Abstract

The paper studies a manager's decision whether to make a disclosure and, if he chooses to do so, by how much to bias the report. The paper illustrates that these decisions are interrelated. In particular, the paper studies a model similar to Myers and Majluf (1984) in which an entrepreneur who is privately informed about the value of his firm's assets in place may issue equity to finance a profitable investment opportunity. In contrast to Myers and Majluf (1984), our model does not assume that the manager is unable to communicate any of his private information. Instead, our model assumes that the manager can voluntarily disclose his private information. However, he is not confined to tell the truth but incurs a personal cost from biasing his report.

The model shows that treating managers’ disclosure and investment decisions both as endogenous yields qualitatively different predictions than when the disclosure and investment decisions are considered separately. In particular, we find that in contrast to traditional voluntary disclosure models, in which there exists a single threshold above which firms choose to disclose their information and below which they withhold it, our model predicts that managers sometimes disclose low and high asset values but do not disclose intermediate asset values. Moreover, in contrast to Myers and Majluf (1984), the manager does not pursue a profitable investment opportunity when the value of the firm’s assets is in an intermediate range while the manager pursues the investment opportunity when the value of his firm’s assets is either low or high. The model also predicts that (i) the underinvestment problem is more severe when the return on investment is low; and (ii) low-performing firms have (weakly) higher cost of capital than high-performing firms.
1 Introduction

Firms’ real decisions and disclosure decisions are closely intertwined. The information firms disclose to capital markets affects firms’ real decisions because it alters the information asymmetry between the firm and investors. As a result, firms’ disclosure decisions determine their access to capital markets and the rate of return investors require when investing with the firm. In turn, the price at which a firm can raise additional capital determines the feasibility and profitability of new investment opportunities. In that way, firms’ disclosure decisions directly affect their investment decisions and the distribution of their future cash flows. That is, firms’ disclosure decisions have “real effects.” The objective of this paper is to study how firms’ voluntary disclosure decisions affect their investment decisions and vice versa.

To illustrate the interdependencies between firms’ disclosure and investment decisions, the paper studies a firm that is lacking the capital to pursue a profitable investment opportunity. Myers and Majluf (1984) have shown that in the presence of information asymmetry between the firm and its potential investors regarding the value of the firm’s assets in place, the firm sometimes forgoes the profitable investment opportunity. This results in inefficient investment behavior. The model in Myers and Majluf (1984) assumes that firms lack the ability to communicate their private information to potential investors. However, in reality, firms can issue reports to inform potential investors, though firms might not be able to issue perfectly credible or verifiable reports. Rather, managers can and do manipulate the reports they issue due to various incentives they face (see for example, Burgstahler and Dichev 1997; Teoh, Welch and Wong 1998a,b). In order to capture these stylized facts, the model assumes that managers can voluntarily issue a report, however, they are not confined to tell the truth and incur a cost from biasing the report. The model demonstrates that the manager’s investment decision and disclosure decision are interdependent. Moreover, the model shows that the equilibrium characteristics of corporate investment and disclosure strategies are qualitatively different when studied jointly than when studied independently. Hence, it is essential to study the corporate investment and disclosure strategies in a model in which both are determined endogenously.

While costly reporting distortions have been widely studied in mandatory disclosure settings

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1 Of course, there exist additional interdependencies between firms’ real decisions and disclosure decisions. For instance, more and better information can make contracts more efficient. For a detailed discussion of the “real effects” perspective on disclosure decisions see Kanodia (2006).
they have been largely ignored in voluntary disclosure settings.\(^2\) Instead, most voluntary disclosure models assume that any disclosure has to be truthful (e.g., Verrecchia 1983, Dye 1985, Jung and Kwon 1988) while some disclosure models take the opposite extreme viewpoint and assume that misreporting is costless (e.g., cheap talk models in Stocken 2000 and Fischer and Stocken 2001). In this paper, we cover the “middle ground” by assuming that disclosures do not have to be truthful and that reporting distortions are costly to the manager. This assumption is descriptive of corporate disclosures which managers have the flexibility to manipulate but incur costs when doing so. In reality, managers generally enjoy reporting discretion due to the forward looking nature of many voluntary disclosures and the inherent flexibility in Generally Accepted Accounting Principles (GAAP).\(^3\) Although managers have the ability to manipulate their reports they incur costs from doing so. These manipulation costs may stem from potential litigation costs or reputation concerns.

In addition to the reporting flexibility, the model is designed to capture two other characteristics of the corporate disclosure environment. First, in the model neither the firm nor the manager incur costs associated with making a disclosure per se (in contrast to, e.g., Verrecchia 1983). In reality, the actual costs of making the disclosure – including, e.g., costs of distributing press releases or holding conference calls – are likely to be small due to modern information technology. For that reason, we assume in the model that these costs are zero. Second, in the model the manager always obtains private information (in contrast to, e.g., Dye 1985 and Jung and Kwon 1988). While in certain settings it is likely that the manager obtains private information probabilistically, the manager is likely to always have private information about certain aspects of the firm’s existing business. Hence, the model assumes that the manager always obtains private information about the value of the firm’s assets in place.

The model illustrates that the manager sometimes withholds his private information in equilibrium even though the manager always obtains private information and there are no costs associated with making a disclosure per se. The reason is that even though the manager has the opportunity to issue a report without incurring any costs, in equilibrium the manager cannot costlessly commu-

\(^2\)Costly state falsification models include models of earnings management by Stein (1989); Fischer and Verrecchia (2000); Sankar and Subramanyam (2001); Dye and Sridhar (2004); Guttman, Kadan and Kandel (2006). As far as we are aware, the only paper studying costly misreporting in a voluntary disclosure setting is Korn (2004).

\(^3\)At times of initial public offering, firms have even more discretion in preparing the report in the offering prospectus, since the current accounting regulation (APB 20) allows IPO firms to change their accounting choices via retroactive restatement for all the financial statements presented in the offering prospectus. An IPO setting increases the discretion of the firm, however, it also increases the potential costs associated with manipulation of their disclosure.
nicate the actual value of his firm’s assets in place. If a manager were to choose the report that does not impose disclosure costs, investors would perceive the firm’s assets to be less valuable than they actually are. The model further illustrates that due to the interaction of investment and disclosure behavior, the manager’s disclosure strategy does not always take the common form of a “threshold” equilibrium in which the manager withholds information when the value of his firm’s assets is below the threshold and discloses information when the value is above the threshold. Instead, the manager sometimes discloses low and high asset values but withholds intermediate asset values. Similarly, the manager’s investment strategy also no longer takes the form of a threshold below which the manager invests and above which he foregoes the profitable investment opportunity — in contrast to Myers and Majluf 1984. This is due to the interdependencies between the manager’s disclosure and investment decision.

As it is often the case in disclosure games with continuous support, multiple equilibria with pooling reports may evolve (see, e.g., Guttman et al. 2006). Solving for the entire set of equilibria that involve such pooling behavior proves intractable. Therefore, as common in the literature (e.g., Riley 1979, Miller and Rock 1985), we study equilibria in which — whenever a manager issues a report — it fully reveals the manager’s private information to investors. In the absence of a report, investors can only infer the average value of the firm’s assets in place for which the manager remains silent. In that sense, this model is qualitatively different from costly signaling games in mandatory disclosure settings in which investors are always able to infer the manager’s private information (e.g., Stein 1989). In the following, we expand on the model’s predictions regarding the manager’s investment and reporting behavior and the information investors can infer from it.

First, the model predicts that if the investment opportunity is sufficiently profitable, the manager always invests in it resulting in efficient investment behavior. This is intuitive. If the profitability of the investment opportunity is high, the investment return outweighs the costs of being undervalued by investors or the costs from biasing the report. Since sufficiently high investment returns lead to the straight-forward case of efficient investment, we focus in the following discussion on the case of less profitable investments. For such investments, the model predicts that a manager does not pursue the investment opportunity when value of the firm’s assets is in an intermediate range while the manager pursues the investment opportunity when the value of his firm’s assets is either low or high. This prediction differs from Myers and Majluf (1984) and is due to the interdependencies between the manager’s investment and disclosure decision. In the model, the
manager raises capital and pursues the investment opportunity when the value of his assets is high because, in that case, it is cost-effective for the manager to voluntarily issue a report that reveals the value of his assets to investors. In contrast, it is not cost effective for the manager to issue a report when the value of his assets is in the intermediate range and, as a result, the manager does not raise capital. The reason for disclosure being cost-effective for high asset values is that the manager needs to bias his report upwards by less in order to communicate the true value of his firm’s assets to investors when the value of his assets in place are high. As a result, the manager incurs lower disclosure costs, making it worthwhile to issue a report and raise capital in order to realize the return of the investment opportunity.

