Sensitivity Disclosures*

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Abstract

The SEC requires that some firms disclose information about risks. Firms may disclose correlations by reporting the sensitivity to market risk factors of cash flows related only to financial instruments and derivatives. We propose a theoretical model that analyzes the consequences of mandating firms to disclose their sensitivity. This model extends previous research on managers' voluntary disclosures of variances of future cash flows and measurement error of disclosures. We derive equilibrium prices and stock returns endogenously in a setting where truthful disclosure of the sensitivity is voluntary and show why investors require an additional risk premium in the absence of sensitivity disclosures. Further, a manager's decision to disclose or withhold the sensitivity may be affected by other firms' disclosures of sensitivity even when sensitivities are uncorrelated. Finally, we show how voluntary sensitivity disclosures affect firms' cost of capital even in the limiting case with infinitely many firms.

* We thank seminar participants at University of Minnesota and Burton Workshop.

1. Introduction

While financial firms have long faced risk-based regulation, some non-financial firms also face mandatory risk disclosure requirements since January 1997 when the U.S. Securities and Exchange Commission issued Financial Reporting Release No. 48 (FRR48).[1] FRR48 requires the disclosure of information about risks in one of three formats: sensitivity, tabular, or Value-at-Risk. If firms choose the sensitivity format, then FRR48 requires disclosure of the sensitivity to market risk factors of future cash flows related only to financial instruments and derivatives. We propose a theoretical model that facilitates analysis of the consequences of mandating firms to disclose their sensitivity. To do this, we follow prior theoretical accounting research and analyze the benchmark of voluntary disclosures to establish a benchmark for the effect of mandating disclosures.

An extensive literature in accounting investigates managers' disclosure incentives in the presence and absence of mandatory disclosures: Verrecchia (1983, 1990) investigate managers' costly discretionary disclosures of future income; Dye (1990), Penno (1996), and Hughes and Pae (2004) investigate managers' choice and voluntary disclosure of precision; Jorgensen and Kirschenheiter (2003) characterize managers' costly discretionary disclosure of firm-specific variances. In this paper, we analyze managers' costly discretionary disclosure of correlations in terms of *sensitivity*. As will be explained in more detail below, we consider a setting where firms' *sensitivity* disclosure under FRR48 are informative of the correlation of the firm's future cash flows with other firms' future cash flows.

We first consider a single firm setting and show that the manager has an incentive to disclose sensitivities in a symmetric interval around zero while withholding sensitivities away from zero. This means that a manager's discretionary disclosure incentives regarding variances and correlations coincide in a single firm setting. We then show that this need not extend to a setting with multiple firms. In a two firm setting where managers make their disclosure decisions sequentially, the last manager will observe the other firm's sensitivity, if disclosed, before making the discretionary

[1] The official title of FRR 48 is "Disclosure of Accounting Policies for Derivative Financial Instruments and Derivative Commodity Instruments and Disclosure of Quantitative Information About Market Risk Inherent in Derivative Financial Instruments, Other Financial Instruments and Derivative Commodity Instruments".

disclosure decision. The last disclosing manager will condition his disclosure decision on the sensitivity previously disclosed by the other firm manager. In contrast, managers' disclosures about firm specific risk are unrelated, even when the disclosure decisions are made sequentially.

We also consider a setting with infinitely many firms. In this setting, each manager's decision whether to disclose precision will affect the firm's cost of capital. This result contributes to current accounting papers on the relation, if any, between the distribution information in the capital markets and firm's cost of equity capital. The existing theoretical literature has not reached agreement on whether the distribution of public and private information in the economy and managers' voluntary disclosures can affect the firms' ex ante cost of equity capital. One avenue for such a link could be the real effects derived from changes in disclosure quality (see Dye (1990), Lambert, Leuz, and Verrecchia (2007a,b) and Hughes, Liu, and Liu (2007)). Hughes et al. (2007) consider the role of private and public information in the limiting case with infinitely many securities. However, none of these papers allow for managers' disclosure decisions to be discretionary.

In our modeling choices, we rely on the extensive literature in finance economics on estimation uncertainty.[2] First, firms' future cash flows are assumed to be normally distributed. This assumption implies that firms have unlimited liability and that stock prices might become negative. Second, all investors are assumed to have constant absolute risk aversion and are price takers such that investors' portfolio decisions lead to endogenously derived market-clearing prices characterized from a representative agent with aggregate risk tolerance as in Wilson (1968). The benefit of these two assumptions is that prices exhibit the mean-variance separability in the absence of estimation risk and further market-clearing prices can be endogenously derived even in the presence of estimation uncertainty.

Third, simple factor models assume that firms' future cash flows are subject to two sources of risk: one market-wide, the other firm-specific. In the estimation risk literature, however, investors are exposed to an additional third source of risk because

[2] These papers include Barry and Brown (1985), Clarkson, Guedes, and Thompson (1996), Coles, and Loewenstein (1988), and Coles, Loewenstein, and Suay (1995).

they do not know one or more parameters that determine the future cash flows. Instead, investors hold common prior beliefs about the distribution of the parameter. Specifically, in our setting, the unknown parameter is the sensitivity of the firm's future cash flows to the market-wide risk factor. Through costly voluntary disclosure about the sensitivity, managers can remove investors' estimation uncertainty. For firms that remove this estimation uncertainty through disclosure, investors require less of a risk premium for holding the stock. However, other firms have unfavorable news (high risk exposure characterized by extreme sensitivities) that do not make discretionary disclosure worthwhile. Rational investors anticipate what type of sensitivities would remain undisclosed and require a higher risk premium for holding such firms' stock. In summary, our paper establishes a link between managers' discretionary disclosures and firms' cost of equity capital.

The paper proceeds as follows: Section 2 outlines current regulation that guides firms' risk disclosures; Section 3 outlines the model; Section 4 solves the benchmark case of a single firm economy; Section 5 describes the model for a two firm economy; Section 6 describes a general multi firm economy; Finally, section 7 summarizes and outlines future research.

2 Regulatory Background

This paper extends our earlier article on risk disclosures where we characterized managers' voluntary disclosures about the firm-specific variance of firms' future cash flows in an equilibrium model without mandatory risk disclosure. While financial firms have long faced risk-based regulation, some non-financial firms also face mandatory risk disclosures since January 1997 when the U.S. Securities and Exchange Commission issued FRR48, which requires the disclosure of information about risks in one of three formats: sensitivity, tabular, or Value-at-Risk. If firms choose the sensitivity format, then FRR48 requires disclosure of the sensitivity to market risk factors of cash flows related only to financial instruments and derivatives. We propose the first theoretical model that analyzes the consequences of mandating firms to disclose their sensitivity.

