a discretionary component per se where the firm or manager observe performance before deciding whether to disclose it. It is reasonably straightforward to interpret variation in $\boldsymbol{\zeta}$, ν, or ε_r as outcomes of an unmodeled partial disclosure subgame insofar as they reduce market participants' ability to infer y from r.

Following Delmas and Burbano (2011), total greenwashing in our setting is the difference between the report, r^*, and the ESG performance, y^*:

$$\begin{aligned} G^* &= r^* - y^* = \left(\frac{\psi}{c_r} + \boldsymbol{\zeta}^T \mathbf{e}^* + \nu \varepsilon_y + \varepsilon_r \right) - \boldsymbol{\eta}^T \mathbf{e}^* - \varepsilon_y \\ &= \left(\boldsymbol{\zeta} - \boldsymbol{\eta} \right)^T \mathbf{e}^* + (\nu - 1)\, \varepsilon_y + \frac{\psi}{c_r} + \varepsilon_r. \end{aligned} \tag{7}$$

[6]See Friedman and Heinle (2016) for a setting where investors are also risk averse with respect to ESG performance. This creates a second risk-premium term, similar to that for cash-flow risk but multiplied by λ.

The total greenwashing in (7) can be decomposed into ex-ante components (associated with the design of the report) and ex-post components (associated with equilibrium efforts and reporting bias). There are two ex-ante components. The first is the degree to which the sensitivity of the report to efforts differs from the sensitivity of ESG to efforts, which reflects ESG incongruence. This is captured by $(\boldsymbol{\zeta} - \boldsymbol{\eta})^T$ multiplying $\mathbf{e}^*$ in the first term. The second is the degree to which the report captures variation in ESG performance outside the manager's control, ε_y. This is reflected in the $(\nu - 1)$ term multiplying ε_y. In our model, the term $(\nu - 1)$ allows us to vary whether the report captures efforts/inputs (i.e., efforts such as amounts spent on carbon mitigation, $\nu = 0$) or performance/outcomes (i.e., actual emissions as reflected in y, $\nu = 1$).

There are also two ex-post components of greenwashing in (7). These two are related to the manager's equilibrium choices regarding reporting and efforts. First, reporting greenwashing, captured by $\frac{\psi}{c_r} + \varepsilon_r$, captures greenwashing driven by the manager's reporting choice, r. Reporting greenwashing depends on the price responsiveness to the report, ψ, the non-random cost of reporting bias, c_r, and the idiosyncratic component of the manager's reporting objective, ε_r. In the Dye and Sridhar (2004) framework, which we adopt, the random reporting incentives ε_r become similar to measurement noise from the perspective of investors. In equilibrium, the manager places a weight of 1 on the random term and simply adds it to the report. The expected reporting greenwashing is given by $\frac{\psi}{c_r}$, which depends on ψ and c_r, but is unwound by investors in equilibrium.

Second, real greenwashing is based on the manager's efforts, $\mathbf{e}^*$. In addition to the bliss point, $\boldsymbol{\phi}$, the manager also considers the marginal impact of effort on price through the disclosed report. In particular, the $\frac{\psi}{c_e}\boldsymbol{\zeta}$ term in (3) incorporates how price changes with the report, ψ, the sensitivity of the report to efforts, $\boldsymbol{\zeta}$, and effort costs, c_e. These jointly determine the manager's deviation from the privately observed bliss efforts, $\boldsymbol{\phi}$. For instance, when the report increases in e_1 but decreases in e_2, i.e., $\zeta_1 > 0 > \zeta_2$, and stock price increases in the report, i.e., $\psi > 0$, the manager will choose higher e_1 and lower e_2 relative to the equilibrium without reporting. This real greenwashing arises because the manager increases ESG efforts that are measured in disclosed reports. As an example, consider a report that captures a firm's carbon emissions but omits information about the production and release of hazardous chemicals such as PFAS (polyfluoroalkyl substances). In this situation, real greenwashing occurs when the manager shifts her efforts towards the reduction of carbon emissions and away from avoiding hazardous chemical releases.

4 Analysis

In this section, we further analyze the four effects that ESG disclosure has on prices. In our model, there are two sets of parameters that affect the equilibrium outcomes. The first set of parameters relates to the signal-to-noise ratio in the ESG report: the amount of variation in the report that is due to the manager's effort, $|\boldsymbol{\zeta}|$; the variance of the reporting noise, σ_r^2; the variance of the exogenous ESG variation, σ_y^2; and whether exogenous ESG variation is measured in the report at all, ν. The second set of parameters relates to the degree to which the report captures each effort, ζ_1 and ζ_2, which determine the congruence of the report with cash flows and ESG output. Because each comparative static is affected by all the other parameters, we conduct our analysis in two steps. First, we isolate the effects of the parameters that affect the signal-to-noise ratio in a setting with a single effort. Then we isolate the effects of congruency in a setting with two efforts but without noise in the report. In both settings, we investigate the price response to the disclosure of the ESG report, the real effect of the ESG report, the effect of disclosure on the expected returns, and the effect on greenwashing.

4.1 Single-effort setting

This section limits the manager's choice set to a single effort, $\mathbf{e} = e$, which allows for scalar representations that focus on the implications of investor preferences, effort costs, and ESG-related sources of uncertainty. To examine the equilibrium, we substitute $\boldsymbol{\eta} = \eta$, $\boldsymbol{\theta} = \theta$, $\boldsymbol{\zeta} = \zeta$, $\Sigma_\phi = \sigma_\phi^2$, $\boldsymbol{\phi} = \phi$, and $\bar{\boldsymbol{\phi}} = \bar{\phi}$ into (3)-(5) and denote the solutions as e_I^*, r_I^*, and p_I^*, where $\psi_I = \frac{dp^*}{dr} = \frac{(\lambda\eta-\theta)\sigma_\phi^2\zeta+\lambda\nu\sigma_y^2}{\sigma_\phi^2\zeta^2+\nu^2\sigma_y^2+\sigma_r^2}$. We present the respective solutions in the appendix.

Corollary 1 *When ESG efforts are one-dimensional, and $\zeta, \eta, \psi_I > 0$, the price response to the report (ψ_I) increases in the fraction of ESG-concerned investors (λ), the sensitivity of ESG to managerial effort (η), and the sensitivity of cash flows to managerial effort ($-\theta$). Furthermore, ψ_I decreases in manager's reporting cost variance, σ_r^2. Finally, ψ_I increases in the uncontrollable ESG risk, σ_y^2, iff*

$$\nu \neq 0 \text{ and } \lambda\left(\zeta^2\sigma_\phi^2 + \sigma_r^2\right) > \zeta\nu\sigma_\phi^2\left(\lambda\eta - \theta\right); \tag{8}$$

increases in the weight on the uncontrollable ESG risk, ν, iff

$$\lambda\left(\zeta^2\sigma_\phi^2 + \sigma_r^2 - \nu^2\sigma_y^2\right) > 2\zeta\nu\sigma_\phi^2\left(\lambda\eta - \theta\right); \tag{9}$$

increases in the sensitivity of the report to managerial effort, ζ, iff

$$\left(\lambda\eta - \theta\right)\left(\nu^2\sigma_y^2 + \sigma_r^2\right) - 2\lambda\nu\sigma_y^2\zeta > \sigma_\phi^2\zeta^2\left(\lambda\eta - \theta\right); \tag{10}$$

and increases in the effort uncertainty, σ_ϕ^2, iff

$$\left(\lambda\eta - \theta\right)\left(\nu^2\sigma_y^2 + \sigma_r^2\right) > \zeta\lambda\nu\sigma_y^2. \tag{11}$$

Corollary 1 shows that when the manager's effort is more important to investors (higher values of λ, η, and $-\theta$) the price response to the report is higher. Similarly, more noise in the report induced by reporting bias, ε_r, reduces the response coefficient. These results follow from the effect of the respective parameters on the signal-to-noise ratio. The effects of the other parameters are more subtle.

First, while ESG interested investors learn about the uncontrollable ESG performance from the report, this random performance is noise from the perspective of an investor who is interested only in cash flows. Which of these two forces (i.e., learning/signal versus noise) dominates depends on parameter values and determines the effects of the amount of uncontrollable ESG performance risk, σ_y^2, and the weight of that risk in the report, ν. For example, when $\lambda = 0$ then the price impact decreases in both σ_y^2 and ν because no investor is interested in ε_y and, thus, all investors treat it as a noise term. When $\lambda\eta = \theta$, the manager's effort does not affect the performance that the average investor cares about. In this situation, an increase in σ_y^2 or ν (for low ν) increases the price impact because the ESG investor still tries to infer ε_y from the report.

The report's sensitivity to the managerial effort and the effort uncertainty, ζ and σ_ϕ^2, are similar to the reporting weight, ν, and the variance of the uncontrollable ESG risk, σ_y^2. Investors, in aggregate, use the report to learn about the manager's effort, which is unknown ex-ante due to uncertainty about her bliss action, ϕ, as well as the uncontrollable portion of the ESG outcome, ε_y. An increase in the sensitivity of the report to the manager's effort makes the report more reflective of effort but less reflective of ε_y. If the ESG and cash flow effects of effort offset, i.e., $(\lambda\eta - \theta) \approx 0$, then the condition in (10) will tend to be negative precisely because higher ζ makes the report less informative about ε_y, which is valued by the λ portion of type-2 investors. When the effect of effort on values-weighted outcomes is large, i.e., $(\lambda\eta - \theta) >> 0$, then learning about effort is important. However, $\frac{d\psi_I}{d\zeta}$ can still be negative, particularly if $\sigma_\phi^2\zeta^2$ is large relative to $\nu^2\sigma_y^2 + \sigma_r^2$. This latter effect occurs because the variance of r is quadratic in ζ (with coefficient σ_ϕ^2), while the use of r to learn about effort is driven by the covariance between efforts and expected outcomes ($E[x]$ and $E[y]$), which is linear in ζ.