At first, it might seem surprising that in equilibrium the manager needs to bias his report upwards by less in order to communicate the true value of his firm’s assets to investors when the value is “high” compared to when the value is “intermediate.” In particular, we might expect the manager’s bias to increase monotonically in his type as it is the case in standard signaling models in which the sender’s payoff depends linearly on investors’ perception of his type (e.g., Riley 1979, Miller and Rock 1985). In contrast, in this paper, the manager’s payoff depends linearly on the fraction of ownership that he has to give up in exchange for the capital investors provide. This causes the manager’s benefit from marginally increasing investors’ perception to be higher when the value of the firm’s assets is lower. The reason is that the effect of making investors believe that the firm’s assets are marginally more valuable is greater when the “size of the pie” is smaller. This, together with the result that the manager does not bias his report when his assets are worth zero, yields the prediction that the bias and biasing costs are initially increasing and then decreasing in the firm’s asset value. Hence, the reporting bias and biasing costs reach their maximum for intermediate asset values. The fact that the bias function that emerges in this paper is different from the bias function in standard signaling models illustrates that modeling specific signaling settings and incorporating institutional details – such as financing needs and investment opportunities – can alter predictions about properties of the sender’s report and signaling costs in a qualitative

4 This applies to cases in which it is too costly for the firms with intermediate asset values to raise capital without disclosing information due to the undervaluation of their assets by investors.

5 To see this, consider a manager whose firm’s assets in place are worth \( x \) and who issues a report such that investors perceive the value of his firm’s assets to be worth \( x + \epsilon \). If investors perceive the value of the firm’s assets in place to be \( x + \epsilon \) they require fraction \( \alpha = \frac{\epsilon}{x + \epsilon} \) of the firm’s ownership shares in exchange for providing capital \( I \) where \( r \) denotes the return on investment. Since the assets are in fact worth \( x \) and not \( x + \epsilon \), the actual value of the shares investors obtain is \( I \frac{x + r}{x + \epsilon + r} \) rendering the manager’s benefit from issuing a report that mimics a firm with assets worth \( x + \epsilon \) to be \( I - I \frac{x + r}{x + \epsilon + r} \). Hence, the manager benefits less from mimicking a firm whose assets are marginally more valuable when the actual value of his firm’s assets is higher.
sense.

Second, the model predicts that there exists an equilibrium in which the manager discloses low and high asset values but does not disclose intermediate asset values. This prediction differs from the more common threshold equilibria in which good news are disclosed while bad news are withheld. The reason for the manager withholding intermediate asset values but not low or high asset values is that the disclosure costs arise endogenously in the form of the biasing costs necessary to communicate the actual value of the firm’s assets to investors (rather than the disclosure costs being exogenous and constant). As discussed above, the manager’s equilibrium bias and biasing costs are highest for intermediate asset values and exceed the return on investment if the investment opportunity is not very profitable. As a result, the manager opts to withhold his information and forego the investment opportunity. Alternatively, the manager might not only withhold intermediate asset values but also sufficiently low asset values, giving rise to two distinct non-disclosure intervals. However, the two non-disclosure intervals differ in terms of the manager’s investment behavior. While he does not raise capital for intermediate asset values, the manager raises capital without issuing a voluntary report if the asset value is sufficiently low. Whether the manager issues a voluntary report for low asset values depends on investors’ inferences when a manager raises capital without disclosing information. If investors believe that the assets are worthless when a manager attempts to raise capital without disclosing information, the manager will issue a report before raising capital even if the value of his assets is low. If, however, investors are less skeptical, the manager will raise capital without disclosing information when the value of his assets is low (and consequently saving the biasing costs).

Third, the model predicts that the underinvestment problem is more severe, in the sense that the manager foregoes the profitable investment opportunity more often, when the return on investment is lower or when the costs from biasing the report are higher. The fact that the underinvestment problem is more severe when the return on investment is lower is intuitive. As the return on investment decreases, the manager is less willing to incur biasing costs in order to communicate the value of his firm’s assets to investors. As a result, it becomes less attractive for the manager to raise capital and invest.

Finally, the model predicts that, if investors are risk-averse and price the uncertainty regarding the firm’s assets in place, firms that voluntarily disclose information prior to raising capital have lower cost of capital than firms that do not issue a report. The reason is that investors face
greater uncertainty about the firm's asset values, and hence, investors require a higher return on their equity investment when the firm does not issue a report. Since in equilibrium only firms with low asset values raise capital without issuing a report, the model predicts a negative association between performance and firms' cost of capital.

As briefly discussed above, the model differs from most voluntary disclosure models because it assumes that the manager may distort his voluntary disclosure but that he incurs costs from doing so. As far as we are aware, the only other paper studying costly misreporting in a voluntary disclosure setting is Korn (2004). Korn (2004) assumes that the manager wants to maximize the firm's share price net of his reporting costs. In equilibrium, the manager issues a voluntary report when his private information is sufficiently favorable. In that sense, Korn (2004) yields predictions similar to voluntary disclosure models in, e.g., Jovanovic (1982), Verrecchia (1983), Dye (1985), Jung and Kwon (1988), and Wagenhofer (1990) that identify a disclosure threshold above which the manager chooses to disclose information and below which the manager prefers to remain silent. While our model is similar to Korn (2004) insofar as we also assume that the manager may induce costly biases into his voluntary disclosures, our model differs from Korn in several important aspects. In contrast to Korn (2004), we consider the manager's voluntary disclosure decision jointly with a decision to raise equity capital for a profitable investment opportunity. In return for the equity capital provided by investors, the manager has to give up a fraction of the firm's ownership to the investors. Considering the need for equity capital to pursue a profitable investment opportunity significantly alters the predictions about the manager's equilibrium disclosure strategy. This model predicts that there does not always exist a threshold equilibrium in which firms only disclose favorable news. Instead, firms may disclose unfavorable news but withhold intermediate news. In fact, there always exists an equilibrium in which unfavorable news is disclosed even though disclosure is costly. As a result, the paper adds to the literature not only by analyzing the effect of costly misreporting on management's voluntary disclosure strategies but also by considering the interdependencies between firms' investment and disclosure decisions.

The remainder of the paper proceeds as follows. Section 2 outlines the setting of the model. Section 3 studies the equilibrium in which bad news are disclosed. In this equilibrium, if the return on the investment is sufficiently high, the manager issues a report and raises capital for all asset values. If the return on the investment is lower, the manager issues a report and raises capital for both low and high asset values, while the manager does no issue a report and does not raise
capital for intermediate asset values. Section 4 studies the equilibrium in which bad news remain undisclosed. This equilibrium differs from the equilibrium studied in Section 3 in that the manager raises capital without issuing a report for low asset values. Section 5 provides concluding remarks. All proofs are delegated to the Appendix.

2 Model setup

This section describes a parsimonious model of investment and voluntary disclosure. We start with a brief outline of the sequence of events in the model.

There are three points in time. An individual (called the “manager” in what follows) owns a firm with assets in place and with a new, non-scalable production technology that requires external financing of $I$. The net return of this investment opportunity is $r > 0$. At $t = 1$, the manager privately learns the value of his firm’s assets in place, $x$. Then, at $t = 2$, the manager decides whether to raise equity capital from outside investors to finance the new investment opportunity and whether to voluntarily issue a report on his firm’s assets value. Both the current assets in place and the new project (if carried out) will generate their final cash flows at $t = 3$.

We next provide more detail on the preceding outline of the model.

The value of the firm’s assets in place is a realization of the random variable $\tilde{x}$ which is distributed over $[0, \infty)$ according to the probability density function $f(x) > 0$ for all $x$. We restrict $x$ to non-negative values based on the rationale that the assets in place have an abandonment option. At $t = 1$, the manager privately learns the realization of the value of his firm’s assets in place.

At $t = 2$, the manager simultaneously decides whether to issue a report to investors and whether to raise capital in the equity market. The investors observe the manager’s report, if one has been issued, prior to the opening of the equity market. If the manager decides to issue a report on his firm’s assets value, $x_R \in [0, \infty)$, he is not confined to tell the truth but may bias his report. We denote the manager’s reporting bias by $b(x) = x_R(x) - x$. If the report differs from the true value

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6 The return on investment does not have to be deterministic but may be random as long as the manager and outside investors are symmetrically informed about it. The results of the model remain unchanged where $r$ then refers to the expected return on investment.