Under the current SEC standard for market risk disclosures, Financial Reporting Release No. 48 (FRR 48), firms apply one of three different formats: Value-at-Risk, sensitivity, and tabular format. The first format, Value-at-Risk, informs financial statement users about downside risks and variances. The 1994 annual report from Swiss Bank Corporation (SBC) illustrates this:

> "The standard deviation of SBC's daily trading revenue in 1994 worked out at CHF 21 million, virtually unchanged from the previous year. Industry practice in computing "value at risk" is to use two standard deviations rather than one, which effectively doubles SBC's "value-at-risk" number to CHF 42 million – a level that is comparable to the ex-ante indicators computed for other firms."

This illustrates the relation between standard deviation (a two-sided risk measure) and Value-at-Risk (a risk measure that exclusively considers downside risk). In addition to the association with the standard deviation of contemporary trading revenues, Jorion (2002) provides empirical evidence that banks' Value-at-Risk disclosures predict the standard deviation of future trading revenues.

The second risk disclosure format allowed under FRR 48 informs financial statement users about the sensitivity of fair values in changes in exogenous risk factors such as commodity prices, equity prices, exchange rates, and interest rates. As an illustration, consider the following disclosures made by Italian automobile maker, Fiat, in item 11, Quantitative and Qualitative Disclosures about Market Risk, of its 20-F filing:

> **"Changes in Market Risk Exposure Compared to 2005**
>
> Our policy on financial risk management has not substantially changed from the preceding year.
>
> *Exchange Rate Risk*
>
> The characteristics and the mix of our financial instruments with exposure to foreign exchange rate risk at December 31, 2006, have not changed substantially from the preceding year. The increase in the potential loss in fair value arising from a hypothetical 10% change in relevant foreign exchange rates (E460 million at December 31, 2006, as compared to E273 million at December 31, 2005) noted above is the result of an increase in the hedging of the Group's main exposures

and of an extension of its hedging policy to certain entities operating in emerging markets.

Interest Rate Risk

The increase in the potential loss arising from a hypothetical 10% change in relevant interest rates in the fair value of fixed rate financial instruments (E105 million at December 31, 2006, as compared to E33 million as December 31, 2005) reflects the greater weight of fixed rate instruments in the Group's debt portfolio, reflecting in particular the bonds issued during the year.

Other Risks from Derivative Financial Instruments

The increase in the potential loss in fair value in the event of a hypothetical, unfavourable and instantaneous change of 10% in the price of the underlying equities (E40 million at December 31, 2006 as compared to E8 million at December 31, 2005) reflects the impact of new agreements entered into during 2006 and the rise in the market value of the Fiat shares during 2006."

The economic magnitudes of different sensitivities are not immediately comparable without knowing the likelihood of a 10% change in interest rates relative to the likelihood of a 10% change in exchange rates. Nonetheless, sensitivities inform financial statement users about the covariance between fair values and common risk factors related to currency and interest among others.

The third risk disclosure format allowed under FRR48 is the tabular format which provides a table with narrative information. This tabular format requires notional amounts for individual exposures for time horizons of 1 through 5 years and thereafter (similar to operating and capital lease disclosures in footnotes). Hodder and McAnally (2001) show how financial statement users can convert risk disclosures from the tabular format into sensitivity format, while the reverse conversion is not possible. Further, the tabular format alone is insufficient to convert to Value-at-Risk format risk disclosures. This suggests that the different formats may have different information content and that managers can avoid providing sensitivity disclosures and remain in compliance with FRR 48 by choosing the Value-at-Risk format. While firms must disclose using one of the three formats, firms are not required to apply the same format to different sources of risk under FRR 48. Further, firms may opt to disclose sensitivity or Value-at-Risk with regards to earnings, cash flows, or fair values.

Jorion (2002) documents that banks Value-at-Risk (VaR) disclosures are correlated with the variability in subsequent income. Liu et al (2004) show a positive relation between VaR and CAPM beta and future realized stock returns as predicted by Jorgensen and Kirschenheiter (2003). The empirical evidence regarding the market effect of risk disclosures (a.k.a. value relevance) is mixed (see Hodder (2005) and Sribunnak and Wong (2006), among others). These mixed findings could be attributable to FRR 48 being a recent regulation, unfamiliar to investors, or to low quality of risk information.

To summarize, while Value-at-Risk disclosures are disclosures informative about standard deviations or variances, sensitivities are disclosures about *co*variation or correlations. Jorgensen and Kirschenheiter (2003) model voluntary disclosures about variances of firm-specific risks, whereas this paper models voluntary disclosures about sensitivities.

3. Model

In a related paper, Jorgensen and Kirschenheiter (2003) characterize what discretionary risk disclosure managers would make about firm-specific cash flow variance in the absence of any disclosure requirements. In their setting, managers' disclosures affect the equilibrium stock prices, equilibrium stock returns, and the firms' ex ante cost of capital when there is a finite number of firms. In this paper, managers can voluntarily make disclosures of the sensitivity of the firm's future cash flows to a common market-wide risk factor. We summarize the sequence of events in Figure 1 and the notation of our model in Table 1. We next present the assumptions of the model.

Assumption 1 (Cash Flows): The exchange economy includes *J* firms indexed by $j = 1,\ldots,J$. Each firm *j* has a risky investment project in place that pays $\tilde{X}_j$ at time 2. Variables without tilde, such as, X_j, denotes the realization of the corresponding random variable $\tilde{X}_j$. We assume a single market-wide factor model describes cash flow uncertainty, that is, $\tilde{X}_j = \mu_j + \tilde{\gamma}_j \tilde{F} + \tilde{\varepsilon}_j$, where μ_j is the expected cash flow, $\tilde{F}$ denotes the market-wide cash flow factor, $\tilde{\gamma}_j$ is the firm-specific factor loading, or sensitivity of firm *j*'s cash flows to the market-wide factor, and the firm-specific cash flow is $\tilde{\varepsilon}_j$ for $j = 1,...,J$. As is standard, we refer to $\tilde{F}$ and $\tilde{\varepsilon}_j$ as the market-wide risk factor and the firm-specific risk for firm *j*, respectively. We let P_j represent the market value of firm *j* and normalize the total supply of the *j*'th risky asset to one share.

Assumption 2 (Random Variables): We assume that the market-wide factor, $\tilde{F}$, and firm-specific cash flows, $\tilde{\varepsilon}_j$, are Normally distributed with mean zero and variances $\sigma_F^2 > 0$ and $\sigma_\varepsilon^2 > 0$, respectively. The conditional distribution of a firm's future cash flows, $\tilde{X}_j \mid \tilde{\gamma}_j = \gamma_j$, is therefore also Normal with mean μ_j and variance $\gamma_j^2 \sigma_F^2 + \sigma_\varepsilon^2$, where γ_j is the realization of a random variable representing the sensitivity. All primitive random variables $\{\tilde{F}, \tilde{\gamma}_1, \tilde{\varepsilon}_1, ..., \tilde{\gamma}_J, \tilde{\varepsilon}_J\}$ are mutually independent.