The following corollary investigates the real effect of changes in the exogenous parameters on managerial effort.

Corollary 2 *The effects in Corollary 1 that increase ψ_I also increase the manager's expected effort ($E[e_I^*]$) with the exception of an increase in ζ. An increase in ζ increases the manager's expected effort when*

$$\lambda\nu\sigma_y^2 + 2\zeta\sigma_\phi^2(\lambda\eta - \theta) > \zeta^2\sigma_\phi^2\lambda\frac{\nu\sigma_y^2}{\sigma_r^2 + \nu^2\sigma_y^2} \tag{12}$$

Generally speaking, the effect of the manager's effort on price depends on the price response coefficient ψ_I. When a shift in an exogenous parameter increases the price response, the manager has stronger incentives to provide effort and thus provides more. However, this intuition can be violated when the manager's impact on the report, ζ, increases. An increase in ζ, assuming $\psi_I > 0$, directly increases effort incentives because the report increases more for every unit of effort. However, as shown in Corollary 1, an increase in ζ can cause a decrease in ψ_I. This effect can dominate the direct effect of an increase in ζ such that a stronger reporting sensitivity to the manager's effort can lead to a lower effort provision. As the expression in (12) shows, when the report measures ESG inputs rather than outputs ($\nu = 0$) an increase in ζ always increases the price response coefficient and, thus, leads to an increase in the manager's effort. However, when the report measures outputs and when investors' are sufficiently interested in the random ESG variation (i.e., $\lambda\sigma_y^2 >> 0$), then increasing the impact of the manager's effort in the report reduces the price response coefficient ψ_I, and can reduce the manager's effort.

Our further analysis in this subsection relates to the expected return around the disclosure of an ESG report. Because investors have rational expectations, the expected report only affects price through the risk premium. Because investors learn about the manager's choices, they face less uncertainty about cash flows and, thus, demand a lower risk premium. In particular, the disclosure reduces the risk premium by $\rho\frac{\zeta^2\theta^2\sigma_\phi^4}{\zeta^2\sigma_\phi^2+\nu^2\sigma_y^2+\sigma_r^2}$, which is the single-effort equivalent of the last term in (6). The following corollary analyzes the expected return around the ESG disclosure.

Corollary 3 *When ESG efforts are one-dimensional and $\zeta, \eta, \psi_I > 0$, the expected return around the ESG disclosure is increasing in ζ, θ, and σ_ϕ^2 and is decreasing in ν, σ_y^2, and σ_r^2.*

Corollary 3 highlights the conflict in learning about cash flows and ESG performance. Because investors in our model are risk neutral with respect to ESG performance, the expected return increases when investors learn more about cash flows. In particular, a higher reporting sensitivity to the effort, a higher productivity of the effort, and more ex-ante uncertainty about the effort increase the expected return. Variation in the report that is due to factors that do not affect cash flows, on the other hand, reduces the expected return.

Finally, we investigate the impact of parameter changes on equilibrium greenwashing. In the single-effort setting the expected total greenwashing (based on the definition in 7) is given by

$$E\left[G_I^*\right] = E\left[r_I^* - y_I^*\right] = (\zeta - \eta)\, E\left[e_I^*\right] + \frac{\psi_I}{c_r} \tag{13}$$

The first term, $(\zeta - \eta)\, E\left[e_I^*\right]$, measures the amount of expected real greenwashing. In the single-effort setting, the manager cannot shift efforts towards measured components. In other words, there is no real greenwashing driven by cross-effort allocation. However, the ESG report can present the firm as overly green when $\zeta > \eta$. The second term reflects the manager's reporting bias, $\frac{\psi_I}{c_r}$, in expectation. Because the price impact of the ESG report is independent of the reporting cost, all comparative statics from Corollary 1 that increase ψ also increase the expected reporting-related greenwashing in (13). This greenwashing, however, is anticipated by investors in equilibrium and unwound from the report.

4.2 Two-effort setting

In this section, we go back to the baseline model in which the manager takes two actions. However, to eliminate the effect of noise in the report (from the perspective of the investors), we assume that $\sigma_y^2 = \sigma_r^2 = 0$. In particular, when $\sigma_r^2 = 0$, there is no uncertainty about the manager's bias. This allows investors to unravel the manager's reporting strategy to infer the linear combination of efforts implied by $\boldsymbol{\zeta}^T\mathbf{e}$. Furthermore, when $\sigma_y^2 = 0$, ESG outcomes are determined entirely by the manager's efforts. Consequently, the report only captures variation in ESG that is associated, via managerial efforts, with cash flows (for $\theta_i \neq 0$). As a result, the parameter ν has no impact on the equilibrium. In sum, we set $\sigma_y^2 = \sigma_r^2 = 0$ to focus on the effect of congruence in the reporting system.

Below, we examine three potential types of reports: cash-flow congruent ($\boldsymbol{\zeta} = -\boldsymbol{\theta}$), ESG congruent ($\boldsymbol{\zeta} = \boldsymbol{\eta}$), and values congruent ($\boldsymbol{\zeta} = \lambda\boldsymbol{\eta} - \boldsymbol{\theta}$). We view these as relevant from the perspective of various standard setters, regulators, firms, and investors. Our approach focuses on intuitive types of congruence rather than optimal congruence derived conditional on a particular objective function, mainly because different parties will have different objectives regarding ESG reporting. Nonetheless, comparisons across report types that differ in $\boldsymbol{\zeta}$ is informative with regard to tradeoffs across outcomes that heterogeneous parties will evaluate differently (e.g., cash flows versus ESG performance versus what is valued by a representative investor). To avoid trivial situations in which the three types of reports are equivalent, we assume throughout that cash-flow and ESG impacts are not perfectly aligned, i.e., there is no constant κ such that $\boldsymbol{\eta} = \kappa\boldsymbol{\theta}$.

4.2.1 Comparing cash-flow congruence and ESG congruence

When $\sigma_y^2 = \sigma_r^2 = 0$ and the report is cash-flow congruent, $\boldsymbol{\zeta} = -\boldsymbol{\theta}$, the equilibrium is given by

$$\mathbf{e}_\theta^* = \boldsymbol{\phi} - \frac{\psi_\theta^*}{c_e}\boldsymbol{\theta},\ r_\theta^* = \frac{\psi_\theta^*}{c_r} - \boldsymbol{\theta}^T\mathbf{e}_\theta^*, \tag{14}$$

$$\psi_\theta^* = 1 - \lambda\frac{\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta}}, \text{ and} \tag{15}$$

$$p_\theta^* = \bar{x} + \left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\left(-\frac{1}{c_e}\psi_\theta^*\boldsymbol{\theta} + \bar{\boldsymbol{\phi}}\right) - \psi_\theta^*\boldsymbol{\theta}^T\left(\boldsymbol{\phi} - \bar{\boldsymbol{\phi}}\right) - \rho\sigma_x^2. \tag{16}$$

The last term in equation (16) is the risk premium based only on $\rho\sigma_x^2$. The risk premium term shows that when there is no measurement noise and the report aggregates the efforts in the same way that they affect cash flows, investors can perfectly infer any variation in cash flows that stems from the manager's efforts. However, equation (15) suggests that investors cannot perfectly infer the impact of the manager's efforts on the ESG output. In particular, the market response coefficient in (15) is equal to 1, for the cash-flow relevant news, minus a term, $\lambda\frac{\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta}}$, that shows that ESG interested investors are not able to infer all ESG variation because the efforts are not aggregated for that purpose. The numerator in this term captures incongruence between ESG and cash flow implications via $\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\theta}$, weighted by the $\boldsymbol{\phi}$ covariance matrix.

When $\sigma_y^2 = \sigma_r^2 = 0$ and the report is ESG congruent, $\boldsymbol{\zeta} = \boldsymbol{\eta}$, the equilibrium is given by

$$\mathbf{e}_\eta^* = \boldsymbol{\phi} + \frac{\psi_\eta^*}{c_e}\boldsymbol{\eta},\ r_\eta^* = \frac{\psi_\eta^*}{c_r} + \boldsymbol{\eta}^T \mathbf{e}_\eta^*, \tag{17}$$

$$\psi_\eta^* = \lambda - \frac{\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\eta}}{\boldsymbol{\eta}^T \Sigma_\phi \boldsymbol{\eta}}, \text{ and} \tag{18}$$

$$p_\eta^* = \bar{x} + \left(\lambda \boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\left(\frac{\psi_\eta^*}{c_e}\boldsymbol{\eta} + \bar{\boldsymbol{\phi}}\right) + \psi_\eta^* \boldsymbol{\eta}^T \left(\boldsymbol{\phi} - \bar{\boldsymbol{\phi}}\right) - \rho\left(\sigma_x^2 + \boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta} - \frac{\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\eta}\boldsymbol{\eta}^T \Sigma_\phi \boldsymbol{\theta}}{\boldsymbol{\eta}^T \Sigma_\phi \boldsymbol{\eta}}\right) \tag{19}$$

Here, the risk premium in (19) shows that when the efforts are aggregated according to their influence on the firm's ESG output, investors are not able to infer the entire variation in cash flows that stems from the manager's efforts. We can write the risk premium as

$$\rho\left(\sigma_x^2 + \boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta} - \frac{\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\eta}\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\eta}}{\boldsymbol{\eta}^T \Sigma_\phi \boldsymbol{\eta}}\right) = \rho\left(\sigma_x^2 + \frac{\boldsymbol{\theta}^T \Sigma_\phi \left(\boldsymbol{\theta}\boldsymbol{\eta}^T - \boldsymbol{\eta}\boldsymbol{\theta}^T\right)\Sigma_\phi \boldsymbol{\eta}}{\boldsymbol{\eta}^T \Sigma_\phi \boldsymbol{\eta}}\right)$$

where $\left(\boldsymbol{\theta}\boldsymbol{\eta}^T - \boldsymbol{\eta}\boldsymbol{\theta}^T\right)$ captures the incongruence between ESG and cash flow effects, and goes to zero as $\boldsymbol{\eta} \to \kappa\boldsymbol{\theta}$, for some scalar κ. In contrast to the response coefficient ψ_θ^* in (15), ψ_η^* in (18) is equal to λ (capturing the fraction of investors who value the ESG relevant information in the report) minus a term, $\frac{\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\eta}}{\boldsymbol{\eta}^T \Sigma_\phi \boldsymbol{\eta}}$, that shows investors' inability to perfectly infer cash flow effects from the ESG-congruent report. As above, this term captures incongruence between ESG and cash flow implications, weighted by the $\boldsymbol{\phi}$ covariance matrix, via $\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\eta}$.