7 We consider asymmetric information with respect to the value of the firm’s assets in place but not with respect to the return of the new investment opportunity. We make this assumption since information asymmetry with respect to the value of the firm’s assets in place is sufficient to obtain the underinvestment problem described in Myers and Majluf (1984). While the underinvestment problem in Myers and Majluf (1984) is robust to the additional information asymmetry with respect to the return on investment, it would add significant complexity in our model. In order to maintain tractability, we therefore assume that the manager and outside investors are symmetrically informed about the return on investment.
of the current assets in place, the manager incurs a personal cost of manipulating the report. This cost is increasing in the distance between the report and the true value of the firm’s assets value, $x$. In particular, we assume that the manager incurs the cost $g(x_R - x)$ where the cost function $g(\cdot)$ is a well behaved U-shaped function, i.e., it is convex with $g(0) = 0$, $g'(0) = 0$. The manager may raise equity capital whether or not he issued a report $x_R$. If the manager decides to raise equity capital, he offers a fraction $\alpha$ of the firm’s ownership to investors in return for investing $I$ with the firm. Investors may accept or reject the offer. We assume that investors are risk-neutral and that they accept the offer when they break even on average. If investors accept the offer they contribute capital $I$ and the manager pursues the investment opportunity.

At $t = 3$, the firm’s final cash flows are realized. If the manager did not pursue the investment opportunity, the firm’s final cash flows equal $x$ and the manager retains all of it. If the manager raised capital and invested into the new production technology the firm’s final cash flows are $x + I + r$, the manager retains fraction $(1 - \alpha)$ and investors are paid fraction $\alpha$ of the final cash flows.

In equilibrium, the manager simultaneously decides whether to issue a report, if so to what extent to bias the report, and whether to raise capital. If the manager decides to raise capital, he rationally anticipates investors’ response and chooses the fraction $\alpha$ of the firm’s ownership he offers to investors in exchange for their investment such that investors break even on average. The fraction $\alpha$ is determined by

$$I = \alpha \left( E[\hat{x}|\Omega] + I + r \right).$$

where $\Omega$ denotes the public information that is available to investors at $t = 2$. All parameters of the model, i.e., $r$ and $I$ as well as the cost function $g(\cdot)$ and the prior distribution of asset values $f(\cdot)$ are common knowledge. Figure 1 summarizes the sequence of events of the model.

In the model, the manager jointly considers his disclosure and investment decision. The reason is that the manager’s disclosure decision depends on his investment decision and vice versa. On the one hand, voluntary disclosure can only be beneficial to the manager if he decides to raise capital. In the absence of equity issuance, outside investors’ perception of the firm value is irrelevant and hence the manager cannot benefit from influencing investors’ beliefs about his firm’s value. On the other hand, the profitability of the new investment opportunity to the manager depends on his voluntary disclosure decision, even though the overall profitability of the new investment opportunity is known.

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8This is a standard assumption in the costly state falsification literature (e.g., Lacker and Weinberg 1989; Stein 1989, Fischer and Verrecchia 2000, Gutman, Kadan, and Kandel 2006).

9In Corollaries 3 and 4, we discuss the implications of investors being risk-averse.
The manager learns the value of his firm’s assets in place, \(x\).

The manager decides whether to raise capital, whether to voluntarily disclose information and if so what report, \(x_{R}\), to issue. If the manager raises capital, he invests into the new production technology.

The firm’s final cash flow is realized and the manager and investors are paid according to their respective claims. If the manager invested capital \(I\) into the new production technology the firm’s cash flows are \(x+I+I\), otherwise they are \(x\).

Figure 1: Timeline

and fixed. The reason is that the manager’s report, or its absence, affects investors’ perception of the firm value and determines the fraction of shares the manager needs to give up to investors in return for the capital they provide. Because of these interdependencies, it is essential to jointly consider firms’ voluntary disclosure decisions and investment decisions. The following analysis studies these interdependencies and illustrates how the manager’s decisions whether to issue a voluntary report – and if so to what extent to distort the report – affect and are affected by the manager’s investment decision.

3 Equilibrium with bad news disclosed

In the model, a manager has to simultaneously decide whether to raise capital, whether to issue a report, and if so to what extent to bias the report. As discussed in the introduction of the paper, two types of equilibrium can evolve: equilibrium with bad news being disclosed (discussed in this section) and equilibrium with bad news not being disclosed (discussed in Section 4). Before analyzing the manager’s joint investment and disclosure decision, we study the manager’s decision to bias his report taking the decision whether to disclose as given.

3.1 Disclosure bias

In this section, we study the extent to which the manager biases his report under the assumption that he always issues a report, i.e., under the assumption of full disclosure. As it is often the case in disclosure games with continuous support, multiple equilibria with pooling reports may evolve (e.g., Guttman et al. 2006). Solving for the entire set of equilibria that involve such pooling behavior proves intractable. As common in the literature, we therefore limit our analysis to a subset of equilibria in which the manager’s report allows investors to perfectly infer the manager’s private
information. In such equilibria, a manager that discloses information does not bear any costs from being pooled with firms whose assets in place are less valuable. Hence, a manager that voluntarily discloses information can fully capture the return on investment, \( r \), and therefore will always seize the profitable investment opportunity.\(^{10}\) The following Lemma formalizes this observation.

**Lemma 1** If a manager voluntarily issues a report \( x_R(x) = x + b(x) \) he also raises capital and invests.

In a full disclosure equilibrium, the manager issues a report for all realizations of asset values and hence always raises capital and pursues the investment opportunity. The report issued by the manager affects his payoff in two ways. On the one hand, the report affects investors’ beliefs regarding the value of the firm’s assets in place, which determine the fraction \( \alpha \) of equity investors require in exchange for providing capital \( I \). On the other hand, the manager incurs disclosure costs whenever his report differs from the true value of assets in place. The manager’s biasing costs increase in the magnitude of the bias. The trade-off between these two factors is reflected in the first order condition of the manager’s optimization problem and determines the extent to which the manager biases his report. The following Lemma characterizes the manager’s bias strategy, \( b(x) \), in a full disclosure equilibrium.

**Lemma 2** In a full disclosure equilibrium, the bias in the manager’s report is given by the solution to the differential equation

\[
b'(x) = \frac{I}{g'(b(x))(x+I+r)} - 1,
\]

(1)

with the boundary condition \( b(0) = 0 \).

The equilibrium bias strategy \( b(x) \) has the following properties: it is continuous, always positive, initially increasing, obtains a unique maximum and converges to zero as the value of the firm’s assets in place goes to infinity.

Figure 2 illustrates the equilibrium bias, \( b(x) \), and the manager’s equilibrium report, \( x_R(x) = x + b(x) \), in a full disclosure equilibrium. The figure is based on a quadratic cost function, \( g(b) = \frac{1}{2}b^2 \), and the parameter values \( I = 1 \) and \( r = 0.25 \).

In equilibrium, the manager biases his report upwards when making a disclosure. The reason is that investors associate higher reports with the firm’s assets in place being more valuable. When

\(^{10}\)Recall that we assume that the manager cannot expropriate the capital raised for himself, i.e., there is no moral hazard problem with respect to the manager’s investment decision once he has raised capital.
investors perceive the firm’s assets in place to be more valuable, investors require a smaller fraction of the firm’s equity in exchange for investing capital $I$. In turn, the manager can keep a larger fraction of the firm’s equity to himself. The extent to which the manager biases his report upwards depends on the benefits and costs associated with reporting a higher value for the assets in place.

Three observations jointly explain the shape of the equilibrium bias function as shown in Figure 2.

First, a manager whose firm’s assets in place are worth 0 does not bias his report. This is intuitive as in equilibrium investors identify him as the lowest type and he is therefore not willing to bear any signaling costs.

Second, the manager’s benefit from investors perceiving his firm’s assets to be marginally more valuable than in fact they are depends on the actual value of the firm’s assets in place. When the value of the firm’s assets in place is lower, the manager benefits more from mimicking a firm whose assets are marginally more valuable. The reason is that the effect of making investors believe that the firm’s assets are marginally more valuable is greater when the “size of the pie” is smaller. To illustrate this further, we consider a manager whose firm’s assets in place are worth $x$ and who issues a report such that investors perceive the value of his firm’s assets to be $\hat{x}$. If investors perceive the value of the firm’s assets in place to be $\hat{x}$ they require $\alpha = \frac{I}{\hat{x} + I + r}$ shares in exchange for providing capital $I$. Hence, the actual value of the shares investors obtain is $I \frac{\hat{x} + I + r}{\hat{x} + I + r}$. Since investors provide capital $I$ in exchange for those shares, they overpay by $I - I \frac{\hat{x} + I + r}{\hat{x} + I + r}$. In turn, the manager’s benefit from issuing a report that mimics a firm with assets worth $\hat{x}$ is also $I - I \frac{\hat{x} + I + r}{\hat{x} + I + r}$. The manager’s benefit from inducing investors to believe that the value of the firm’s assets in place
are worth marginally more than in fact they are is
\[
\lim_{\hat{x} \to x} \frac{\partial}{\partial \hat{x}} \left( I - I \frac{x + I + r}{x + I + r} \right) = \frac{I}{x + I + r},
\]
which decreases in the value of the firm’s assets, \( x \). Hence, the manager benefits less from mimicking a firm whose assets are marginally more valuable when the actual value of his firm’s assets is higher.