Assumption 3 (Managers): At time 0, firm *j*'s manager privately observes the realized sensitivity, γ_j. He then chooses whether to truthfully disclose v_j, incurring exogenous costs $C_j > 0$ if he does disclose and no cost if he does not disclose. Manager *j*'s disclosure strategy is characterized by $N_j \subseteq \Re^+$, the set of values of the sensitivity that the manager *j* will not disclose. Let $\underline{N}$ denote the vector of disclosure strategies for all *J* managers and let $\underline{N}_{-j}$ denote the *(J-1)*-dimensional vector of all firms' disclosure strategies except firm *j*. Each manager holds the same beliefs about other managers' disclosure strategies as do the investors.

Assumption 4 (Investors): There are *I* individuals in the market indexed by $i = 1,...,I$ who have constant absolute risk aversion, $a_i > 0$. Each investor allocates his initial wealth of W_i^0 between purchasing S_{ij} shares of the *j*'th stock and investing B_i in riskless bonds. Investors take as given the share price – or market value – of each firm, P_j, and the return on the bond, R_f. Let $\hat{N}_j \subseteq \Re^+$ denote the set of values of the sensitivity in the firm's future cash flows that the investors believe is manager *j*'s non-disclosure set, where the circumflex or "hat" over the N_j denotes investors' inferences concerning the disclosure strategy that the manager has adopted rather than the manager's actual disclosure strategy. Let $\underline{\hat{N}}$ denote the *J*-dimensional vector of these inferred disclosure strategies. If investors expect full disclosure but a manager does not disclose, investors infer the worst, that is, the investors believe that the manager observed but did not disclose the highest or lowest possible sensitivity. Finally, let $P_j = P_j\left(\gamma_j, N_j \mid \underline{N}_{-j}, \underline{\hat{N}}\right)$ denote price of firm *j*'s shares.

To simplify the presentation, we suppress in our discussion the distinction between investors' beliefs regarding the managers' non-disclosure decision and managers' actual choice of non-disclosure. We also suppress the investors' information that determines prices.

Given the above assumptions, an equilibrium for this exchange economy is characterized by stock prices $\underline{P}^* = \left(P_1^*, ..., P_J^*\right)$ and a set of investor's optimal demand for shares, $\underline{S_i}^* = \left(S_{i1}^*\left(\underline{P}^*\right), ..., S_{iJ}^*\left(\underline{P}^*\right)\right)$, where asterisk superscripts indicate that these are equilibrium values. In equilibrium, market clearing must result, that is, $\sum_{i=1}^{I} S_{ij}^*\left(\underline{P}^*\right) = 1$ for all $j = 1\mathbf{,...,}\, J$.

As a baseline case, we next show the derivation of a manager's discretionary disclosure in an economy with a single firm, where sensitivity disclosures are conceptually the same as disclosures about firm-specific variances.

4. Single Firm Economy

To illustrate our model, this section considers the special case where there is a single firm $J = 1$ and hence we suppress the firm subscript j. We let $\Im$ denote the investors' public information. Following Huang and Litzenberger (1988), we consider the portfolio choice problem face by an individual investor who maximizes his expected utility

$$\max_{B_i, S_i} \quad E\left[U_i\left(\tilde{W}_i\right)\middle|\Im\right] = E\left[-e^{-a_i \tilde{W}_i}\middle|\Im\right] = E\left[-e^{-a_i\left(S_i\left(\tilde{X} - 1_{\{\gamma \notin N\}} C\right) + B_i R_f\right)}\middle|\Im\right]$$

$$s.t. \quad B_i + S_i P \leq W_i^0 \tag{1}$$

Where U_i is the utility function of investor i characterized by his constant risk tolerance, a_i^{-1}. Since each investor's initial budget constraint is binding, we can substitute out the number of bonds through $B_i = W_i^0 - S_i P$. Hence, future wealth can be rewritten as

$$\tilde{W}_i = \left(W_i^0 - S_i P\right)R_f + S_i\left(\tilde{X} - C\right) = W_i^0 R_f + S_i\left(\tilde{X} - 1_{\{\gamma \notin N\}} C - PR_f\right). \tag{2}$$

Each investor's portfolio choice problem now reduces to:

$$\begin{aligned}
&\max_{S_i} \quad E\left[U\left(\tilde{W}_i\right)\middle|\Im\right] = E\left[-e^{-a_i\left(S_i\left(\tilde{X} - 1_{\{\gamma \notin N\}} C - PR_f\right) + W_i^0 R_f\right)}\middle|\Im\right] \\
&= -e^{-a_i W_i^0 R_f} e^{-a_i S_i\left(\mu - 1_{\{\gamma \notin N\}} C - PR_f\right)} E\left[e^{-a_i S_i\left(\tilde{\gamma}\tilde{F} + \tilde{\varepsilon}\right)}\middle|\Im\right] \\
&= -e^{-a_i W_i^0 R_f} e^{-a_i S_i\left(\mu - 1_{\{\gamma \notin N\}} C - PR_f\right)} E\left[e^{-a_i S_i \tilde{\gamma}\tilde{F}}\middle|\Im\right] E\left[e^{-a_i S_i \tilde{\varepsilon}}\right] \\
&= -e^{-a_i W_i^0 R_f} e^{-a_i S_i\left(\mu - 1_{\{\gamma \notin N\}} C - PR_f\right)} E\left[e^{\frac{a_i^2}{2}(S_i\tilde{\gamma})^2 VAR\left[\tilde{F}\middle|\Im\right]}\middle|\Im\right] e^{\frac{a_i^2}{2} S_i^2 VAR[\tilde{\varepsilon}]} \\
&= -e^{-a_i W_i^0 R_f} e^{-a_i S_i\left(\mu - 1_{\{\gamma \notin N\}} C - PR_f\right)} E\left[e^{\frac{a_i^2}{2}(S_i\tilde{\gamma})^2 \sigma_F^2}\middle|\Im\right] e^{\frac{a_i^2}{2} S_i^2 \sigma_\varepsilon^2}
\end{aligned} \tag{3}$$

Given the public information $\Im$, each investor's certainty equivalent $\left(CE_i\middle|\Im\right)$ can be written as:

$$\left(CE_i\middle|\Im\right) = W_i^0 R_f + S_i\left(\mu - 1_{\{\gamma \notin N\}} C - PR_f\right) - a_i^{-1} \ln\left(E\left[e^{\frac{a_i^2}{2}(S_i\tilde{\gamma})^2 \sigma_F^2}\middle|\Im\right]\right) - \frac{a_i}{2} S_i^2 \sigma_\varepsilon^2 \tag{4}$$

Prior to trading, investors will either (i) observe disclosure of $\tilde{\gamma} = \gamma$ or, alternatively, (ii) observe no disclosure and infer that $\tilde{\gamma} = \gamma \in N$. We consider each subgame (i) and (ii) in sequence.

(i) Manager discloses sensitivity $\tilde{\gamma} = \gamma$.