The following corollary compares the response coefficients across the two regimes, which allows us to discuss differences in expected efforts and bias.

Corollary 4 *When $\sigma_y^2 = \sigma_r^2 = 0$ and $\Sigma_\phi = \sigma_\phi^2 \mathbf{I}$, (i) the response coefficient for an ESG-congruent report is only higher than that for a cash-flow congruent report when the manager's effort has a greater effect on the firm's ESG output than on the firm's cash flows, i.e., $\psi_\eta^* > \psi_\theta^*$ iff $\lambda\left(1 + \frac{\theta_1\eta_1 + \theta_2\eta_2}{\theta_1^2 + \theta_2^2}\right) > 1 + \frac{\theta_1\eta_1 + \theta_2\eta_2}{\eta_1^2 + \eta_2^2}$; (ii) the firm's expected ESG output (the firm's expected cash flow) is weakly higher (weakly lower) for an ESG-congruent report than for a cash-flow congruent report, i.e., $\boldsymbol{\eta}^T\left(\mathbf{e}_\eta^* - \mathbf{e}_\theta^*\right) \geq 0$ and $-\boldsymbol{\theta}^T\left(\mathbf{e}_\eta^* - \mathbf{e}_\theta^*\right) \leq 0$; and (iii) expected price is higher for an ESG-congruent report than for a cash-flow congruent report if and only if*

$\lambda^2 > \frac{\theta_1^2+\theta_2^2}{\eta_1^2+\eta_2^2}\left(1 + c_e\rho\sigma_\phi^2\right)$.

In Corollary 4, part (i) shows that because all investors care about cash flows, the market response to a report that focuses on cash-flow implications tends to be higher. The market response to a report that focuses on ESG outputs is higher if the efforts that show up in the report have a stronger impact on the ESG outputs. That is, in a scenario without reporting noise, the response coefficient is largely dictated by three components: how much there is to learn about each effort (σ_ϕ^2); how congruent the report is to the interest of the respective investor; and how important the respective task is. In turn, this implies that when the vectors $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ are of equal length ($|\boldsymbol{\theta}| = |\boldsymbol{\eta}|$), then $\sigma_y^2 = \sigma_r^2 = 0$ and $\Sigma_\phi = \sigma_\phi^2\mathbf{I}$ imply that the market response is stronger to a cash-flow oriented ESG report.

Part (ii) of Corollary 4 effectively echoes the adage "you get what you measure." In particular, when the response coefficient is positive, the manager has incentives to provide a higher aggregate report. This implies that when the report is congruent to the firm's ESG output, then the manager has incentives to allocate more effort to a task that has a higher impact on the ESG output. ESG output is the same in both reporting systems when $\lambda = 0$. That is, when ESG is priced only for its effects on cash flows, different reporting systems do not yield different expected ESG outcomes. Furthermore, ESG and cash flow converge under both reporting systems as $\frac{\theta_1}{\theta_2} \to \frac{\eta_1}{\eta_2}$, that is, when the relative sensitivities of cash flows and ESG to the manager's efforts converge or, alternatively, when $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ become congruent such that $\boldsymbol{\theta} \to \kappa\boldsymbol{\eta}$ for a scalar κ.

Finally, part (iii) shows that the firm's expected stock price is higher under an ESG-congruent reporting system when the fraction of investors that value ESG is sufficiently large. Corollary 4 part (iii) also shows that the threshold for λ increases in the risk aversion and the variance of the effort incentives. As we discuss above, investors in our model are risk averse with respect to cash flows, but risk neutral with respect to ESG. For this reason, the risk premium embedded in price is lower when the report is cash-flow congruent, as investors can use the cash-flow congruent report to infer the effort-induced variation in cash flows. A lower investor risk aversion (or effort uncertainty), lowers the price benefit of a cash-flow congruent report. Furthermore, part (iii) shows that the threshold for λ increases in the length of the vector $\boldsymbol{\theta}$ but decreases in the length of the vector $\boldsymbol{\eta}$. When cash flow is relatively more sensitive to the efforts measured in the ESG report (i.e., a high value for $\theta_1^2 + \theta_2^2 = \boldsymbol{\theta}^T\boldsymbol{\theta} = |\boldsymbol{\theta}|^2$), a cash-flow congruent report has a more beneficial effect on cash flows, which makes it more likely that a cash-flow congruent reporting system leads to higher expected prices.

When considering ESG reporting options, Corollary 4 (ii) implies that when the goal is to motivate stronger ESG performance, the report should focus directly on the ESG outcome and should define materiality (i.e., what should be included in the report) in terms of the ESG outcome, rather than in terms of the firm's cash flows. Put another way, ex ante ESG congruence discourages ex post real greenwashing. However, disclosure standards that are ex ante ESG congruent will come at a cost to cash flows. From part (iii) of Corollary 4, managers who seek to maximize expected stock price may nevertheless advocate for ESG-congruent reporting standards when there are enough ESG sensitive investors in the market.

The analysis of the expected ESG performance, y, also relates to real greenwashing to the extent that the manager shifts efforts towards measured activities. When the report is ESG congruent, shifting effort towards measured activities is beneficial to the firm's ESG performance, which leads to the above results. Expected greenwashing, from (7), also reflects reporting bias for both cash flow- an ESG-congruent reports, which are

$$E\left[G_\theta^*\right] = -\left(\boldsymbol{\theta} + \boldsymbol{\eta}\right)^T E\left[\mathbf{e}_\theta^*\right] + \frac{\psi_\theta}{c_r} \text{ and } E\left[G_\eta^*\right] = \frac{\psi_\eta}{c_r}, \tag{20}$$

respectively. The expressions in (20) show that the effects on ψ_θ in Corollary 4 (i) carry over to the comparison of the reporting bias for the two reporting systems, assuming that c_r are the same. However, total expected greenwashing also differs between the two reporting systems due to the real greenwashing that occurs in the cash flow-congruent system while being absent from the ESG-congruent system. ESG-congruent reporting eliminates the misrepresentation of the manager's impact on the firm's ESG performance. Thus, total expected greenwashing can be higher under cash flow-congruent reporting than ESG-congruent reporting even when prices are more responsive to ESG-congruent reports (i.e., when $\psi_\eta > \psi_\theta$ and $-(\boldsymbol{\theta}+\boldsymbol{\eta})^T E[\mathbf{e}^*_\theta] > \frac{\psi_\eta-\psi_\theta}{c_r}$). Conversely, total expected greenwashing can be lower under cash flow-congruent reporting than ESG-congruent reporting even when prices are more responsive to cash flow congruent reports (i.e., when $\psi_\theta > \psi_\eta$ and $(\boldsymbol{\theta}+\boldsymbol{\eta})^T E[\mathbf{e}^*_\theta] > \frac{\psi_\theta-\psi_\eta}{c_r}$). Price responses alone, therefore, are not sufficient statistics for total greenwashing, despite their ability to inform about reporting-related greenwashing.

4.2.2 Values congruence

The discussion so far shows that congruence of the report to the firm's ESG output and to cash flows come with different costs. However, both of these reporting systems are incongruent to the interests of an average, representative investor who values efforts according to $\lambda\boldsymbol{\eta}-\boldsymbol{\theta}$. In this section, we investigate a values-congruent reporting system ($\boldsymbol{\zeta}=\lambda\boldsymbol{\eta}-\boldsymbol{\theta}$), maintaining our assumptions of $\sigma_y^2=\sigma_r^2=0$ and $\Sigma_\phi=\sigma_\phi^2\mathbf{I}$, to focus on congruity and effort-sensitivity issues. Here, the equilibrium is given by

$$\mathbf{e}^*_V = \boldsymbol{\phi}+\frac{1}{c_e}(\lambda\boldsymbol{\eta}-\boldsymbol{\theta}),\; r^*_V=\frac{1}{c_r}+(\lambda\boldsymbol{\eta}-\boldsymbol{\theta})^T\mathbf{e}^*, \tag{21}$$

$$\psi^*_V = \frac{dp^*}{dr}=1, \text{ and} \tag{22}$$

$$\begin{aligned} p^*_V &= \bar{x}+(\lambda\boldsymbol{\eta}-\boldsymbol{\theta})^T\left(\frac{1}{c_e}(\lambda\boldsymbol{\eta}-\boldsymbol{\theta})+\bar{\boldsymbol{\phi}}\right)+(\lambda\boldsymbol{\eta}-\boldsymbol{\theta})^T(\boldsymbol{\phi}-\bar{\boldsymbol{\phi}}) \\ &\quad -\rho\left(\sigma_x^2+\sigma_\phi^2\frac{\lambda^2(\theta_1\eta_2-\theta_2\eta_1)^2}{(\lambda\eta_1-\theta_1)^2+(\lambda\eta_2-\theta_2)^2}\right). \end{aligned} \tag{23}$$

Because the report is not congruent with cash flows (for $\lambda>0$ and $\theta_1\eta_2\neq\theta_2\eta_1$), the report does not eliminate investors' uncertainty about the effects of effort on cash flows, leaving a higher risk premium in (23) than with cash-flow congruent reporting in (16). However, a values congruent report leads to an even higher risk premium than an ESG-congruent report when $2\lambda > \frac{\theta_1^2+\theta_2^2}{\theta_1\eta_1+\theta_2\eta_2}$. Notice that when one effort increases cash flows ($\theta_i < 0$) and the other effort decreases cash flows ($\theta_{-i} > 0$), while both efforts increase ESG output ($\eta_1, \eta_2 > 0$) the values congruent report will receive most of its variation from the effort that increases cash flows. As a result, investors will not be able to learn about the effort that decreases cash flows and may, thus, end up with a higher posterior uncertainty.