Third, the costs the manager incurs from marginally increasing investors’ beliefs about the value of the firm’s assets depends (i) on the overall magnitude of the bias (due to the convexity of the cost function \( g(b) \)) and (ii) on the sensitivity of investors’ inferences to changes in the manager’s report (which determines the additional bias necessary in order to make investors believe that the value of his firm’s assets in place is marginally higher than in fact they are). Letting \( \hat{x} \) once again denote investors’ perception of the firm’s asset value, \( \frac{\partial x_R}{\partial \hat{x}} \) measures the additional bias necessary to marginally increase investors’ beliefs about the firm’s asset value and \( \frac{\partial g(x_R - x)}{\partial \hat{x}} = g'(b(x)) \frac{\partial x_R}{\partial \hat{x}} \) measures the additional costs the manager incurs from marginally increasing investors’ beliefs. In equilibrium, investors’ inferences have to be consistent with the manager’s reporting strategy, i.e.,
\[
\frac{\partial x_R}{\partial \hat{x}} = \frac{\partial x_R}{\partial x} \quad \text{or} \quad \frac{\partial g(x_R - x)}{\partial \hat{x}} = g'(b(x)) \frac{\partial x_R}{\partial x} = 1 + b'(x). \quad (11)
\]

These three observations jointly explain the shape of the equilibrium bias function in Figure 2: Initially, the bias is zero due to the fact that a manager with assets worth zero does not bias his report. Managers with assets slightly more valuable are willing to incur signaling costs and therefore bias their reports upwards. Since the bias is still relatively small, the marginal costs of biasing the report are also relatively small. In equilibrium, the manager’s marginal costs of biasing his report must equal his marginal benefits from biasing his report. This implies that the marginal benefits of a manager with low asset values must be relatively low. Since the marginal benefits of a manager with low asset values from increasing investors’ beliefs is high, it must be that investors’ beliefs are relatively insensitive to the manager’s report. Investors’ beliefs are relatively insensitive to the manager’s report, when investors attribute most of an increase in the report to an increase in the manager’s bias and only a small part to an increase in the asset value. This provides the intuition for the fact that the bias function increases at a high rate when asset values are relatively low. As the bias continues to increase, the marginal costs of biasing the report increases. At the same time,

\[\text{Note that these three observations are equivalent to Lemma 2. The first observation provides the boundary condition } b(0) = 0. \quad \text{The second observation gives the marginal benefit from inducing investors to believe that the value of the firm’s assets in place are worth marginally more than in fact they are.} \]

\[\text{while the third observation provides the marginal costs, } \frac{\partial g(x_R - x)}{\partial x} = g'(b(x)) (1 + b'(x)). \quad \text{Equating the marginal benefits and costs and rearranging terms yields the differential equation in Lemma 2.} \]
as the value of the firm’s assets increases the marginal benefits from increasing investors’ beliefs decreases. In order for the marginal costs to equal the marginal benefit from increasing the bias, investors’ beliefs must become more sensitive to the report. This implies that the rate of increase in the manager’s bias must decrease. Since the marginal benefits from increasing investors’ beliefs eventually approaches zero the equilibrium bias decreases from one point on.\textsuperscript{12} In the limit, when the value of the firm’s assets in place is very high the manager’s benefit from changing investors’ beliefs diminishes and therefore the manager biases his report upward by a diminishing amount ($\lim_{x \to \infty} b(x) = 0$).

The equilibrium bias described in Lemma 2 shows that – as standard in costly signaling settings – truth-telling is not an equilibrium and the manager ends up paying signaling costs even though he does not mislead investors in equilibrium. As a result, the manager always bears disclosure costs (except when his firm’s asset in place are worth 0). The signaling costs the manager incurs differ from the signaling costs in standard signaling settings in which the sender (manager) maximizes his perceived type net off his signaling costs (e.g., Riley 1979, Miller and Rock 1985). In the standard signaling models, which consider only the disclosure decision of the manager, the marginal benefit of the manager from increasing investors’ beliefs about his type is constant. This property combined with a convex cost function yields increasing signaling costs that converge to a finite upper bound as the sender’s type goes to infinity. Our model studies a different setting in which a manager that considers raising capital in order to finance an investment opportunity makes an investment decision in addition to the disclosure decision. The differences in the setting give rise to a qualitatively different disclosure behavior. This illustrates that modeling specific signaling settings and incorporating institutional details into the model can alter predictions about equilibrium properties of the sender’s message and signaling costs in a qualitative sense.

\textsuperscript{12} Assume that at $x = x^*$ the manager’s bias is (weakly) decreasing. To see that for any $x > x^*$ the manager’s bias is monotonically decreasing, suppose to the contrary that at some $x' > x^*$ the bias function starts to increase in $x$. The fact that the bias function starts to increase at $x'$ has two implications. First, the manager’s marginal cost from biasing the report is increasing in $x$ at $x'$. Second, the bias function is increasing and convex in $x$ at $x'$, which implies that the sensitivity of investors’ beliefs to the report decreases at $x'$. The decreased sensitivity of investors’ beliefs combined with the fact that the benefit from marginally increasing investors’ belief is decreasing in $x$ implies that the manager’s marginal benefit from increasing his bias is decreasing at $x'$. This leads to a contradiction, since in equilibrium the manager’s marginal benefit from increasing the bias in his report must equal his marginal costs from doing so for any $x$. 

13
3.2 Joint investment and disclosure decision

The previous section characterized the properties of the manager’s bias and resulting signaling costs if he decides to issue a report. However, the manager is not required to issue a report. When the manager does not issue a report but raises capital on the equity market, investors draw inferences about the value of the firm’s assets from the fact that the manager did not make a disclosure. If investors expect the manager to sometimes raise capital without making a disclosure (i.e., raising capital without making a disclosure is part of the manager’s equilibrium strategy), investors update their beliefs according to Bayes’ Rule. This case is analyzed in Section 4. If, however, investors do not expect the manager to ever raise capital without making a disclosure (i.e., raising capital without making a disclosure is not part of the manager’s equilibrium strategy), we have to specify investors’ off-equilibrium beliefs. This case is analyzed in this section.

Raising capital without making a disclosure is not part of the manager’s equilibrium strategy only if investors infer from it that the firm’s assets are worth zero.\textsuperscript{13} While these off-equilibrium beliefs might seem “extreme” in the sense that they assign a probability of one to the lowest asset value, they are in fact robust to D1 or D2 refinements. In addition, the equilibrium characterized in this section shares many of the aspects of the equilibrium discussed in Section 4, in which raising capital without making a disclosure is part of the manager’s equilibrium strategy.

Setting the off-equilibrium beliefs to zero also guarantees that no manager prefers to deviate and raise capital without disclosing information over raising capital with issuing the report characterized in Lemma 2. Hence, the manager is left with the choice of whether to issue a report and raise capital, or whether to remain silent and save the disclosure cost but forgo the profitable investment opportunity. Since the sole purpose of the disclosure is to convince outside investors of the value of the firm’s assets, the manager will prefer not to issue a report if the signaling costs outweigh the return the manager earns from investment. That is, if the equilibrium bias is such that it were to impose costs \( g(b(x)) \) that exceed the investment return, \( r \), the manager is better off withholding his information and foregoing the investment opportunity.

For more profitable investment opportunities (i.e., for higher values of \( r \)), the manager is willing to bear higher biasing costs in order not to forego the return on investment. This is reflected in

\textsuperscript{13} To see that suppose investors’ off-equilibrium beliefs are such that they infer the firm’s assets to be worth \( x’ > 0 \) if the manager raises capital without making a disclosure. Then, a manager with assets worth less than \( x’ \) can not only save on the disclosure costs by raising capital without making a disclosure but his firm’s assets will also be perceived as more valuable than in fact they are. As a result, the manager would prefer to deviate and raise capital without making a disclosure.
a higher threshold of bias, $g^{-1}(r)$, up to which the manager is willing to bias his report. While this seems to suggest that the manager is less likely to withhold information when the return on investment is higher, we can draw such conclusion only after considering the effect of $r$ on the equilibrium bias $b(x)$. It turns out that the effect of $r$ on the equilibrium bias $b(x)$ is such that higher returns on investments cause the equilibrium bias to be lower for any given $x > 0$. The reason is that as $r$ – which is common across all firms – increases, the difference in value of assets in place across firms becomes relatively less important. As a result, the manager is less willing to bear signaling costs resulting in a lower equilibrium bias $b(x)$. This together with the threshold $g^{-1}(r)$ being higher for higher returns on investments implies that it is in fact the case that the manager is less likely to withhold information when the return on investment is higher.