If the manager decides to disclose the sensitivity, then $\{\tilde{\gamma} = \gamma\} \in \Im$ and the investor's certainty equivalent reduces to the familiar mean-variance form:

$$\left(CE_i \middle| \tilde{\gamma} = \gamma\right) = W_i^0 R_f + S_i\left(\mu - C - PR_f\right) - a_i \frac{S_i^2}{2}\left(\gamma^2 \sigma_F^2 + \sigma_\varepsilon^2\right) \qquad \text{(4-i)}$$

The first order condition for an interior solution is:

$$0 = \frac{\partial}{\partial S_i}\left(CE_i \middle| \tilde{\gamma} = \gamma\right) = \left(\mu - C - PR_f\right) - a_i S_i\left(\gamma^2 \sigma_F^2 + \sigma_\varepsilon^2\right) \qquad \text{(5-i)}$$

Such that the demand for shares of each investor i,

$$S_i(P) = a_i^{-1} \frac{\left(\mu - C - PR_f\right)}{\left(\gamma^2 \sigma_F^2 + \sigma_\varepsilon^2\right)}$$

is linear in the price of shares. Aggregating over all investors on both sides, we find the aggregate demand for stock is:

$$\sum_{i=1}^{I} S_i(P) = \sum_{i=1}^{I} a_i^{-1} \frac{\left(\mu - C - PR_f\right)}{\left(\gamma^2 \sigma_F^2 + \sigma_\varepsilon^2\right)}.$$

To ensure market clearing, the right hand side is equal to the aggregate supply of shares:

$$1 = a^{-1} \frac{\left(\mu - C - PR_f\right)}{\left(\gamma^2 \sigma_F^2 + \sigma_\varepsilon^2\right)}$$

which we normalized to one without loss of generality and where $a^{-1} = \sum_{i=1}^{I} a_i^{-1}$ is the aggregate risk tolerance. Rearranging terms, we find that the market clearing price,

$$P^* = \frac{\mu - C - a\left(\gamma^2 \sigma_F^2 + \sigma_\varepsilon^2\right)}{R_f},$$

which is also of the familiar mean-variance separable form. By substituting this market-clearing price into each investor's demand function, we find that $S_i^* = S_i\left(P^*\right) = a_i^{-1} a$, consistent with two fund-separation theorems in finance.

<u>*Proposition 1 (i):*</u>

After investors observe that the manager disclosed sensitivity $\tilde{\gamma} = \gamma$, each investor's demand for shares is $S_i^* = a_i^{-1} a$ and the market clearing price of stock is

$$P^*(\gamma) = \frac{\mu - C - a\left(\gamma^2 \sigma_F^2 + \sigma_\varepsilon^2\right)}{R_f}. \qquad \text{(6-i)}$$

As expected, the market value is lower the higher the exposure to risks.

(ii) Manager does *not* disclose the sensitivity, investors infer that $\tilde{\gamma} = \gamma \in N$.

If the manager decides to disclose the sensitivity, then, with slight abuse of notation, $\{\tilde{\gamma} = \gamma \in N\} \in \Im$. In the absence of disclosure of sensitivity, each investor i chooses to maximize his certainty equivalent:

$$\left(CE_i \middle| \tilde{\gamma} = \gamma \in N\right)$$
$$= W_i^0 R_f + S_i\left(\mu - PR_f\right) - a_i^{-1} \ln\left(E\left[e^{\frac{a_i^2}{2}(S_i\tilde{\gamma})^2\sigma_F^2} \middle| \tilde{\gamma} = \gamma \in N\right]\right) - \frac{a_i}{2}\left(S_i^2\sigma_\varepsilon^2\right) \tag{4-ii}$$

The associated first order condition for an interior maximum is

$$0 = \frac{\partial}{\partial S_i}\left(CE_i \middle| \tilde{\gamma} = \gamma \in N\right) = \left(\mu - PR_f\right) - a_i S_i \Lambda_i\left(N, S_i\right)\sigma_F^2 - a_i S_i \sigma_\varepsilon^2 \tag{5-ii}$$

Where

$$\Lambda_i\left(N, S_i\right) = \frac{E\left[\tilde{\gamma}^2 e^{\frac{a_i^2}{2}(S_i\tilde{\gamma})^2\sigma_F^2} \middle| \tilde{\gamma} = \gamma \in N\right]}{E\left[e^{\frac{a_i^2}{2}(S_i\tilde{\gamma})^2\sigma_F^2} \middle| \tilde{\gamma} = \gamma \in N\right]}$$

Even though this is no longer reducing to the familiar mean-variance form, it is easily verified that the following results in a market clearing:

<u>Proposition 1 (ii):</u>

After investors observe that the manager does not disclose sensitivity, that is, $\tilde{\gamma} = \gamma \in N$, each investor's demand for shares is $S_i^* = a_i^{-1}a$ and the market clearing price of stock is

$$P^*\left(\tilde{\gamma} = \gamma \in N\right) = \frac{\mu - a\left(\Lambda(N)\sigma_F^2 + \sigma_\varepsilon^2\right)}{R_f} \tag{6-ii}$$

where

$$\Lambda(N) = \frac{E\left[\tilde{\gamma}^2 e^{\frac{a^2\sigma_F^2}{2}\tilde{\gamma}^2} \middle| \tilde{\gamma} = \gamma \in N\right]}{E\left[e^{\frac{a^2\sigma_F^2}{2}\tilde{\gamma}^2} \middle| \tilde{\gamma} = \gamma \in N\right]}. \tag{7}$$

To characterize the manager's discretionary disclosure decision, we compare equations (6-i) and (6-ii). When the support of sensitivities are bounded, then managers will (not) make discretionary disclosures of sensitivities when the disclosure costs are sufficiently low (high). In contrast, when the support of sensitivities is unbounded, a partial disclosure equilibrium always exists. We focus on the partial disclosure equilibrium, which is characterized by one (or two) disclosure threshold(s) determined by finding the $\tilde{\gamma} = \bar{\gamma}$ such that the disclosure price coincides with the no disclosure price, that is,

$$\frac{\mu - C - a\left(\bar{\gamma}^2\sigma_F^2 + \sigma_\varepsilon^2\right)}{R_f} = \frac{\mu - a\left(\Lambda(N)\sigma_F^2 + \sigma_\varepsilon^2\right)}{R_f}$$

which reduces to

$$\bar{\gamma}^2 + a^{-1}C\sigma_F^{-2} = \Lambda(N).$$

<u>Proposition 2:</u>

When J=1, the disclosure set supporting any partial disclosure equilibrium is $(-\bar{\gamma}, \bar{\gamma})$.

It is intuitive that the disclosure set includes zero. Clearly, a manager who observes the most favorable news, corresponding to no exposure to the market wide risk, $\tilde{\gamma} = 0$, is the most immediate candidate for disclosure. Second, the symmetry of the disclosure set is seen from the observation that (6-i) depends on the sensitivity squared.