The following corollary compares outcomes for the values-congruent reporting system with those from the cash-flow and ESG-congruent reporting systems.

Corollary 5 *When $\sigma_y^2 = \sigma_r^2 = 0$ and $\Sigma_\phi = \sigma_\phi^2\mathbf{I}$, a values-congruent report yields: (i) weakly higher expected ESG output and cash flows than either an ESG-congruent or a cash-flow-congruent report, i.e., $\boldsymbol{\eta}^T\left(\mathbf{e}_V^* - \mathbf{e}_\eta^*\right) = 0$, $\boldsymbol{\eta}^T\left(\mathbf{e}_V^* - \mathbf{e}_\theta^*\right) \geq 0$, $-\boldsymbol{\theta}^T\left(\mathbf{e}_V^* - \mathbf{e}_\eta^*\right) \geq 0$, and $-\boldsymbol{\theta}^T\left(\mathbf{e}_V^* - \mathbf{e}_\theta^*\right) = 0$; (ii) a higher price than an ESG-congruent report when $\frac{1}{c_e}\frac{(\theta_1\eta_2-\theta_2\eta_1)^2}{\eta_1^2+\eta_2^2} - \rho\sigma_\phi^2\left(\frac{\lambda^2(\theta_1\eta_2-\theta_2\eta_1)^2}{(\lambda\eta_1-\theta_1)^2+(\lambda\eta_2-\theta_2)^2} - \frac{(\theta_1\eta_2-\theta_2\eta_1)^2}{\eta_1^2+\eta_2^2}\right) > 0$; and (iii) a higher price than a cash-flow-congruent report when $\frac{1}{c_e}\lambda^2\frac{(\theta_1\eta_2-\theta_2\eta_1)^2}{\theta_1^2+\theta_2^2} - \rho\sigma_\phi^2\left(\frac{\lambda^2(\theta_1\eta_2-\theta_2\eta_1)^2}{(\lambda\eta_1-\theta_1)^2+(\lambda\eta_2-\theta_2)^2}\right) > 0$.*

Corollary 5 (i) shows the strong result that ESG output and cash flows are at least as high with the values congruent report than with either of the other two reports. Parts (ii) and (iii) illustrate tradeoffs related to the reports' effects on the risk premium versus their real effects on efforts. In particular, when the risk aversion and the effort uncertainty are sufficiently large (such that the conditions in the corollary are violated) and when the condition $2\lambda > \frac{\theta_1^2+\theta_2^2}{\theta_1\eta_1+\theta_2\eta_2}$ holds, then the values congruent report leads to lower prices due to the loss of cash-flow relevant information.

Expected greenwashing in the setting with a values-congruent report is given by

$$E\left[G_V^*\right] = -\left(\boldsymbol{\theta} + (1-\lambda)\,\boldsymbol{\eta}\right)^T E\left[\mathbf{e}_V^*\right] + \frac{1}{c_r}. \tag{24}$$

Clearly, values-congruent reports, while encouraging higher levels of expected ESG performance and cash flows, do not eliminate expected greenwashing. Presence of real greenwashing may increase expected greenwashing in a values-congruent report compared to an

ESG-congruent report. However, if price response to the ESG-congruent report is small enough $\left(-(\boldsymbol{\theta}+(1-\lambda)\boldsymbol{\eta})^T E[e^*] > \frac{\psi_{\boldsymbol{\eta}}-1}{c_r}\right)$, a report aligned with ESG outcomes can lead to more greenwashing. Whether the values-congruent report increases greenwashing respective to the cash-flow-congruent report depends on the real greenwashing for both reports $(E[e^*], E[e^*_{\boldsymbol{\theta}}])$, the importance of real greenwashing to investors, and the magnitude of price response to a cash-flow-congruent report $(\phi_{\boldsymbol{\theta}})$.

5 Financial report

In the main model, the ESG report is the only source of information to investors.[7] However, investors also have access to financial reports that convey earnings and cash flow information. To investigate the interaction between financial and ESG reports, we extend the main model by introducing an additional financial report: $\tilde{f} = \tilde{x} + \tilde{\varepsilon}_f$, with $\tilde{\varepsilon}_f \sim N(0, \sigma_f^2)$. We assume it is disclosed simultaneously with the ESG report, r, and, in contrast to the ESG report, is not subject to managerial discretion.

Similar to the benchmark in the main model, we first analyze a setting where the ESG report is not disclosed. Proposition 3 derives the manager's effort and the equilibrium price when only the financial report is disclosed.

Proposition 3 *In the equilibrium with disclosure of only the financial report, the optimal effort and the stock market price are given by*

$$e_f^\dagger = \boldsymbol{\phi} - \frac{\psi_{f'}}{c_e}\boldsymbol{\theta} \tag{25}$$

$$p_f^\dagger = \bar{x} + (\lambda\boldsymbol{\eta} - \boldsymbol{\theta})^T\left(-\frac{\psi_{f'}}{c_e}\boldsymbol{\theta} + \bar{\boldsymbol{\phi}}\right) \tag{26}$$

$$+\psi_{f'}\left(f - \left(\bar{x} + \boldsymbol{\theta}^T\boldsymbol{\theta}\frac{\psi_{f'}}{c_e} - \boldsymbol{\theta}^T\bar{\boldsymbol{\phi}}\right)\right) \tag{27}$$

$$-\rho\left(\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\theta} + \sigma_x^2 - \frac{\left(\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\theta} + \sigma_x^2\right)^2}{\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\theta} + \sigma_x^2 + \sigma_f^2}\right), \tag{28}$$

[7] Effectively, the main model assumes that all other information is subsumed in investors' prior beliefs.

where $\psi_{f'} = \frac{\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\theta} + \sigma_x^2 + \lambda \boldsymbol{\eta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\theta}}{\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\theta} + \sigma_x^2 + \sigma_f^2}$.

Similar to Proposition 2, where the firm only discloses an ESG report, the manager's effort choices are driven by the manager's preferences, $\boldsymbol{\phi}$, and the sensitivity of price to the manager's efforts relative to their cost, $-\frac{\psi_{f'}}{c_e}\boldsymbol{\theta}$. The coefficient $\psi_{f'}$ measures the impact of the financial report on price, and the coefficient $-\boldsymbol{\theta}$ is the sensitivity of the financial report to the manager's actions. The risk premium in equation (28) shows that because the financial report is perfectly congruent with cash flows, only the measurement noise, σ_f^2, prevents investors from perfectly learning the firm's cash flows.

When the firm discloses both a financial report and an ESG report, the manager chooses her effort knowing that both reports influence the firm's price. Thus, the sensitivities of both reports to managerial efforts affect equilibrium efforts, and through efforts, cash flows and ESG performance. In the proof to Proposition 4 we derive the full equilibrium for the setting with financial and ESG reporting. Relative to Proposition 3, the addition of the ESG report affects the manager's effort, greenwashing, and how investors use both reports.

Proposition 4 *The expected return at the disclosure of the ESG and the financial report in tandem is higher than at the disclosure of the financial report only.*

Because the two reports both aggregate the manager's efforts, they are correlated with each other and investors use both reports to inform their trading strategies. In the knife-edge case of a financial report without noise ($\sigma_f^2 \to 0$), cash-flow oriented investors put a weight of 1 on the financial report and do not use the ESG report at all. However, ESG-oriented investors continue to use both reports to (potentially) learn about random ESG variation.

Notably, the result that the risk premium of a firm that discloses both reports is lower than the risk premium of a firm that only discloses a financial report holds only when there is noise in the financial report. In particular, the difference in risk premia is given by

$$\rho\sigma_f^4 \frac{\left(\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\zeta}\right)^2}{\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\theta} + \sigma_x^2 + \sigma_f^2} \frac{1}{\left(\boldsymbol{\zeta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2\right)\left(\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\theta} + \sigma_x^2 + \sigma_f^2\right) - \left(\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\zeta}\right)^2}.$$

This term shows that investors demand a higher risk premium in the absence of the ESG report but that the difference goes away as $\sigma_f^2 \to 0$.

Adding an ESG report to a financial report in the single-effort setting of Section 4.1 is tantamount to decreasing the prior uncertainty about all random variables (except for the ESG related noise terms, ε_y and ε_r). However, how the relative sensitivities of the ESG report affect the equilibrium is a function of the financial report. In deriving the following corollary, we return to the setting of Section 4.2 and set $\sigma_y^2 = \sigma_r^2 = 0$ and $\Sigma_\phi = \sigma_\phi^2 \mathbf{I}$. For further simplicity, we set $\sigma_f^2 = 0$ such that the financial report is noiseless. The following corollary analyzes the impact of the congruence of the ESG report to either cash flows or ESG on the firm's expected cash flows and ESG.