Since the bias and as a result the biasing costs are small for both sufficiently small and large asset values (see Figure 2), no disclosure occurs – if at all – for intermediate asset values. Hence, either there exists an equilibrium in which the manager issues a report for all realizations of asset values (if the return of investment is sufficiently high) or there exists an equilibrium in which the manager issues a report for low and high asset values but not for intermediate asset values (if the return on investment is sufficiently low). Proposition 1 formalizes this result.

**Proposition 1** There exists a threshold $r^* > 0$ such that

(i) if $r \geq r^*$ there exists a full disclosure equilibrium in which the manager discloses a report and invests for all $x \geq 0$.

(ii) if $r < r^*$ there exists an equilibrium with intermediate news undisclosed. The equilibrium can be characterized by $0 < x_1^D < x_2^D$ such that for $x \in [0, x_1^D] \cup [x_2^D, \infty)$ the manager makes a disclosure, raises capital and invests; and for $x \in (x_1^D, x_2^D)$ the manager does not make a disclosure and does not raise capital and consequently does not invest.

When a manager makes a disclosure, he biases his report according to the bias described by the differential equation in Lemma (2) with the boundary condition $b(0) = 0$.

If, in equilibrium, the manager prefers to remain silent for some values of assets in place then investors expect not to observe certain reports. For such off-equilibrium reports, we need to specify investors’ (off-equilibrium) beliefs. “Extreme” off-equilibrium inferences can support “extreme” reporting behavior. For instance, if we assume that investors infer from any report issued that
the firm’s assets are worth zero, we can support an equilibrium without disclosure. However, such equilibria seem hardly realistic. For that reason, we assume off-equilibrium beliefs that, in our view, are more realistic. In particular, we assume that investors’ beliefs are continuous and increasing\textsuperscript{14} and that if investors observe reports they did not expect to observe in equilibrium, they infer that the manager was forced to make a disclosure for some reason. For instance, the manager might have – by mistake – privately communicated the information to one stakeholder and is therefore forced to make a public disclosure under Regulation Fair Disclosure\textsuperscript{15}. When the manager is forced to make a disclosure, it is optimal for him to follow the same biasing strategy as in the full disclosure equilibrium discussed above. That is, his bias can be described by the differential equation in (1) and the boundary condition $b(0) = 0$. Given these off-equilibrium beliefs, the manager’s equilibrium bias when he decides to voluntary disclose information will also be described by the differential equation in (1) and the boundary condition $b(0) = 0$.

Proposition 1 illustrates the importance of jointly considering real effects and firms’ disclosure decisions. Both the investment and disclosure strategy in Proposition 1 are qualitatively different from the equilibrium strategies that emerge when investment and disclosure decisions are considered separately. The following discussion further examines these differences.

In models that exclusively focus on firms’ voluntary disclosure decisions, we know that disclosure costs (mostly) cause firms to disclose information when it is sufficiently favorable and withhold information when it is sufficiently unfavorable (e.g., Jovanovic 1982, Verrecchia 1983, Wagenhofer 1990, Korn 2004). In contrast, our model predicts that firms may disclose unfavorable news but withhold intermediate news. In fact, there always exists an equilibrium in which unfavorable news is disclosed even though disclosure is costly. The different prediction results from the fact that our model studies a specific disclosure setting in which the manager may issue a report and raise equity capital in order to finance an investment opportunity. In return for the equity capital provided by investors, the manager has to give up a fraction of the firm’s ownership to the investors. Hence, the manager’s payoff linearly depends on the fraction of the firm that investors require in return for their equity investment. In contrast, most other disclosure models assume that the manager’s payoff

\textsuperscript{14}That is the value of the firm’s assets that investors infer from a report, $\hat{x}(x_R)$, is a continuous and increasing function for $x_R \in [0, \infty)$.

\textsuperscript{15}We could explicitly model this scenario by assuming that the manager can decide whether to make a disclosure in most states of the world but is mandated to make a disclosure with a (small) probability $p$. This would avoid the problem of off-equilibrium reports and guarantee that all reports, $x_R \geq 0$, are on the equilibrium path. As a result, it would implement the same voluntary disclosure and investment behavior.
is linear in investors’ perception of the firm’s value/type. The differences in the models’ predictions underscore the importance of considering context-specific factors in modeling disclosure decisions.

The investment strategy in Proposition 1 also differs from the investment strategy in Myers and Majluf (1984), in which only managers whose firm’s assets in place are worth less than a certain threshold raise capital and invest. In the full disclosure equilibrium in part (i) of Proposition 1, the information asymmetry that hinders efficient investment in the model of Myers and Majluf (1984) disappears entirely and managers do not forego any profitable investment opportunities. However, the full disclosure equilibrium in part (i) of Proposition 1 exists only when the return on investment is sufficiently high. The reason is that the manager never makes a disclosure that costs him more than the return on the investment. In other words, the reporting behavior depicted in Figure 2 only constitutes an equilibrium if the biasing costs are less than the return on investment, \( r = 0.25 \), for all values of assets in place or – equivalently – if the bias is always less than \( g^{-1}(r) = \sqrt{2r} = 0.71 \). For \( r = 0.25 \), it is indeed the case that the biasing costs are less than \( r \) for all values of assets in place and hence full disclosure constitutes an equilibrium. Figure 3 illustrates the manager’s equilibrium disclosure and investment strategy for \( r = 0.25 \). When the investment is less profitable, e.g., \( r = 0.12 \) instead of \( r = 0.25 \), the bias exceeds \( g^{-1}(r) = \sqrt{2r} = 0.48 \) for intermediate asset values. This is illustrated in Figure 4. Since the manager prefers not to make a disclosure for all asset values \( x \) for which \( b(x) > g^{-1}(r) \), in equilibrium intermediate types do not make a disclosure while low and high types voluntarily disclose information. Hence, inefficient investment behavior continues to occur when the investment opportunity is not too profitable. In contrast to Myers and Majluf (1984), it is the firms with intermediate asset values that forego the profitable investment opportunity and not the firms with high asset values. The reason is that for firms with high values of assets in place disclosure costs are relatively low such that they are outweighed by the return on investment.

In contrast, for intermediate asset values the disclosure costs are higher because the equilibrium bias is greater. If the return on the investment opportunity is not sufficiently high to justify the disclosure costs, firms with intermediate asset values are better off not disclosing information. Proposition 1 shows that if for intermediate values of the firm’s assets in place the manager does not voluntarily disclose information, he also does not invest. At first, this behavior might appear suboptimal. The reason is that firms forego return \( r \) when they do not pursue the investment opportunity. However, if a subset of firms with asset values between \( x_1^D \) and \( x_2^D \) were to raise
capital and invest without disclosing information, then all firms with $x < x_1^D$ would prefer to mimic them by also raising capital without disclosing information. Hence, in equilibrium, all firms with asset values between $x_1^D$ and $x_2^D$ do not raise capital and investors’ off-equilibrium beliefs are such that they infer that the firm is of the lowest type if it attempts to raise capital without disclosing information. The boundaries of the non-disclosure interval $(x_1^D, x_2^D)$ are uniquely defined by the cost of disclosure being such that they are exactly offset by the return on investment, i.e., $g(b(x_1^D)) = g(b(x_2^D)) = r$.

Figure 3: Full disclosure equilibrium: disclosure and investment strategy for highly profitable investments ($r = 0.25$)

Figure 4: Equilibrium with intermediate types undisclosed: disclosure and investment strategy for less profitable investments ($r = 0.12$)

While the investment behavior characterized in Proposition 1 differs from the investment behavior in Myers and Majluf (1984), it is still the case that the underinvestment problem is less
severe, in the sense that the manager foregoes the profitable investment opportunity less often, when the return on investment is higher. This is intuitive. As the return on investment increases, the manager is willing to incur higher costs (in the form of disclosure costs in this model or in the form of price discounts due to pooling in Myers and Majluf 1984) in order to raise capital and invest. The following Corollary formalizes this observation.

**Corollary 1** For \( r < r^* \), \( x_1^{D} \) is increasing in \( r \) and \( x_2^{D} \) is decreasing in \( r \). Hence, in the equilibrium characterized in Proposition 1, the underinvestment problem is less severe, in the sense that the probability of the manager raising capital and investing is higher, for higher returns on investment.