Because we only considered a single firm economy so far, the above findings are essentially a reformulation of Jorgensen and Kirschenheiter (2003) where the manager, instead, could disclose firm specific cash flow variances. The remainder of the paper will demonstrate how sensitivity disclosures differ qualitatively from disclosures about firm-specific variances in a multi firm economy in two respects. In the next section, we show that sequential discretionary sensitivity disclosures create spill over effects between firms that disclose sensitivity early and subsequent discretionary disclosure decisions. Then in section 6, we consider the limiting case of infinitely many stocks and show that discretionary disclosures still have an effect on prices and stock returns.

5. Two Firm Economy

Consider the special case where there are two firms, that is, $J = 2$. After substitution of initial binding budget constraint, $B_i = W_i^0 - \sum_{j=1}^{2} S_{ij} P_j$, each investor's terminal wealth is

$$\tilde{W}_i = \left(W_i^0 - \sum_{j=1}^{2} S_{ij} P_j \right) R_f + \sum_{j=1}^{2} S_{ij} \left(\tilde{X}_j - 1_{\{\gamma_j \notin N_j\}} C_j \right)$$

$$= W_i^0 R_f + \sum_{j=1}^{2} S_{ij} \left(\tilde{X}_j - 1_{\{\gamma_j \notin N_j\}} C_j - P_j R_f \right)$$

The investor's expected utility optimization problem is:

$$\max_{S_i} \quad E\left[U_i\left(\tilde{W}_i\right) \middle| \Im \right] = E\left[-e^{-a_i\left(W_i^0 R_f + \sum_{j=1}^{2} S_i\left(\tilde{X}_j - PR_f\right)\right)} \middle| \Im \right]$$

$$= -e^{-a_i W_i^0 R_f} e^{-a_i S_{i1}\left(\mu_1 - 1_{\{\gamma_1 \notin N_1\}} C_1 - P_1 R_f\right) + S_{i2}\left(\mu_2 - 1_{\{\gamma_2 \notin N_2\}} C_1 - P_2 R_f\right)} E\left[e^{-a_i\left(S_{i1}\left(\tilde{\gamma}_1 \tilde{F} + \tilde{\varepsilon}_1\right) + S_{i2}\left(\tilde{\gamma}_2 \tilde{F} + \tilde{\varepsilon}_2\right)\right)} \middle| \Im \right]$$

Where $\Im$ is the investors' public information. To evaluate the expected values, note that

$$E\left[e^{-a_i\left(S_{i1}\left(\tilde{\gamma}_1 \tilde{F} + \tilde{\varepsilon}_1\right) + S_{i2}\left(\tilde{\gamma}_2 \tilde{F} + \tilde{\varepsilon}_2\right)\right)} \middle| \Im \right]$$

$$= E\left[e^{-a_i\left(S_{i1}\tilde{\gamma}_1 + S_{i2}\tilde{\gamma}_2\right)\tilde{F}} \middle| \Im \right] E\left[e^{-a_i\left(S_{i1}\tilde{\varepsilon}_1 + S_{i2}\tilde{\varepsilon}_2\right)} \right]$$

$$= E\left[e^{\frac{a_i^2}{2}\left(S_{i1}\tilde{\gamma}_1 + S_{i2}\tilde{\gamma}_2\right)^2 VAR\left[\tilde{F} \middle| \Im\right]} \middle| \Im \right] e^{\frac{a_i^2}{2}\left(S_{i1}^2 VAR[\tilde{\varepsilon}_1] + S_{i2}^2 VAR[\tilde{\varepsilon}_2]\right)}$$

$$= E\left[e^{\frac{a_i^2}{2}\left(S_{i1}\tilde{\gamma}_1 + S_{i2}\tilde{\gamma}_2\right)^2 \sigma_F^2} \middle| \Im \right] e^{\frac{a_i^2}{2}\left(S_{i1}^2 + S_{i2}^2\right)\sigma_\varepsilon^2}$$

It follows that each investor i maximizes his certainty equivalent

$$\left(CE_i \middle| \Im\right) = W_i^0 R_f + \sum_{j=1}^{2} S_{ij}\left(\mu_j - 1_{\{\gamma_j \notin N_j\}} C_j - P_j R_f\right) - a_i^{-1} \ln\left(E\left[e^{\frac{a_i^2}{2}\left(\sum_{j=1}^{2} S_{ij}\tilde{\gamma}_j\right)^2 \sigma_F^2} \middle| \Im \right] \right) - \frac{a_i}{2}\left(\sum_{j=1}^{2} S_{ij}^2 \right)\sigma_\varepsilon^2$$

When investors trade, they are in one of the following four possible subgames:

(I) Both firms disclose and investors observe $\tilde{\gamma}_1 = \gamma_1$ and $\tilde{\gamma}_2 = \gamma_2$,

(II) Only firm 1 discloses and investors observe $\tilde{\gamma}_1 = \gamma_1$ and infer $\tilde{\gamma}_2 = \gamma_2 \in N_2$,

(III) Only firm 2 discloses and investors observe $\tilde{\gamma}_2 = \gamma_2$ and infer $\tilde{\gamma}_1 = \gamma_1 \in N_1$,

(IV) Neither firm disclose and investors infer that $\tilde{\gamma}_1 = \gamma_1 \in N_1$ and $\tilde{\gamma}_2 = \gamma_2 \in N_2$.

Again, we consider each subgame separately noting that (II) and (III) are parallel.

(I) Both managers discloses sensitivities $\tilde{\gamma}_1 = \gamma_1$ and $\tilde{\gamma}_2 = \gamma_2$.

We can rewrite each investor i's certainty equivalent on mean-variance separable form:

$$\begin{aligned}\left(CE_i \middle| \tilde{\gamma}_1 = \gamma_1, \tilde{\gamma}_2 = \gamma_2\right) &= W_i^0 R_f + \sum_{j=1}^{2} S_{ij}\left(\mu_j - C_j - P_j R_f\right) \\ &\quad - \frac{a_i}{2}\left\{S_{i1}^2\left(\gamma_1^2\sigma_F^2 + \sigma_\varepsilon^2\right) + S_{i2}^2\left(\gamma_2^2\sigma_F^2 + \sigma_\varepsilon^2\right) + 2S_{i1}S_{i2}\gamma_1\gamma_2\sigma_F^2\right\}\end{aligned} \tag{4-I}$$

The first order conditions for an interior maximum are

$$\begin{aligned}0 &= \frac{\partial}{\partial S_{i1}}\left(CE_i \middle| \tilde{\gamma}_1 = \gamma_1, \tilde{\gamma}_2 = \gamma_2\right) = \left(\mu_1 - C_1 - P_1 R_f\right) - a_i S_{i1}\left(\gamma_1^2\sigma_F^2 + \sigma_\varepsilon^2\right) - a_i S_{i2}\gamma_1\gamma_2\sigma_F^2 \\ 0 &= \frac{\partial}{\partial S_{i2}}\left(CE_i \middle| \tilde{\gamma}_1 = \gamma_1, \tilde{\gamma}_2 = \gamma_2\right) = \left(\mu_2 - C_2 - P_2 R_f\right) - a_i S_{i2}\left(\gamma_2^2\sigma_F^2 + \sigma_\varepsilon^2\right) - a_i S_{i1}\gamma_1\gamma_2\sigma_F^2\end{aligned} \tag{5-I}$$

Proceeding as in case (i) of section 4, we can show that the equilibrium for (I) is

$$\begin{aligned}P_1^* &= \frac{\left(\mu_1 - C_1\right) - a\left(\gamma_1^2\sigma_F^2 + \sigma_\varepsilon^2 + \gamma_1\gamma_2\sigma_F^2\right)}{R_f} \\ P_2^* &= \frac{\left(\mu_2 - C_2\right) - a\left(\gamma_2^2\sigma_F^2 + \sigma_\varepsilon^2 + \gamma_1\gamma_2\sigma_F^2\right)}{R_f}\end{aligned} \tag{6-I}$$

By substitution, it is easily verified that $S_{i1}\left(P_1^*, P_2^*\right) = S_{i2}\left(P_1^*, P_2^*\right) = a_i^{-1}a$.