Corollary 6 *When $\sigma_y^2 = \sigma_r^2 = \sigma_f^2 = 0$ and $\Sigma_\phi = \sigma_\phi^2 \mathbf{I}$, expected cash flow, $E[x]$, is the same under cash-flow-congruent and ESG-congruent reports; expected ESG, $E[y]$, is weakly higher with an ESG-congruent report.*

Corollary 6 shows that when a high-quality (i.e., $\sigma_f^2 = 0$) financial report is available, the firm's expected cash flows do not suffer for an ESG-congruent ESG report. In essence, the high-quality financial report provides sufficient incentives to increase cash flows. However, the proposition also shows that the firm's expected ESG performance can suffer when the firm provides an ESG report that is cash-flow-congruent report rather than ESG-congruent. This shows a potential downside of focusing on cash flow implications of ESG related activities, conditional on the prior existence of high-quality financial reporting.

6 Conclusion

ESG reporting is of interest to investors for multiple reasons: some investors inherently care about the firm's ESG, and all investors plausibly care about the potential cash-flow consequences of the firm's ESG efforts. These reasons cause investors to incorporate information gleaned from ESG reports into their trading decisions, which in turn provides a channel, via stock price, that motivates firms and managers to shift their real efforts. Additionally, price motives can push managers to engage in activities that affect only the report (e.g., greenwashing via reporting bias). Our model characterizes the frictions central to the setting of ESG reporting to investors and develops predictions for how features of the ESG report as well as characteristics of firms and investors affect ESG reports, market pricing, corporate actions, and greenwashing.

In particular, we develop implications related to whether the report reflects the firm's ESG performance, the firm's ESG efforts, or the impacts of ESG actions on cash flows. Reporting that better captures cash-flow (ESG) effects tilts corporate actions towards increasing cash flows (ESG). Reports that are congruent with investors' values, on average, motivate managers to improve expected cash flows and ESG. Additionally, several features discussed above affect market responses to disclosure of ESG reports. When market price responses are stronger, managers tend to have greater incentives to increase the report, either through real ESG-related efforts or through report biasing. Broadly, greenwashing is a natural outcome of investor concern over firms' ESG performance. While it is not desirable in and of itself, increased greenwashing may be viewed as a side effect of an increase in societal concerns over corporate ESG performance.

Because our model features effects on both cash flows and ESG, the predictions from our model could help guide future analyses that incorporate data on financial, ESG, and market performance. We show that reporting standards that lead to lower cash flows need not lead to lower stock prices when investors directly value ESG, and that divergence between cash flow effects and market performance may reflect investors' ESG preferences. As a result, regulators, standard setters, and researchers should consider more than stock price reactions or future cash flows independently when evaluating the materiality of ESG reports, particularly when materiality is interpreted as affecting investor decisions rather than through a lens focused on financial performance per se. Furthermore, the optimal ESG report properties will depend, naturally, on the objective function being maximized. Firms or investors seeking to maximize cash flows should prefer different reports relative to regulators interested in maximizing ESG performance or standard setters looking to maximize the amount of information available to dispersed investors.

Stepping back, we view our framework as a stepping stone (rather than a first step) on the long and intertwined paths of research related to ESG reporting. Prior studies have generated useful foundations, but there is much additional work to be done, and our framework provides some guideposts. For instance, little is known as to how preferences over ESG performance are formed and change over time, or whether investors are risk averse, risk neutral, or risk seeking with regards to corporate ESG. While our model focuses on a one-epoch single-firm setting, additional frictions and forces plausibly arise with multiple periods and firms. Additional dimensions of investor heterogeneity, including information asymmetry and differences in wealth or risk aversion are also likely to lead to additional interesting results. In modeling the firm, we have also focused on a manager who seeks to maximize price. Contracting on ESG related performance measures opens up additional avenues for ESG reports to be relevant to firm performance and risk. While we have focused on a single ESG report, potentially in the presence of extant financial reports, the landscape is characterized by a broad and shifting set of ESG information providers. Our results related to investor learning suggest that multiple ESG reports capturing different dimensions may be beneficial to investors, rather than just reflecting disagreement between vendors or rent seeking. Stepping back, we view ESG reporting as a fertile field likely to sprout many interesting studies relevant to researchers, regulators, and practitioners.

Appendix

Proof of Proposition 2

Below, we derive a linear equilibrium in which

1. The risky asset's price is a linear function of the report: $p = a + br$;

2. The manager's choice of effort e_i is linear in ϕ, i.e., $\mathbf{e} = \boldsymbol{\alpha}_e + \boldsymbol{\beta}_e \boldsymbol{\phi}$, where $\boldsymbol{\alpha}_e = (\alpha_{e1}, \alpha_{e2})^T$ and $\boldsymbol{\beta}_e = \begin{pmatrix} \beta_{e11} & \beta_{e12} \\ \beta_{e21} & \beta_{e21} \end{pmatrix}$;

3. The report is linear in e, ε_y, ε_r, i.e., $r = \alpha_r + \beta_{r1} e_1 + \beta_{r2} e_2 + \gamma_y \nu \varepsilon_y + \gamma_r \varepsilon_r = \alpha_r + \boldsymbol{\beta}_r^T \mathbf{e} + \gamma_y \nu \varepsilon_y + \gamma_r \varepsilon_r$, where $\boldsymbol{\beta}_r = (\beta_{r1}, \beta_{r2})^T$.

Proceeding via backward induction, we start with the competitive market for firm shares in period 2, conditional on the report, r, provided to the market. All investors observe r. Firm financial and ESG outputs are:

$$\begin{aligned} \tilde{x} &= \bar{x} - \boldsymbol{\theta}^T \mathbf{e} + \tilde{\varepsilon}_x \text{ and} \\ \tilde{y} &= \boldsymbol{\eta}^T \mathbf{e} + \tilde{\varepsilon}_y \end{aligned}$$

Denote $\mu_x = E[x|r]$, $\Sigma_x = Var[x|r]$, and $\mu_y = E[y|r]$. Using the manager's strategy $\mathbf{e} = \boldsymbol{\alpha}_e + \boldsymbol{\beta}_e \boldsymbol{\phi}$, investor demands are given by

$$\begin{aligned} q_1 &= \frac{\mu_x - p}{\rho \Sigma_x} \text{ and} \\ q_2 &= \frac{\mu_x + \mu_y - p}{\rho \Sigma_x}. \end{aligned}$$

Market clearing implies $\lambda q_2 + (1 - \lambda) q_1 = 1$ and solving for p gives

$$p = \mu_x + \lambda \mu_y - \rho \Sigma_x,$$

such that price is a linear function of the report, r.

Next, we derive the manager's reporting strategy conditional on the market pricing function and a chosen e:

$$
\begin{aligned}
r^* &\in \arg\max_r p - \frac{c_e}{2}\sum_{i\in\{1,2\}}(e_i-\phi_i)^2 - \frac{c_r}{2}\left(r-\zeta_1 e_1-\zeta_2 e_2-\nu\varepsilon_y-\varepsilon_r\right)^2 \\
&= \arg\max_r \mu_x+\lambda\mu_y-\rho\Sigma_x-\frac{c_r}{2}\left(r-\zeta_1 e_1-\zeta_2 e_2-\nu\varepsilon_y-\varepsilon_r\right)^2
\end{aligned}
$$

Note from above that μ_x and μ_y are linear in r, while λ and $\rho\Sigma_x$ are independent of r. The FOC implies that:

$$
\begin{aligned}
0 &= \frac{d\mu_x}{dr}+\lambda\frac{d\mu_y}{dr}-c_r\left(r^*-\zeta_1 e_1-\zeta_2 e_2-\nu\varepsilon_y-\varepsilon_r\right) \\
\Rightarrow r^* &= \frac{-\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r\right)+\lambda\left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\gamma_y\nu\sigma_y^2\right)}{c_r\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\gamma_y^2\nu^2\sigma_y^2+\gamma_r^2\sigma_r^2\right)}+\boldsymbol{\zeta}^T\mathbf{e}+\nu\varepsilon_y+\varepsilon_r
\end{aligned}
$$

and the SOC is satisfied for any $c_r>0$.

Matching coefficients implies $\boldsymbol{\beta}_r=\boldsymbol{\zeta}$, $\gamma_r=\gamma_y=1$, and $\alpha_r=\frac{-\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}\right)+\lambda\left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\nu\sigma_y^2\right)}{c_r\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)}$ so that

$$
r^\dagger=\frac{\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\lambda\nu\sigma_y^2}{c_r\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)}+\boldsymbol{\zeta}^T\mathbf{e}+\nu\varepsilon_y+\varepsilon_r
$$

Plugging these into the pricing function gives

$$
p=\mu_x^\dagger+\lambda\mu_y^\dagger-\rho\Sigma_x^\dagger,
$$

where $\mu_x^\dagger$, $\mu_y^\dagger$, and $\Sigma_x^\dagger$ are μ_x, μ_y, and Σ_x with $\boldsymbol{\beta}_r$, γ_r, γ_y and α_r, respectively, substituted.