### 4 Equilibrium with bad news undisclosed

In the previous section, we characterized the equilibrium in which the manager voluntarily discloses bad news. In that equilibrium, the manager raises capital and invests only when voluntarily disclosing information. In addition to the equilibrium discussed in the previous section, there exists an equilibrium in which the manager does not disclose bad news. However, the manager still raises capital and invests for those relatively low asset values for which does not issue a report. Before we analyze the manager’s joint investment and disclosure decision in Section 4.2, we study the manager’s decision to raise capital and invest without making a disclosure in Section 4.1.

#### 4.1 Investment decision

In this section, we discuss the manager’s investment decision while taking his disclosure strategy as given. In the equilibria we study, investors are able to infer the value of the firm’s assets in place when the manager voluntarily discloses information. In such equilibrium, a manager that voluntarily issues a report always raises capital and invests (see Lemma 1). However, the manager may not issue a voluntary report for some asset values. We denote the set of asset values for which the manager does not issue a voluntary report by \( X_{nd} \). In this section, we take the set of asset values for which the manager does not issue a voluntary report, \( x \in X_{nd} \), as given and solve for the manager’s investment strategy when he remains silent. This analysis is similar to the analysis in Myers and Majluf (1984).

When the manager does not voluntarily disclose information, the information asymmetry between the manager himself and investors is not fully resolved. While the manager knows the precise value of the firm’s assets in place, investors can only make inferences about the average
value of the assets in place. Based on their inferences, investors will require a share \( \alpha_{nd} \) in the firm’s equity in return for providing capital \( I \). In equilibrium, the equity share \( \alpha_{nd} \) will guarantee that the investors break even on average.

Although, investors on average correctly value the equity capital they invest in, investors overvalue some and undervalue other issues of equity capital. When investors undervalue the equity issued in exchange for their investment, the firm’s manager must give up a larger fraction of equity ownership to new investors than seems necessary based on the manager’s private information. As a result, when the actual value of the firm’s assets is sufficiently higher than investors’ inferences, the manager chooses not to raise capital even though this means foregoing the profitable investment opportunity. Hence, whenever the value of the firm’s asset in place exceeds a threshold \( x_I \), the manager will prefer passing up on the profitable investment opportunity over being pooled with firms that also invest without disclosing information but whose assets in place are worth less.\(^{16}\) Of course, investors anticipate that a manager will only raise equity capital when the firm’s assets in place are not particularly valuable and price new equity issues (as reflected in \( \alpha_{nd} \)) accordingly. In equilibrium, the manager’s investment and disclosure strategy and the share \( \alpha_{nd} \) investors demand in exchange for their investment \( I \) when the manager does not issue a voluntary report satisfy the following condition.

\[
I = \alpha_{nd} E \left[ \bar{x} + I + r \mid \bar{x} < x_I, x \in X_{nd} \right].
\]  

A manager whose assets in place are worth \( x_I \) has to be indifferent between investing without disclosing information and his next best option. When the manager invests without disclosing information, he retains an equity stake worth \((1 - \alpha_{nd})(x_I + I + r)\). The manager’s next best option is either to voluntarily disclose information and invest, yielding an expected payoff of \( x_I + r - g(b(x_I)) \), or not to raise capital and forego the profitable investment opportunity, in which case the manager simply keeps \( x_I \). In equilibrium \( x_I \) must satisfy

\[
\max \{ x_I + r - g(b(x_I)), x_I \} = (1 - \alpha_{nd})(x_I + I + r)
\]  

and only firms whose assets in place are worth less than \( x_I \) will raise capital and invest if they choose not to voluntarily provide information. Lemma (3) summarizes firms’ investment behavior for a given disclosure strategy.

\(^{16}\) \( x_I \) does not necessarily have to be in the set \( X_{nd} \). If \( x_I > x \) for all \( x \in X_{nd} \) then all firms that remain silent raise capital and invest. If \( x_I < x \) for all \( x \in X_{nd} \) then no firm that remains silent raises capital and invests.
Lemma 3

(a) If a firm voluntarily issues a report \( x_R (x) \) it also raises capital and invests. The manager’s payoff is \( x + r - g (b (x)) \).

(b) For any set of firms, \( X_{nd} \), for which investors anticipate that the manager does not issue a report, there exists \( \alpha_{nd} \in (0, 1) \) and \( x_I \in (0, \infty) \) that jointly satisfy equations (2) and (3). In equilibrium,

- for \( x \geq x_I \) the manager does not raise capital and consequently does not invest for \( x \in X_{nd} \). The manager’s payoff is \( x \).
- for \( x < x_I \) the manager raises capital and invests for \( x \in X_{nd} \). The manager’s payoff is \( (1 - \alpha_{nd}) (x + I + r) \).

Equation (2) illustrates that the fraction of equity investors demand in return for investing capital in the amount of \( I \) depends on their beliefs about the value of the firm’s assets for which the manager does not provide information but issues equity to raise capital. If the intersection of \( \{ x | x < x_I \} \) and \( X_{nd} \) is empty, we assume that investors believe that a firm that raises capital without voluntarily disclosing information has the lowest possible value of assets in place, i.e., \( x = 0 \). We analyzed this scenario in Section 3.2. When the intersection of \( \{ x | x < x_I \} \) and \( X_{nd} \) is non-empty, i.e., when the manager as part of his equilibrium strategy sometimes raises capital without making a disclosure, investors update their beliefs following Bayes’ Rule. The following section analyzes the manager’s equilibrium disclosure and investment strategy in this case.

4.2 Joint investment and disclosure decision

Since the manager makes both an investment and a disclosure decision, he can implement four qualitatively distinct strategies depending on whether he discloses and/or whether he invests. As we’ve argued before, the manager never discloses without raising capital because disclosure is (weakly) costly and the sole purpose of making a disclosure is to gain access to cheaper capital by convincing investors of the value of the firm’s assets in place. Hence, for any value of the assets in place the manager pursues one of the following three options: disclose/invest; not disclose/invest; not disclose/not invest. Before we characterize the manager’s equilibrium strategy for all values of assets in place, we study when the manager prefers to invest without making a disclosure. Lemma
4 shows that if it is optimal for the manager to raise capital and invest without making a disclosure when his assets are worth \( x' \) then it is also optimal for him to raise capital and invest without making a disclosure when his assets are worth less than \( x' \).

**Lemma 4** In equilibrium, if the manager invests without making a disclosure when the firm’s assets are worth \( x' \) then the manager also invests without making a disclosure when the firm’s assets are worth \( x < x' \).

In the following, we denote the highest asset value for which the manager prefers investing without issuing a report by \( x_I \). Lemma 4 establishes that if \( x_I > 0 \) the manager prefers raising capital without making a disclosure for all \( x < x_I \). The reason is as follows. In equilibrium, investors rationally infer the average asset value for which the manager raises capital without issuing a report. While investors price the firm’s assets correctly on average, they overvalue assets that are worth little and undervalue assets that are worth more. When his assets are undervalued, it is conceivable that rather than raising capital without issuing a report the manager may prefer to either forego the profitable investment opportunity or to issue a (costly) report that allows investors to infer the actual value of his firm’s assets. However, Lemma 4 establishes that this is never the case when the manager prefers raising capital without issuing a report for some higher asset values.

It is straightforward to see that the manager never prefers to forego the profitable investment opportunity for \( x < x_I \). When the manager raises capital without issuing a report, the undervaluation of the firm when its assets are worth \( x \) is less than when its assets are worth \( x_I \). Since the return on investment is sufficient to compensate the manager for the undervaluation when the assets are worth \( x_I \) it is more than sufficient if the assets are worth \( x < x_I \). In additions, the manager never prefers to issue a report for any \( x < x_I \) even though he will avoid the discount from being pooled with less valuable asset realizations. To see that, suppose the highest asset value (less than \( x_I \)) for which the manager preferred to issue a report were \( x' < x_I \). Then, for asset values worth slightly more than \( x' \) the manager would prefer to issue the same report \( x_R(x') \) over raising capital without making a disclosure – contradicting the assumption that \( x' \) is the highest asset value for which the manager prefers to issue a report. The intuition for that is as follows. If the manager preferred to issue a report \( x_R(x') \) when the assets are worth \( x_I \) then his benefit from retaining a larger fraction of the firm must outweigh his disclosure costs. The costs of issuing a report \( x_R(x') \) are lower for asset values slightly higher than \( x_I \) than they are for \( x_I \) because the
manager has to bias his report upwards by less. The benefit from the increased fraction that the manager would be able to retain when he issued the report $x_R(x')$ is larger when the assets are worth more. Hence, for asset values slightly higher than $x'$ the manager would strictly prefer to issue the report $x_R(x')$ over raising capital without issuing a report.