(II) Only firm 1 discloses and investors observe $\tilde{\gamma}_1 = \gamma_1$ and infer $\tilde{\gamma}_2 = \gamma_2 \in N_2$.

And the individual investor's certainty equivalent is

$$\begin{aligned}&\left(CE_i \middle| \tilde{\gamma}_1 = \gamma_1, \tilde{\gamma}_2 = \gamma_2 \in N_2\right) \\ &= W_i^0 R_f + S_{i1}\left(\mu_1 - C_1 - P_1 R_f\right) + S_{i2}\left(\mu_2 - P_2 R_f\right) - \frac{a_i}{2} S_{i1}^2 \gamma_1^2 \sigma_F^2 - \frac{a_i}{2}\left(S_{i1}^2 + S_{i2}^2\right)\sigma_\varepsilon^2 \\ &- a_i^{-1}\ln\left(E\left[e^{a_i^2\left(S_{i1}S_{i2}\gamma_1\right)\tilde{\gamma}_2\sigma_F^2 + \frac{a_i^2}{2}S_{i2}^2\tilde{\gamma}_2^2\sigma_F^2} \middle| \tilde{\gamma}_2 = \gamma_2 \in N_2\right]\right)\end{aligned} \tag{4-II}$$

The first order conditions for an interior solution are

$$0=\frac{\partial}{\partial S_{i1}}\left(CE_i\middle|\tilde{\gamma}_1=\gamma_1,\tilde{\gamma}_2=\gamma_2\in N_2\right)$$

$$=\left(\mu_1-C_1-P_1R_f\right)-a_iS_{i1}\gamma_1^2\sigma_F^2-a_iS_{i1}\sigma_\varepsilon^2-a_i^{-1}\frac{E\left[a_i^2S_{i2}\gamma_1\tilde{\gamma}_2\sigma_F^2e^{a_i^2(S_{i1}S_{i2}\gamma_1)\tilde{\gamma}_2\sigma_F^2+\frac{a_i^2}{2}S_{i2}^2\tilde{\gamma}_2^2\sigma_F^2}\middle|\tilde{\gamma}_2=\gamma_2\in N_2\right]}{E\left[e^{a_i^2(S_{i1}S_{i2}\gamma_1)\tilde{\gamma}_2\sigma_F^2+\frac{a_i^2}{2}S_{i2}^2\tilde{\gamma}_2^2\sigma_F^2}\middle|\tilde{\gamma}_2=\gamma_2\in N_2\right]} \qquad (5\text{-II})$$

And

$$0=\frac{\partial}{\partial S_{i2}}\left(CE_i\middle|\tilde{\gamma}_1=\gamma_1,\tilde{\gamma}_2=\gamma_2\in N_2\right)$$

$$=\left(\mu_2-P_2R_f\right)-a_iS_{i2}\sigma_\varepsilon^2-a_i^{-1}\frac{E\left[a_i^2\left(S_{i1}\gamma_1\tilde{\gamma}_2+S_{i2}\tilde{\gamma}_2^2\right)\sigma_F^2e^{a_i^2(S_{i1}S_{i2}\gamma_1)\tilde{\gamma}_2\sigma_F^2+\frac{a_i^2}{2}S_{i2}^2\tilde{\gamma}_2^2\sigma_F^2}\middle|\tilde{\gamma}_2=\gamma_2\in N_2\right]}{E\left[e^{a_i^2(S_{i1}S_{i2}\gamma_1)\tilde{\gamma}_2\sigma_F^2+\frac{a_i^2}{2}S_{i2}^2\tilde{\gamma}_2^2\sigma_F^2}\middle|\tilde{\gamma}_2=\gamma_2\in N_2\right]} \qquad (5\text{-II})$$

These first order conditions reduce to

$$\left(\mu_1-C_1-P_1R_f\right)a_i^{-1}=S_{i1}\left(\gamma_1^2\sigma_F^2+\sigma_\varepsilon^2\right)+S_{i2}\gamma_1\Lambda_{i1}\sigma_F^2$$
$$\left(\mu_2-P_2R_f\right)a_i^{-1}=S_{i2}\sigma_\varepsilon^2+\left(S_{i1}\gamma_1\Lambda_{i1}+S_{i2}\Lambda_{i2}\right)\sigma_F^2$$

Where

$$\Lambda_{i1}\left(\gamma_1,N_2;S_{i1},S_{i2}\right)=\frac{E\left[\tilde{\gamma}_2e^{a_i^2(S_{i1}S_{i2}\gamma_1)\tilde{\gamma}_2\sigma_F^2+\frac{a_i^2}{2}S_{i2}^2\tilde{\gamma}_2^2\sigma_F^2}\middle|\tilde{\gamma}_2=\gamma_2\in N_2\right]}{E\left[e^{a_i^2(S_{i1}S_{i2}\gamma_1)\tilde{\gamma}_2\sigma_F^2+\frac{a_i^2}{2}S_{i2}^2\tilde{\gamma}_2^2\sigma_F^2}\middle|\tilde{\gamma}_2=\gamma_2\in N_2\right]}$$

$$\Lambda_{i2}\left(\gamma_1,N_2:S_{i1},S_{i2}\right)=\frac{E\left[\tilde{\gamma}_2^2e^{a_i^2(S_{i1}S_{i2}\gamma_1)\tilde{\gamma}_2\sigma_F^2+\frac{a_i^2}{2}S_{i2}^2\tilde{\gamma}_2^2\sigma_F^2}\middle|\tilde{\gamma}_2=\gamma_2\in N_2\right]}{E\left[e^{a_i^2(S_{i1}S_{i2}\gamma_1)\tilde{\gamma}_2\sigma_F^2+\frac{a_i^2}{2}S_{i2}^2\tilde{\gamma}_2^2\sigma_F^2}\middle|\tilde{\gamma}_2=\gamma_2\in N_2\right]}$$