Finally, we solve for the manager's choice of $\mathbf{e}$.

$$\begin{aligned}
\mathbf{e}^* &\in \arg\max_{\mathbf{e}} E\left[\mu_x^\dagger + \lambda\mu_y^\dagger - \rho\Sigma_x^\dagger - \frac{c_e}{2}\sum_{i\in\{1,2\}}(e_i - \phi_i)^2 - \frac{c_r}{2}\left(r - \zeta_1 e_1 - \zeta_2 e_2 - \nu\varepsilon_y - \varepsilon_r\right)^2\right] \\
&= \arg\max_{\mathbf{e}} E\left[\mu_x^\dagger + \lambda\mu_y^\dagger - \rho\Sigma_x^\dagger - \frac{c_e}{2}\sum_{i\in\{1,2\}}(e_i - \phi_i)^2 - \frac{c_r}{2}\left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_r\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right)^2\right]
\end{aligned}$$

The FOC is given by

$$\begin{aligned}
0 &= \frac{d}{d\mathbf{e}}\frac{\partial}{\partial r}E\left[\mu_x^\dagger + \lambda\mu_y^\dagger - \rho\Sigma_x^\dagger\right] - \frac{c_e}{2}\frac{d}{d\mathbf{e}}\left(\mathbf{e}^* - \boldsymbol{\phi}\right)^T\left(\mathbf{e}^* - \boldsymbol{\phi}\right) \\
0 &= \frac{\partial}{\partial r}E\left[\mu_x^\dagger + \lambda\mu_y^\dagger\right]\frac{dr}{d\mathbf{e}} - c_e\left(\mathbf{e}^* - \boldsymbol{\phi}\right) \\
&\Rightarrow \mathbf{e}^* = \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_e\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\boldsymbol{\zeta} + \boldsymbol{\phi}
\end{aligned}$$

which implies $\boldsymbol{\beta}_e$ is a 2×2 identity matrix and $\boldsymbol{\alpha}_e = \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_e\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\boldsymbol{\zeta}$. Therefore, in equilibrium, we have

$$p^* = \mu_x^* + \lambda\mu_y^* - \rho\Sigma_x^*,$$

where μ_x^*, μ_y^*, and Σ_x^* are $\mu_x^\dagger$, $\mu_y^\dagger$, and $\Sigma_x^\dagger$ with $\boldsymbol{\beta}_e$ and α_e substituted.

Let

$$\psi = \frac{dp^*}{dr} = \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2}.$$

Then we can write p^* as

$$\begin{aligned}
p^* &= \bar{x} + \left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\left(\frac{\psi}{c_e}\boldsymbol{\zeta} + \bar{\boldsymbol{\phi}}\right) + \psi\left(\boldsymbol{\zeta}^T\left(\boldsymbol{\phi} - \bar{\boldsymbol{\phi}}\right) + \nu\varepsilon_y + \varepsilon_r\right) \\
&\quad - \rho\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta} + \sigma_x^2 - \frac{\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\right)\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\right)}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right)
\end{aligned}$$

Substituting efforts and the report from above, the equilibrium expressions in (3) and (4) are:

$$\begin{aligned}
\mathbf{e}^* &= \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta}+\lambda\nu\sigma_y^2}{c_e\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)}\boldsymbol{\zeta}+\boldsymbol{\phi} = \frac{\psi}{c_e}\boldsymbol{\zeta}+\boldsymbol{\phi} \text{ and} \\
r^* &= \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta}+\lambda\nu\sigma_y^2}{c_r\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)}+\boldsymbol{\zeta}^T\mathbf{e}+\nu\varepsilon_y+\varepsilon_r = \frac{\psi}{c_r}+\frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta}+\boldsymbol{\zeta}^T\boldsymbol{\phi}+\nu\varepsilon_y+\varepsilon_r.
\end{aligned}$$

Single-dimension setting

Substituting $\boldsymbol{\eta}=\eta$, $\boldsymbol{\theta}=\theta$, $\boldsymbol{\zeta}=\zeta$, $\Sigma_\phi=\sigma_\phi^2$, $\boldsymbol{\phi}=\phi$, and $\bar{\boldsymbol{\phi}}=\bar{\phi}$ into (3)-(5) yields

$$e_I^* = \phi+\frac{\psi_I}{c_e}\zeta, \tag{29}$$

$$r_I^* = \frac{\psi_I}{c_r}+\zeta e^*+\nu\varepsilon_y+\varepsilon_r \tag{30}$$

$$= \frac{\psi_I}{c_r}+\frac{\psi_I}{c_e}\zeta^2+\zeta\phi+\nu\varepsilon_y+\varepsilon_r, \text{ and} \tag{31}$$

$$\begin{aligned}
p_I^* &= \bar{x}+(\lambda\eta-\theta)\left(\frac{\psi_I}{c_e}\zeta+\bar{\phi}\right)+\psi_I\left(\boldsymbol{\zeta}\left(\phi-\bar{\phi}\right)+\nu\varepsilon_y+\varepsilon_r\right) \\
&\quad -\rho\left(\frac{\theta^2\sigma_\phi^2\left(\nu^2\sigma_y^2+\sigma_r^2\right)}{\zeta^2\sigma_\phi^2+\nu^2\sigma_y^2+\sigma_r^2}+\sigma_x^2\right)
\end{aligned} \tag{32}$$

where I indicates the one-dimensional effort setting and

$$\psi_I = \frac{dp^*}{dr} = \frac{(\lambda\eta-\theta)\,\sigma_\phi^2\zeta+\lambda\nu\sigma_y^2}{\sigma_\phi^2\zeta^2+\nu^2\sigma_y^2+\sigma_r^2}. \tag{33}$$

Proof of Proposition 3

In this section, we derive a linear equilibrium where $e=\alpha_e+\beta_e\phi$, $p=a+bf$. We solve for the manager's effort:

$$e^\dagger \in \quad argmax \quad E[\mu_x+\lambda\mu_y-\rho\Sigma_x-\frac{c_e}{2}\left(e-\bar{\phi}\right)^T\left(e-\bar{\phi}\right)].$$

The FOC implies:

$$0 = \frac{\partial}{\partial f} E[\mu_x + \lambda \mu_y] \frac{d}{de} - c_e \left(e^{\dagger} - \phi \right)^T .$$

Thus,

$$\begin{aligned}
e^{\dagger} &= \phi - \left(\frac{\boldsymbol{\theta}^T \beta_e \boldsymbol{\Sigma}_\phi \beta_e^T \boldsymbol{\theta} + \sigma_x^2 - \lambda \boldsymbol{\eta}^T \beta_e \boldsymbol{\Sigma}_\phi \beta_e^T \boldsymbol{\theta}}{c_e(\boldsymbol{\theta}^T \beta_e \boldsymbol{\Sigma}_\phi \beta_e^T \boldsymbol{\theta} + \sigma_x^2 + \sigma_f^2)} \right) \boldsymbol{\theta}, \\
\beta_e &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \text{ and} \\
\alpha_e &= - \left(\frac{\boldsymbol{\theta}^T \beta_e \boldsymbol{\Sigma}_\phi \beta_e^T \boldsymbol{\theta} + \sigma_x^2 - \lambda \boldsymbol{\eta}^T \beta_e \boldsymbol{\Sigma}_\phi \beta_e^T \boldsymbol{\theta}}{c_e(\boldsymbol{\theta}^T \beta_e \boldsymbol{\Sigma}_\phi \beta_e^T \boldsymbol{\theta} + \sigma_x^2 + \sigma_f^2)} \right) \boldsymbol{\theta}.
\end{aligned}$$

Firm price and equilibrium effort are as given in the proposition.

Proof of Proposition 4

In this section, we derive a linear equilibrium where $r = \alpha_r + \beta_{r_1} e_1 + \beta_{r_2} e_2 + \gamma_y \nu \varepsilon_y + \gamma_r \varepsilon_r$, $p = a + br + cf$, $\mathbf{e} = \boldsymbol{\alpha}_e + \boldsymbol{\beta}_e \boldsymbol{\phi}$.

Similar to above,

$$r^* = \frac{1}{c_r} \left(\frac{\partial \mu_x}{\partial r} + \lambda \frac{\partial \mu_y}{\partial r} \right) + \boldsymbol{\zeta}^T \mathbf{e} + \nu \varepsilon_y + \varepsilon_r$$

Matching coefficients yields that in equilibrium $\beta_r = \zeta$, $\gamma_y = \gamma_r = 1$, and

$$\begin{aligned}
\alpha_r = & \frac{1}{c_r} \frac{-\boldsymbol{\theta}^T \boldsymbol{\beta}_e \boldsymbol{\Sigma}_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}}{\left(\boldsymbol{\beta}_r^T \boldsymbol{\beta}_e \boldsymbol{\Sigma}_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y^2 \nu^2 \sigma_y^2 + \gamma_r^2 \sigma_r^2 \right) \left(\boldsymbol{\theta}^T \boldsymbol{\beta}_e \boldsymbol{\Sigma}_\phi \boldsymbol{\beta}_e^T \boldsymbol{\theta} + \sigma_x^2 + \sigma_f^2 \right) - \left(\boldsymbol{\theta}^T \boldsymbol{\beta}_e \boldsymbol{\Sigma}_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r \right)^2} \\
& + \frac{1}{c_r} \frac{\lambda \left(\left(\boldsymbol{\eta}^T \boldsymbol{\beta}_e \boldsymbol{\Sigma}_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_r \nu \sigma_y^2 \right) \left(\boldsymbol{\theta}^T \boldsymbol{\beta}_e \boldsymbol{\Sigma}_\phi \boldsymbol{\beta}_e^T \boldsymbol{\theta} + \sigma_x^2 + \sigma_f^2 \right) - \boldsymbol{\eta}^T \boldsymbol{\beta}_e \boldsymbol{\Sigma}_\phi \boldsymbol{\beta}_e^T \boldsymbol{\theta} \boldsymbol{\theta}^T \boldsymbol{\beta}_e \boldsymbol{\Sigma}_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r \right)}{\left(\boldsymbol{\beta}_r^T \boldsymbol{\beta}_e \boldsymbol{\Sigma}_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y^2 \nu^2 \sigma_y^2 + \gamma_r^2 \sigma_r^2 \right) \left(\boldsymbol{\theta}^T \boldsymbol{\beta}_e \boldsymbol{\Sigma}_\phi \boldsymbol{\beta}_e^T \boldsymbol{\theta} + \sigma_x^2 + \sigma_f^2 \right) - \left(\boldsymbol{\theta}^T \boldsymbol{\beta}_e \boldsymbol{\Sigma}_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r \right)^2}
\end{aligned}$$

Then, firm price is

$$p = \mu_x^{\dagger} + \lambda \mu_y^{\dagger} - \rho \Sigma_x^{\dagger}$$

where $\mu_x^{\dagger}, \mu_y^{\dagger}$, and $\Sigma_x^{\dagger}$ are μ_x, μ_y, and Σ_x with $\boldsymbol{\beta}_r, \gamma_r, \gamma_y, \alpha_r$ substituted.