What Lemma 4 does not specify is the highest asset value for which the manager prefers to raise capital without making a disclosure, i.e., $x_I$. Since $x_I$ is the highest asset value for which the manager prefers to raise capital without making a disclosure, the firm’s assets will be undervalued by investors when $x = x_I$. Depending on the extent of the undervaluation, the manager is willing to incur disclosure costs in order to communicate the true value of his firm’s assets to investors. For any $x$, we let $b_I(x)$ denote the maximum bias a manager whose firm’s assets are worth $x$ is willing to pay for in order to separate himself from firms with lower asset values that also raise capital without disclosing information. The function $b_I(x)$ will prove useful in determining $x_I$. The following provides a formal definition of the function $b_I(x)$.

**Definition 1** $b_I(x)$ denotes the bias that renders a manager whose asset in place are worth $x$ indifferent between the following: (1) issuing a revealing report with a bias $b_I(x)$ and pursuing the investment opportunity; and (2) not disclosing but investing when investors expect the firm’s asset in place to be worth $E(\tilde{x}|\tilde{x} < x)$. Formally, $b_I(x)$ is defined by

$$
\left(1 - \frac{I}{x + I + r}\right)(x + I + r) - g(b_I(x)) = \left(1 - \frac{I}{E(\tilde{x}|\tilde{x} < x) + I + r}\right)(x + I + r)
$$

In equilibrium, if the value of the assets in place is $x_I$ the manager must be indifferent between raising capital without disclosing information and the next best option. The next best option is either to issue a report $x_R(x_I)$, which yields the manager a payoff of $x_I + r - g(b(x_I))$, or not to disclose and forego the investment opportunity, which yields the manager a payoff of $x_I$. Hence, in equilibrium the maximum costs that the manager is willing to bear in order to separate himself from lower realization of asset values is $g(b(x_I))$ or $g(r)$ whichever is lower. In other words, in equilibrium, the bias in the report for $x = x_I$ is the lower of the equilibrium bias $b(x_I)$ and $g^{-1}(r)$. This guarantees that the manager is indifferent between raising capital without disclosing information and the next best option for $x = x_I$, and strictly prefers to raise capital without disclosing information for $x < x_I$.

So far, we characterized the equilibrium behavior when the value of assets in place are sufficiently low such that the manager raises capital without issuing a report. In order to fully characterize
the equilibrium, we still need to characterize the manager’s equilibrium strategy for higher asset values. The analysis of the equilibrium strategy for such higher asset values follows closely the analysis of the equilibrium in Section 3.2. Using the definition of \( b_I(x) \), Proposition 2 provides a full characterization of the equilibrium in which for sufficiently low asset values \( x < x_I \) the manager raises capital without making a disclosure.

**Proposition 2**  There exists a threshold \( r^* > 0 \) such that

(i) if \( r \geq r^* \) there exists an equilibrium with bad news undisclosed which is characterized by a threshold \( x_I > 0 \). For \( x \in [0, x_I) \) the manager does not make a disclosure but raises capital and invests; and for \( x \in [x_I, \infty) \) the manager makes a disclosure, raises capital and invests.

(ii) if \( r < r^* \) there exists either

a. An equilibrium with bad news undisclosed which is characterized by the threshold \( x_I > 0 \). For \( x \in [0, x_I) \) the manager does not make a disclosure but raises capital and invests; and for \( x \in [x_I, \infty) \) the manager makes a disclosure, raises capital and invests;

b. An equilibrium with bad news undisclosed which is characterized by two thresholds, \( x_I \) and \( x_D \), where \( 0 < x_I \leq x_D \). For \( x \in [0, x_I) \) the manager does not make a disclosure but raises capital and invests; for \( x \in [x_I, x_D) \) the manager does not make a disclosure and does not raise capital and consequently does not invest; and for \( x \in [x_D, \infty) \) the manager makes a disclosure, raises capital and invests;

c. An equilibrium with two non-disclosure intervals which is characterized by three thresholds \( 0 < x_I < x_D^1 < x_D^2 \). In equilibrium, (a) for \( x \in [0, x_I) \) the manager does not make a disclosure but raises capital and invests; (b) for \( x \in [x_I, x_D^1) \) the manager makes a disclosure, raises capital and invests; (c) for \( x \in (x_D^1, x_D^2) \) does not make a disclosure and does not raise capital and consequently does not invest; and (d) for \( x \in [x_D^2, \infty) \) the manager makes a disclosure, raises capital and invests.

When a manager makes a disclosure, he biases his report according to the bias described by the differential equation in Lemma (2) with the boundary condition \( b(0) = 0 \). The thresholds \( x_I, x_D^1 \) and \( x_D^2 \) are uniquely defined by \( x_I = \min \{ x | b_I(x) = \min \{ g^{-1}(r), b(x) \} \} \) and \( b(x_D^1) = b(x_D^2) = g^{-1}(r) \) and \( x_D^1 < x_D^2 \).
Proposition 2 part (i) shows that if the return on investment is sufficiently high, the manager raises capital and invests for any value of the assets in place. For asset values lower than $x_I$ the manager does not issue a report, while for higher asset values he issues a report. Figure 5 illustrates such an equilibrium.

![Equilibrium with bad news undisclosed (r = 0.25)](image)

**Figure 5: Equilibrium with bad news undisclosed (r = 0.25)**

If the return on investment is lower, the characteristics of the equilibrium are determined by how the thresholds $x_I$, $x_1^D$ and $x_2^D$ compare. $x_I$ is determined by the intersection of the bias function and the function $b_I(x)$. If $x_I > x_2^D$ then $[0, x_I)$ is the unique non-disclosure interval because managers whose asset values exceed $x_2^D$ prefer to disclose and invest. This case is characterized in part (ii) a of Proposition 2 and illustrated in Figure 6. If the intersection of $b(x)$ and $b_I(x)$ is such that $x_I \in (x_1^D, x_2^D)$ there is also a single non-disclosure interval but it spans $[0, x_2^D)$ rather than $[0, x_I)$. Firms with $x \in [0, x_I)$ invest while firms with $x \in (x_I, x_2^D)$ do not invest. This case is characterized in part (ii) b of Proposition 2 and illustrated in Figure 7. Finally, if $x_I < x_1^D$, there are two distinct non-disclosure intervals. For $x < x_I$ the manager does not make a disclosure but raises capital. For $x \in (x_1^D, x_2^D)$ the manager does not make a disclosure but also does not raise capital. This case is characterized in part (ii) c of Proposition 2 and illustrated in Figure 8.

When the return on investment is sufficiently low ($r < r^*$) one of the three equilibria in Figures 6-8 (or, equivalently in parts a-c in Proposition 2) will exist. However, we have so far not yet specified the conditions under which each of the three equilibria exists. Which equilibrium exists will depend on how the threshold $x_I$ compares to $x_1^D$ and $x_2^D$. The location of $x_I$ is determined by the properties of the prior distribution of $x$. When the prior distribution of $x$ assigns relatively
low weight to small asset values, the manager is willing to pay less (as reflected in an downward shift of $b_I(x)$) in order to separate himself from lower types. As a result, $x_I$ is farther to the right. In contrast, $x_I^D$ and $x_2^D$ are independent of the prior distribution of asset values $x$ because both $g^{-1}(r)$ and $b(x)$ do not vary with the prior distribution of $x$.

Corollary 2 formalizes this intuition and establishes that $x_I$ is higher for distribution $h$ than for distribution $f$ if $h$ is a convex transformation of $f$.\footnote{While the corollary assumes a convex transformation of the form $h(x) = \frac{xf(x)}{\int_0^x f(x)dx}$, we conjecture that the result holds for more general convex transformations.} If $h$ is a convex transformation of $f$ than $E_h[\tilde{x}|\tilde{x} < x'] > E_f[\tilde{x}|\tilde{x} < x']$ for all $x' > 0$. As a result, a firm with asset value $x'$ is undervalued by less if investors' prior beliefs are that firms' assets values are distributed according to $h$ rather than $f$. Then, a firm with asset value $x'$ is willing to pay less for separating itself from firms with lower
asset values. This is reflected in the function $b_I(x)$ (which indicates the maximum bias for which a firm is willing to bear the costs in order to separate itself from firms with lower asset values) being lower, which maps into a higher value of $x_I$.

**Corollary 2** If the pdf $h$ is a convex transformation of $f$ in the sense that $h(x) = \frac{xf(x)}{\int_0^\infty xf(x)dx}$ then $x^g_I > x^F_I$.