In equilibrium for subgame (II), two fund separation still holds $S_{i1}\left(P_1^*,P_2^*\right)=S_{i2}\left(P_1^*,P_2^*\right)=a_i^{-1}a$ and market clearing prices are:

$$P_1^*=\frac{\mu_1-C_1-a\left\{\left(\gamma_1^2\sigma_F^2+\sigma_\varepsilon^2\right)+\gamma_1\Lambda_1\sigma_F^2\right\}}{R_f}$$
$$P_2^*=\frac{\mu_2-a\left\{\sigma_\varepsilon^2+\left(\gamma_1\Lambda_1+\Lambda_2\right)\sigma_F^2\right\}}{R_f}$$

where

$$\Lambda_1(\gamma_1, N_2) = \frac{E\left[\tilde{\gamma}_2 e^{a^2\gamma_1\tilde{\gamma}_2\sigma_F^2 + \frac{a^2}{2}\tilde{\gamma}_2^2\sigma_F^2} \middle| \tilde{\gamma}_2 = \gamma_2 \in N_2\right]}{E\left[e^{a^2\gamma_1\tilde{\gamma}_2\sigma_F^2 + \frac{a^2}{2}\tilde{\gamma}_2^2\sigma_F^2} \middle| \tilde{\gamma}_2 = \gamma_2 \in N_2\right]}$$

$$\Lambda_2(\gamma_1, N_2) = \frac{E\left[\tilde{\gamma}_2^2 e^{a^2\gamma_1\tilde{\gamma}_2\sigma_F^2 + \frac{a^2}{2}\tilde{\gamma}_2^2\sigma_F^2} \middle| \tilde{\gamma}_2 = \gamma_2 \in N_2\right]}{E\left[e^{a^2\gamma_1\tilde{\gamma}_2\sigma_F^2 + \frac{a^2}{2}\tilde{\gamma}_2^2\sigma_F^2} \middle| \tilde{\gamma}_2 = \gamma_2 \in N_2\right]}$$

Or

$$\Lambda_1(\gamma_1, N_2) = E^*[\tilde{\gamma}_2]$$
$$\Lambda_2(\gamma_1, N_2) = E^*[\tilde{\gamma}_2^2]$$

Where expectations are taken over the following probability measure:

$$q^*(\gamma_2 \mid \gamma_1) = \frac{e^{a^2\gamma_1\tilde{\gamma}_2\sigma_F^2 + \frac{a^2}{2}\tilde{\gamma}_2^2\sigma_F^2}}{E\left[e^{a^2\gamma_1\tilde{\gamma}_2\sigma_F^2 + \frac{a^2}{2}\tilde{\gamma}_2^2\sigma_F^2} \middle| \tilde{\gamma}_2 = \gamma_2 \in N_2\right]} \quad for\ \gamma_2 \in N_2 .$$

In a sequential disclosure setting, where manager 1 has already disclosed his firm's sensitivity, manager 2's non-disclosure set is $N_2 = \{\gamma_2 \le \underline{\gamma}_2(\gamma_1)\} \cup \{\gamma_2 \ge \bar{\gamma}_2(\gamma_1)\}$. Where the disclosure threshold(s), $\bar{\gamma}_2(\gamma_1)$ and $\underline{\gamma}_2(\gamma_1)$, are determined by:

$$(\gamma_1\Lambda_1(\gamma_1, N_2) + \Lambda_2(\gamma_1, N_2)) = C_2 a^{-1}\sigma_F^{-2} + \left(\{\bar{\gamma}_2(\gamma_1)\}^2 + \gamma_1\bar{\gamma}_2(\gamma_1)\right)$$

and

$$(\gamma_1\Lambda_1(\gamma_1, N_2) + \Lambda_2(\gamma_1, N_2)) = C_2 a^{-1}\sigma_F^{-2} + \left(\{\underline{\gamma}_2(\gamma_1)\}^2 + \gamma_1\underline{\gamma}_2(\gamma_1)\right).$$

If $\gamma_1 = 0$ then these disclosure threshold(s) reduce to the single firm case (J=1).

__Proposition 3:__

Suppose that the manager of firm 1 (representing the rest of the market) had disclosed sensitivity before manager 2 makes his disclosure decision, then manager 2's disclosure decision may depend on the sensitivity of firm 1 even though the distribution of the sensitivities are independent.

6. Limiting Case of Infinite Firm Economy

For the general case where there are J firms, the proof is parallel to what is outlined above. In the limit, as the number of firms and investors go to infinity, each firm's decision to either disclose or not disclose can make a difference. Then, for intermediate values of disclosure costs, managers with low sensitivity will disclose, while manager's with high sensitivity will withhold their information about sensitivities. Consider the case where sensitivities take one of two values, low (say zero) and high. Even in this degenerate case where non-disclosure does not carry any additional risk, still the decision to disclose or withhold information about sensitivities will affect the equilibrium price of each firm.

7. Summary and Future Research

This paper considers disclosures about correlations in the form of sensitivities. First, we relate disclosures about correlations to disclosures about variances. Second, we consider a sequential disclosure setting with two firms where the second manager to disclose sensitivity would prefer to disclose sensitivities that are of opposite sign of sensitivity previously disclosed by the other manager. Third, we document how discretionary disclosures about sensitivities can affect a firm's cost of equity capital even in the limiting case of infinitely many firms where firm-specific risks are perfectly diversifiable. Consequently, managers' discretionary disclosures about correlations can affect the firm's cost of capital even though managers' disclosures about the variance of firm-specific cash flow risk, that is diversifiable, would not affect each firm's cost of capital.

The current paper was set in a stylized setting without real effects. Recent papers, including Kanodia et al. (2000) and Magee (2006), among others, document real effect from disclosure regulation on managers' hedging, operating, and financing decisions. Based on that literature, we would expect that one manager's discretionary disclosures of sensitivities can have spillover effects on other firms' real decisions.

Prior research on risk disclosures considers managers' discretionary disclosure of a perfect signal about the firm's risks. Yet, the models for risk measurement continue to evolve and some regulators have expressed concerns about whether we have appropriate risk measurements.[3] One possible extension would be to introduce imperfect signals about sensitivity disclosures and investigate the quality of sensitivity disclosures.

APPENDIX: Proof of Equilibrium for General Case of J>1

To analyze the general case, we need to introduce more notation. First, we use the indicator function $1_{\{\gamma_j \notin N_j\}}$ to denote whether manager j did disclose his firm's sensitivity. Second, we let the J-dimensional vector $\underline{\Im}$ summarize the investors' information set, where its j'th element takes the value γ_j if the manager disclosed, and otherwise Ø.