Next, solve for the optimal effort. F.O.C. implies:

$$\begin{aligned}
0 &= \frac{\partial}{\partial e} E\left[\mu_x^\dagger + \lambda \mu_y^\dagger\right] \boldsymbol{\zeta} + \frac{\partial}{\partial e} E\left[\mu_x^\dagger + \lambda \mu_y^\dagger\right] \boldsymbol{\theta} - c_e\left(\mathbf{e}^* - \boldsymbol{\phi}\right), \\
\mathbf{e}^* &= \boldsymbol{\phi} + \frac{1}{c_e}\left(-\psi^f \boldsymbol{\theta} + \psi \boldsymbol{\zeta}\right), \\
\psi &= \frac{\partial}{\partial r}\left(\mu_x^\dagger + \lambda \mu_y^\dagger\right), \text{ and} \\
\psi^f &= \frac{\partial}{\partial f}\left(\mu_x^\dagger + \lambda \mu_y^\dagger\right).
\end{aligned}$$

Thus, $\boldsymbol{\beta}_e$ is 2×2 identity matrix,

$$\begin{aligned}
\boldsymbol{\alpha}_e &= \frac{1}{c_e}\left(-\psi^f \boldsymbol{\theta} + \psi \boldsymbol{\zeta}\right), \text{ and} \\
\alpha_r &= \frac{1}{c_r} \frac{-\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\zeta} \sigma_f^2 + \lambda\left(\left(\boldsymbol{\eta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\zeta} + \nu \sigma_y^2\right)\left(\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\theta} + \sigma_x^2 + \sigma_f^2\right) - \boldsymbol{\eta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\theta} \boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\zeta}\right)}{\left(\boldsymbol{\zeta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2\right)\left(\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\theta} + \sigma_x^2 + \sigma_f^2\right) - \left(\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\zeta}\right)^2} = \frac{\psi}{c_r}
\end{aligned}$$

Price in equilibrium is:

$$p^* = \mu_x^* + \lambda \mu_y^* - \rho \Sigma_x^*,$$

where μ_x^*, μ_y^*, and Σ_x^* are $\mu_x^\dagger, \mu_y^\dagger$, and $\Sigma_x^\dagger$ with $\boldsymbol{\beta}_e$ and α_e substituted. Gathering coefficients, we obtain firm price, manager's effort, and ESG report in equilibrium:

$$\begin{aligned}
&\mathbf{e}^* = \boldsymbol{\phi} + \frac{1}{c_e}(-\psi^f \boldsymbol{\theta} + \psi \boldsymbol{\zeta}) \\
&r^* = \frac{\psi}{c_r} + \boldsymbol{\zeta}^T \mathbf{e}^* + \nu \varepsilon_y + \varepsilon_r \\
&p^* = \bar{x} + (\lambda \boldsymbol{\eta} - \boldsymbol{\theta})^T (\bar{\boldsymbol{\phi}} + \frac{1}{c_e}(-\psi^f \boldsymbol{\theta} + \psi \boldsymbol{\zeta})) \\
&+\psi(r^* - \frac{\psi}{c_r} - \boldsymbol{\zeta}^T \frac{1}{c_e}(-\psi^f \boldsymbol{\theta} + \psi \boldsymbol{\zeta}) - \boldsymbol{\zeta}^T \bar{\boldsymbol{\phi}}) \\
&+\psi^f (f - \bar{x} + \boldsymbol{\theta}^T \frac{1}{c_e}(-\psi^f \boldsymbol{\theta} + \psi \boldsymbol{\zeta}) + \boldsymbol{\theta}^T \bar{\boldsymbol{\psi}}) \\
&-\rho(\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\theta} + \sigma_x^2) \\
&+\rho \frac{(\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\zeta})^2 \sigma_f^2 - (\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\zeta})^2 (\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\theta} + \sigma_x^2) + (\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\theta} + \sigma_x^2)^2 (\boldsymbol{\zeta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2)}{(\boldsymbol{\zeta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2)(\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\theta} + \sigma_x^2 + \sigma_f^2) - (\boldsymbol{\theta}^T \boldsymbol{\Sigma}_\phi \boldsymbol{\zeta})^2}
\end{aligned}$$

where $\psi = \frac{\partial p^*}{\partial r} = \frac{(\lambda\boldsymbol{\eta}-\boldsymbol{\theta})^T\boldsymbol{\Sigma}_\phi\boldsymbol{\zeta}\sigma_f^2+\lambda\nu\sigma_y^2\left(\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\theta}+\sigma_x^2+\sigma_f^2\right)+\lambda\left(\boldsymbol{\eta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\zeta}\left(\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\theta}+\sigma_x^2\right)-\left(\boldsymbol{\eta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\theta}\right)^2\right)}{\left(\boldsymbol{\zeta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)\left(\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\theta}+\sigma_x^2+\sigma_f^2\right)-\left(\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\zeta}\right)^2}$ and $\psi^f = \frac{\partial p^*}{\partial f} = \frac{\left(-\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\zeta}\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\zeta}+\left(\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\theta}+\sigma_x^2\right)\left(\boldsymbol{\zeta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)\right)+\lambda\left(-\left(\boldsymbol{\eta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\zeta}+\nu\sigma_y^2\right)\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\zeta}+\boldsymbol{\eta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\theta}\left(\boldsymbol{\zeta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)\right)}{\left(\boldsymbol{\zeta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)\left(\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\theta}+\sigma_x^2+\sigma_f^2\right)-\left(\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi\boldsymbol{\zeta}\right)^2}$.

Proof of Corollary 6

Set $\sigma_r^2 = \sigma_y^2 = \sigma_f^2 = 0$ and $\boldsymbol{\Sigma}_\phi = \sigma_\phi^2\mathbf{I}$. When $\boldsymbol{\zeta} = \boldsymbol{\eta}$: $\psi = \lambda$, $\psi\boldsymbol{\zeta} = \lambda\boldsymbol{\eta}$, $-\boldsymbol{\theta}^T\left(\alpha_e + \frac{\psi^f}{c_e}\boldsymbol{\theta}\right) = -\frac{\lambda}{c_e}\boldsymbol{\theta}^T\boldsymbol{\eta}$. When $\boldsymbol{\zeta} = -\boldsymbol{\theta}$: $\psi\boldsymbol{\zeta} = \lambda\frac{\boldsymbol{\eta}^T\boldsymbol{\theta}}{\boldsymbol{\theta}^T\boldsymbol{\theta}}\boldsymbol{\theta}$, $-\boldsymbol{\theta}^T\left(\alpha_e + \frac{\psi^f}{c_e}\boldsymbol{\theta}\right) = -\frac{\lambda}{c_e}\boldsymbol{\eta}^T\boldsymbol{\theta}$. Thus, $E[x]|_{\boldsymbol{\zeta}=\boldsymbol{\eta}} = E[x]|_{\boldsymbol{\zeta}=-\boldsymbol{\theta}}$.

$E[y]|_{\boldsymbol{\zeta}=\boldsymbol{\eta}}+\frac{1}{c_e}\boldsymbol{\theta} = \boldsymbol{\eta}^T\bar{\phi}+\frac{\lambda}{c_e}\boldsymbol{\eta}^T\boldsymbol{\eta} = \boldsymbol{\eta}^T\bar{\phi}+\frac{\lambda}{c_e}\left(\eta_1^2+\eta_2^2\right), E[y]|_{\boldsymbol{\zeta}=-\boldsymbol{\theta}}+\frac{1}{c_e}\boldsymbol{\theta} = \boldsymbol{\eta}^T\bar{\phi}+\frac{\lambda}{c_e}\frac{(\eta_1\theta_1+\eta_2\theta_2)^2}{\theta_1^2+\theta_2^2}$.

$$\begin{aligned}
&\frac{c_e\left(\theta_1^2+\theta_2^2\right)}{\lambda}\left(E[y]|_{\boldsymbol{\zeta}=\boldsymbol{\eta}} - E[y]|_{\boldsymbol{\zeta}=-\boldsymbol{\theta}}\right)\\
= &\left(\eta_1^2+\eta_2^2\right)\left(\theta_1^2+\theta_2^2\right) - \left(\eta_1\theta_1+\eta_2\theta_2\right)^2\\
= &\eta_1^2\theta_2^2+\eta_2^2\theta_1^2-2\eta_1\eta_2\theta_1\theta_2\\
= &\left(\eta_1\theta_2-\eta_2\theta_1\right)^2 \geq 0
\end{aligned}$$

Thus, $E[y]|_{\boldsymbol{\zeta}=\boldsymbol{\eta}} \geq E[y]|_{\boldsymbol{\zeta}=-\boldsymbol{\theta}}$.

6.1 Correlated ε_x and ε_y

In this extension, suppose $Cov\left(\varepsilon_x,\varepsilon_y\right) = \Sigma_{xy} = \begin{pmatrix}\sigma_x^2 & \sigma_{xy}\\ \sigma_{xy} & \sigma_y^2\end{pmatrix}$, with determinant: $\sigma_x^2\sigma_y^2 - \sigma_{xy}^2 > 0$ such that Σ_{xy} is positive definite. Below, we derive a linear equilibrium in which

1. The risky asset's price is a linear function of the report: $p = a + br$;

2. The manager's choice of effort e_i is linear in ϕ, i.e., $\mathbf{e} = \boldsymbol{\alpha}_e+\boldsymbol{\beta}_e\boldsymbol{\phi}$, where $\boldsymbol{\alpha}_e = (\alpha_{e1},\alpha_{e2})^T$ and $\boldsymbol{\beta}_e = \begin{pmatrix}\beta_{e11} & \beta_{e12}\\ \beta_{e21} & \beta_{e21}\end{pmatrix}$;

3. The report is linear in e, ε_y, ε_r, i.e., $r = \alpha_r + \beta_{r1} e_1 + \beta_{r2} e_2 + \gamma_y \nu \varepsilon_y + \gamma_r \varepsilon_r = \alpha_r + \boldsymbol{\beta}_r^T \mathbf{e} + \gamma_y \nu \varepsilon_y + \gamma_r \varepsilon_r$, where $\boldsymbol{\beta}_r = (\beta_{r1}, \beta_{r2})^T$.

Proceeding via backward induction, we start with the competitive market for firm shares in period 2, conditional on the report, r, provided to the market. All investors observe r. For simplicity, we set $\Sigma_\phi = \begin{pmatrix} \sigma_\phi^2 & 0 \\ 0 & \sigma_\phi^2 \end{pmatrix}$ in the remainder.

Firm financial and ESG outputs are:

$$\begin{aligned} \tilde{x} &= \bar{x} - \boldsymbol{\theta}^T \mathbf{e} + \tilde{\varepsilon}_x \text{ and} \\ \tilde{y} &= \boldsymbol{\eta}^T \mathbf{e} + \tilde{\varepsilon}_y \end{aligned}$$

Taking the manager's strategy $\mathbf{e} = \boldsymbol{\alpha}_e + \boldsymbol{\beta}_e \boldsymbol{\phi}$ as given, we have the following joint distributions:

$$\begin{pmatrix} x \\ r \end{pmatrix} \sim N\left(\begin{pmatrix} \bar{x} - \boldsymbol{\theta}^T \boldsymbol{\alpha}_e - \boldsymbol{\theta}^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \\ \alpha_r + \boldsymbol{\beta}_r^T \boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \end{pmatrix}, \begin{pmatrix} \boldsymbol{\theta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\theta} + \sigma_x^2 & -\boldsymbol{\theta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y \nu \sigma_{xy} \\ -\boldsymbol{\theta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y \nu \sigma_{xy} & \boldsymbol{\beta}_r^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y^2 \nu^2 \sigma_y^2 + \gamma_r^2 \sigma_r^2 \end{pmatrix} \right)$$

and

$$\begin{pmatrix} y \\ r \end{pmatrix} \sim N\left(\begin{pmatrix} \boldsymbol{\eta}^T \boldsymbol{\alpha}_e + \boldsymbol{\eta}^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \\ \alpha_r + \boldsymbol{\beta}_r^T \boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \end{pmatrix}, \begin{pmatrix} \boldsymbol{\eta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\eta} + \sigma_y^2 & \boldsymbol{\eta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y \nu \sigma_y^2 \\ \boldsymbol{\eta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y \nu \sigma_y^2 & \boldsymbol{\beta}_r^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y^2 \nu^2 \sigma_y^2 + \gamma_r^2 \sigma_r^2 \end{pmatrix} \right).$$

Between the above and below, the solution repeats the solution in 6.

When $\sigma_{xy} \neq 0$, we have

$$\begin{aligned}
p^* &= \bar{x} \\
&\quad + \frac{1}{c_e}\left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2 + \nu\sigma_{xy}}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right) \\
&\quad \left(\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) - \left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2 + \nu\sigma_{xy}}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right)\boldsymbol{\zeta}^T\right)\boldsymbol{\zeta} && (34) \\
&\quad + \left(\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) - \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2 + \nu\sigma_{xy}}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\boldsymbol{\zeta}^T\right)\bar{\boldsymbol{\phi}} && (35) \\
&\quad - \frac{1}{c_r}\left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2 + \nu\sigma_{xy}}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right)^2 && (36) \\
&\quad - \rho\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta} + \sigma_x^2 - \frac{\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu\sigma_{xy}\right)\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu\sigma_{xy}\right)}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right) && (37) \\
&\quad + \left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2 + \nu\sigma_{xy}}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right) r && (38)
\end{aligned}$$

The terms in p^* are as follows:

- In line (34) and the line below it, we have the expected cash flow, $\bar{x}$, and a term that goes to zero if either $c_e \to \infty$ or $\boldsymbol{\zeta}$ goes to zero.
- In line (35), we have a term that captures the contribution from the manager's bliss actions, which goes to zero if $\bar{\boldsymbol{\phi}} \to \mathbf{0}^2$, where, abusing notation slightly, $\mathbf{0}^2 = (0,0)^T$.
- In line (36), we have a term that captures a loss from a lack of costs that discipline reporting. This term is negative, but goes to zero as $c_r \to \infty$, i.e., as misreporting relative to what the manager observed gets prohibitively costly.
- In line (37), we have the risk premium term that goes to zero as investors become risk-neutral in cash flows, i.e., $\rho \to 0$. Recall that investors are risk-neutral with respect to impact, y.
- In line (38), we have a term that captures the sensitivity of price to the report. This

term goes to zero as reporting noise gets large, i.e., $\sigma_r^2 \to \infty$.

Let

$$\psi = \frac{dp^*}{dr} = \frac{\left(\lambda \boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) \Sigma_\phi \boldsymbol{\zeta} + \lambda \nu \sigma_y^2 + \nu \sigma_{xy}}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2}.$$

Then we can write p^* as

$$\begin{aligned}
p^* \;=\;& \bar{x} + \left(\lambda \boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) \left(\frac{\psi}{c_e} \boldsymbol{\zeta} + \bar{\boldsymbol{\phi}}\right) && (39)\\
& + \left(\lambda \boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) \Sigma_\phi \boldsymbol{\zeta} \frac{1}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2} r && (40)\\
& + \frac{\lambda \nu \sigma_y^2}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta}_r + \nu^2 \sigma_y^2 + \sigma_r^2} r + \frac{\nu \sigma_{xy}}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta}_r + \nu^2 \sigma_y^2 + \sigma_r^2} r + && (41)\\
& - \psi \left(\frac{\psi}{c_r} + \frac{\psi}{c_e} \boldsymbol{\zeta}^T \boldsymbol{\zeta} + \boldsymbol{\zeta}^T \bar{\boldsymbol{\phi}}\right) && (42)\\
& - \rho \left(\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta} + \sigma_x^2 - \frac{\left(\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\zeta} + \nu \sigma_{xy}\right)\left(\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\theta} + \nu \sigma_{xy}\right)}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2}\right) && (43)
\end{aligned}$$

Expected price can be written as

$$\begin{aligned}
E\left[p^*\right] \;&=\; \bar{x} + \left(\lambda \boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) \left(\frac{\psi}{c_e} \boldsymbol{\zeta} + \bar{\boldsymbol{\phi}}\right) - \rho \left(\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta} + \sigma_x^2 - \frac{\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\zeta} \boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\theta}}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2}\right)\\
&=\; \bar{x} + \left(\lambda \boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) \left(\frac{\psi}{c_e} \boldsymbol{\zeta} + \bar{\boldsymbol{\phi}}\right)\\
&\quad - \rho \left(\frac{\boldsymbol{\theta}^T \Sigma_\phi \left(\boldsymbol{\theta} \boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} - \boldsymbol{\zeta} \boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\theta}\right) + \left(\nu^2 \sigma_y^2 + \sigma_r^2\right) \boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta}}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2} + \sigma_x^2\right)
\end{aligned}$$

The sensitivity of p^* to r, ψ, affect expected price only through its effect on expected effort. However, the sensitivity of the report to effort, $\boldsymbol{\zeta}$, also affects the degree to which investors can use r to learn about $\mathbf{e}^*$ and, consequently, reduce the posterior variance of cash flows. Rewriting the risk premium as

$$\frac{\boldsymbol{\theta}^T \Sigma_\phi \left(\boldsymbol{\theta} \boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} - \boldsymbol{\zeta} \boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\theta}\right) + \left(\nu^2 \sigma_y^2 + \sigma_r^2\right) \boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta}}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2} + \sigma_x^2$$

shows how the first term in the numerator goes to zero as the report becomes congruent with cash flows, i.e., as $\boldsymbol{\zeta} \to \boldsymbol{\theta}$, we have

$$\frac{\boldsymbol{\theta}^T \Sigma_\phi \left(\boldsymbol{\theta} \boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} - \boldsymbol{\zeta} \boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\theta} \right) + \left(\nu^2 \sigma_y^2 + \sigma_r^2 \right) \boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta}}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2} + \sigma_x^2 \to \frac{\left(\nu^2 \sigma_y^2 + \sigma_r^2 \right) \boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta}}{\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta} + \nu^2 \sigma_y^2 + \sigma_r^2} + \sigma_x^2$$

This does not necessarily minimize the risk premium, which in turn goes to $\left(\nu^2 \sigma_y^2 + \sigma_r^2 \right) + \sigma_x^2$ if $\boldsymbol{\zeta} \to \boldsymbol{\theta} \to \infty$.

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