The terminal wealth of an individual investor is:

$$\tilde{W}_i = \left(W_i^0 - \sum_{j=1}^{J} S_{ij}P_j\right)R_f + \sum_{j=1}^{J} S_{ij}\left(\tilde{X}_j - 1_{\{\gamma_j \notin \hat{N}_j\}}C_j\right) = W_i^0 R_f + \sum_{j=1}^{J} S_{ij}\left(\tilde{X}_j - 1_{\{\gamma_j \notin \hat{N}_j\}}C_j - P_j R_f\right)$$

and this investor's expected utility optimization problem is:

$$\max_{S_i} \quad E\left[U_i\left(\tilde{W}_i\right)\middle|\underline{\Im}\right] = E\left[-e^{-a_i\left(W_i^0 R_f + \sum_{j=1}^{J} S_{ij}\left(\tilde{X}_j - 1_{\{\gamma_j \notin \hat{N}_j\}}C_j - PR_f\right)\right)}\middle|\underline{\Im}\right]$$

$$= -e^{-a_i W_i^0 R_f} e^{-a_i \sum_{j=1}^{J} S_{ij}\left(\mu_j - 1_{\{\gamma_j \notin \hat{N}_j\}}C_j - P_j R_f\right)} E\left[e^{-a_i\left(\sum_{j=1}^{J} S_{ij}\left(\tilde{\gamma}_j \tilde{F} + \tilde{\varepsilon}_j\right)\right)}\middle|\underline{\Im}\right]$$

$$= -e^{-a_i W_i^0 R_f} e^{-a_i \sum_{j=1}^{J} S_{ij}\left(\mu_j - 1_{\{\gamma_j \notin \hat{N}_j\}}C_j - P_j R_f\right)} E\left[e^{-a_i\left(\sum_{j=1}^{J} S_{ij}\tilde{\gamma}_j\right)\tilde{F}}\middle|\underline{\Im}\right] \prod_{j=1}^{J} E\left[e^{-a_i S_{ij}\tilde{\varepsilon}_j}\right]$$

$$= -e^{-a_i W_i^0 R_f} e^{-a_i \sum_{j=1}^{J} S_{ij}\left(\mu_j - 1_{\{\gamma_j \notin \hat{N}_j\}}C_j - P_j R_f\right)} E\left[e^{\frac{a_i^2}{2}\left(\sum_{j=1}^{J} S_{ij}\tilde{\gamma}_j\right)^2 \sigma_F^2}\middle|\underline{\Im}\right] \prod_{j=1}^{J} e^{\frac{a_i^2}{2} S_{ij}^2 VAR[\tilde{\varepsilon}_j]}$$

It follows that each investor i maximizes his certainty equivalent

$$CE_i = W_i^0 + \sum_{j=1}^{J} S_{ij}\left(\mu_j - 1_{\{\gamma_j \notin \hat{N}_j\}}C_j - P_j R_f\right) - a_i^{-1}\ln\left(E\left[e^{\frac{a_i^2}{2}\left(\sum_{j=1}^{J} S_{ij}\tilde{\gamma}_j\right)^2 \sigma_F^2}\middle|\underline{\Im}\right]\right) - \frac{a_i}{2}\left(\sum_{j=1}^{J} S_{ij}^2\right)\sigma_\varepsilon^2$$

[3] For example, Sheila Bair, chairman of the U.S. Federal Deposit Insurance Corporation, said that the recent financial market turmoil "reinforced her worries about the 'untried' advanced approaches to assessing bank risks under the contentious international Basel II bank capital adequacy rules" which will determine banks' minimum capital requirement. Source: GRR News Service: OTS's Reich says turmoil will aid effectiveness of Basel II models. October 4, 2007.

The first order condition for each stock j is

$$0=\frac{\partial CE_i}{\partial S_{ij}}=\left(\mu_j-1_{\{\gamma_j\notin\hat{N}_j\}}C_j-P_jR_f\right)-a_i^{-1}\frac{E\left[a_i^2\tilde{\gamma}_j\sum_{k=1}^{J}S_{ik}\tilde{\gamma}_k\sigma_F^2 e^{\frac{a_i^2}{2}\left(\sum_{q=1}^{J}S_{iq}\tilde{\gamma}_q\right)^2\sigma_F^2}\middle|\underline{\Im}\right]}{E\left[e^{\frac{a_i^2}{2}\left(\sum_{q=1}^{J}S_{iq}\tilde{\gamma}_q\right)^2\sigma_F^2}\middle|\underline{\Im}\right]}-a_iS_{ij}\sigma_\varepsilon^2$$

This reduces to

$$\left(\mu_j-1_{\{\gamma_j\notin\hat{N}_j\}}C_j-P_jR_f\right)=a_i\sum_{k=1}^{J}S_{ik}\frac{E\left[\tilde{\gamma}_j\tilde{\gamma}_k e^{\frac{a_i^2}{2}\left(\sum_{q=1}^{J}S_{iq}\tilde{\gamma}_q\right)^2\sigma_F^2}\middle|\underline{\Im}\right]}{E\left[e^{\frac{a_i^2}{2}\left(\sum_{q=1}^{J}S_{iq}\tilde{\gamma}_q\right)^2\sigma_F^2}\middle|\underline{\Im}\right]}\sigma_F^2+a_iS_{ij}\sigma_\varepsilon^2.$$

Define

$$\Lambda_{jk}\left(\underline{\Im}\right)=\frac{E\left[\tilde{\gamma}_j\tilde{\gamma}_k e^{\frac{a^2}{2}\left(\sum_{q=1}^{J}\tilde{\gamma}_q\right)^2\sigma_F^2}\middle|\underline{\Im}\right]}{E\left[e^{\frac{a^2}{2}\left(\sum_{q=1}^{J}\tilde{\gamma}_q\right)^2\sigma_F^2}\middle|\underline{\Im}\right]}$$

then the market-clearing prices are

$$P_j^*R_f=\mu_j-1_{\{\gamma_j\notin\hat{N}_j\}}C_j-a\left(\sum_{k=1}^{J}\Lambda_{jk}\left(\underline{\Im}\right)\sigma_F^2+\sigma_\varepsilon^2\right)$$

for all j, and two fund separation holds such that $S_{ij}^*=a_i^{-1}a$ for a. Note that when both sensitivities are disclosed, $\Lambda_{jk}=\gamma_j\gamma_k$. The rest is as in case (IV) of J=2 (not presented above).

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Table 1: Notation

I	number of investors
i	investor index
J	number of firms
j	firm index
a_i	constant absolute risk aversion of investor i
a^{-1}	aggregate risk tolerance of representative e investor
B_i	wealth invested in safe bonds by investor i
C_j	disclosure costs for firm j
F	common risk factor k, $F \sim N\left(0, \sigma_F^2\right)$
P_j	market value of firm j after sensitivity disclosures may be made if any
R_f	gross return on the safe bonds
S_{ij}	fraction of shares held in stock j by investor i
W_i^0	initial wealth of investor i
X_j	total cash flows of firm j (gross of disclosure costs), $X_j \mid \gamma_j \sim N\left(\mu_j, \gamma_j^2 \sigma_F^2 + \sigma_\varepsilon^2\right)$
ε_j	firm-specific cash flow risk of firm j, $\varepsilon_j \sim N\left(0, \sigma_\varepsilon^2\right)$
μ_j	expected cash flows from firm j
γ_j	risk sensitivity of firm j to factor k or "factor loading"

Figure 1: Timeline