Proposition 2 establishes that there always exists an equilibrium in which low types raise capital and invest without disclosing information. When the firm raises capital without disclosing information, investors can infer the expected value of the firm’s assets in place – and, equivalently, can predict the average return on their investment in the firm’s shares – but investors are not able to predict the exact return on their investment in the firm’s share. In contrast, when the firm makes a disclosure before raising capital, investors are able to infer the exact value of the firm’s assets in place and therefore can predict the exact return on their investment in the firm’s shares. So far, we assumed that investors are risk-neutral. Risk-neutral investors are indifferent whether they earn a certain return or whether they earn the same return in expectations with the actual return sometimes being lower and sometimes higher. If investors are risk-averse and the variation in the return on their equity investment contributes to their exposure to systematic risk, then investors will price equity offerings that are not accompanied by a voluntary disclosure at a discount. Due to the discount associated with non-disclosure, the firm is (weakly) less likely to withhold information.
when investors are risk-averse.\textsuperscript{18}

**Corollary 3** If investors are risk-averse, the firm’s expected cost of capital is lower when it makes a voluntary disclosure than when it does not. As investors’ risk-aversion increases, the likelihood of the firm raising capital without disclosing information decreases.

This prediction is consistent with some empirical studies analyzing the association between disclosures, financial reporting quality, and the cost of capital. Recent work documents that more extensive pre-IPO disclosures are associated with lower under-pricing using several proxies for the level of disclosure a firm engages in prior to its IPO. For a sample of Canadian IPOs, Jog and McConomy (2003) use voluntary earnings forecasts that the Canadian securities commission allows, but does not require, issuers of IPOs to include in their prospectus as their measure of voluntary disclosure and provide evidence that firms that provide a voluntary earnings forecast experience less severe underpricing. Schrand and Verrecchia (2005) use the announcements contained in firms’ press releases as a measure of disclosure level and find evidence consistent with lower underpricing of firms that voluntarily provide more information about their operating, investing and financing activities to the public in press releases prior to its IPO. Leone, Rock and Willenborg (2007) measure the detail an IPO issuer provides regarding their intended use of cash proceeds and first-day under-pricing and also find evidence of a negative association between the level of detail provided and first-day underpricing. Together, the evidence in these papers indicates that voluntary disclosure have a favorable and noticeable impact on the degree of underpricing and the post-issue return performance despite the comprehensive disclosures required by securities commissions prior to an IPO. This suggests that adverse selection is at least in part a determinant of cross-sectional variation in underpricing and that voluntary disclosure can reduce the adverse selection effect consistent with the prediction of this model.

**Corollary 4** If investors are risk-averse, the firm’s expected cost of capital are higher for low-performing firms than for high-performing firms.

---

\textsuperscript{18}Equity offerings being priced at a discount in the absence of a voluntary report causes the $b_t(x)$ function to be higher. The reason is that the manager is willing to bear even higher costs in order to separate himself from the pool of firms when investors are risk-averse in order to avoid the risk-premium.
Appendix

4.2.1 Proof of Lemma 2

In equilibrium, the marginal benefit from biasing the report equals the marginal cost of biasing the report. The marginal benefit is

$$\frac{\partial}{\partial x} \left( 1 - \frac{I}{x - \hat{b}(x) + I + r} \right) (x + I + r) = \frac{I (x + I + r) \left( 1 - \frac{\partial \hat{b}(x)}{\partial x} \right)}{(x - \hat{b}(x) + I + r)^2}$$

On the equilibrium path we can substitute $x - \hat{b}(x) = x$. This yields the FOC (in terms of $\hat{b}(x)$)

$$\frac{I}{x - \hat{b}(x) + I + r} \left( 1 - \frac{\partial \hat{b}(x)}{\partial x} \right) - g'(\hat{b}(x)) = 0$$

(4)

We can expand $\frac{\partial \hat{b}(x)}{\partial x}$ as $\frac{\partial \hat{b}(x + b(x))}{\partial x} = \frac{\partial \hat{b}(x)}{\partial x} (1 + b'(x))$ which has to equal $b'(x)$ on the equilibrium path. Hence, we can substitute $\frac{b'(x)}{1 + b'(x)}$ for $\frac{\partial \hat{b}(x)}{\partial x}$. In addition, $\hat{b}(x) = b(x)$ and $x - \hat{b}(x) = x$ on the equilibrium path. This yields the FOC (in terms of $b(x)$)

$$\frac{I}{x + I + r} \left( 1 - \frac{b'(x)}{1 + b'(x)} \right) - g'(b(x)) = 0$$

(5)

Rearranging yields (1).

The equilibrium bias function is given by the solution to (1) with the boundary condition $b(0) = 0$. We next want to show that there exists a solution to this initial value problem. We cannot invoke the Fundamental Theorem of Differential Equations in order to show that the solution to (1) exists and unique since the RHS of (1) is not finite at $b(0) = 0$. In order to show that the solution exists, we substitute $c(x)$ for $\frac{1}{2} (b(x))^2$. This implies that $b(x) = \sqrt{2|c(x)|}$ and $b'(x) = \frac{c'(x)}{\sqrt{2|c(x)|}}$. Rewriting the differential equation in (1) in terms of $c$ yields

$$c'(x) = \frac{\sqrt{2|c(x)|}}{g'(\sqrt{2|c(x)|})} \frac{I}{x + I + r} - \sqrt{2|c(x)|}$$

(6)

with the boundary condition $c(0) = 0$. Let $F(x, c) = g'\left(\sqrt{2|c|}\right) x + I + r - \sqrt{2|c|}$ denote the RHS of (6). We want to show that $F(x, c)$ is continuous in $x$ and $c$ at $(0,0)$. While $F(x, c)$ is clearly continuous in $x$, it is only continuous in $c$ if $\lim_{c \to 0} F(x, c)$ is finite.

$$\lim_{c \to 0} F(x, c) = \lim_{c \to 0} \frac{\frac{\partial}{\partial c} \sqrt{2|c|}}{g'(\sqrt{2|c|})} \frac{I}{x + I + r} = 0 = \lim_{c \to 0} \frac{1}{\sqrt{2|c|}} \frac{I}{x + I + r} = \frac{1}{g''(0)} \frac{I}{x + I + r}$$
which is finite because \( g(\cdot) \) is strictly convex everywhere. Hence, there exists a continuous and differentiable solution to (6) with the boundary condition \( c(0) = 0 \). Next, we show that \( c(x) \) provides a solution to the manager’s disclosure problem. That requires \( c(x) \geq 0 \) such that \( b(x) \) is real. First note that \( c(x) \geq 0 \) for \([0, \varepsilon]\) and \( \varepsilon \) sufficiently small because \( c'(0) = F'(0,0) = \frac{1}{g(0)} f_{\varepsilon} > 0 \). Suppose there existed \( x' \) such that \( c(x') < 0 \). Since \( c(x) \) is continuous, there must exist \( x'' < x' \) such that \( c(x'') = 0 \) and \( c'(x'') < 0 \). However, \( c(x'') = 0 \) implies that \( c'(x'') > 0 \). Hence, \( c(x) \geq 0 \) for all \( x \in [0, \infty) \). As a result, \( b(x) = \sqrt{2c(x)} \) is real and provides a solution to the manager’s disclosure problem. Moreover, the solution is unique. Suppose it were not unique and there existed two solutions, \( b_1(x) \) and \( b_2(x) \) which both satisfied the differential equation in (5) and \( b_1(0) = b_2(0) = 0 \). Further, since \( b_1(x) \) and \( b_2(x) \) differ and are differentiable there must exist an interval \((x', x'')\) for which \( b_1(x) > b_2(x) \) and \( b'_1(x) > b'_2(x) \). However, for a given \( x \), \( b' \) is lower for higher values of \( b \). As a result, \( b_1(x) > b_2(x) \) and \( b'_1(x) > b'_2(x) \) cannot hold for any \( x \) and the solution to the initial value problem is unique.

We next want to show that the equilibrium bias \( b(x) \) has the following properties: it is continuous, always positive, initially increasing, obtains a unique maximum and converges to zero as the value of the firm’s assets in place goes to infinity.

We have already shown that the equilibrium bias is continuous and strictly positive for \( x > 0 \). From this and the boundary condition \( b(0) = 0 \), it also follows that the bias is initially increasing. However, \( b(x) \) cannot always be increasing. Suppose it were the case that \( b'(x) > 0 \) for all \( x \). Then, we know that \( 1 - \frac{b'(x)}{1+b'(x)} \in (0, 1) \). Hence, the marginal benefit, \( \frac{f}{x+1+r} \left( 1 - \frac{b'(x)}{1+b'(x)} \right) \), converges to 0 for \( x \to \infty \). In equilibrium, the marginal cost must then also converge to zero. This cannot be the case if \( b(x) \) is positive and always increasing for any \( x \). As a result, for \( x \) sufficiently large \( b(x) \) is decreasing \((-1 < b'(x) < 0)\) and hence \( 1 - \frac{b'(x)}{1+b'(x)} > 1 \). Further, since \( b(x) > 0 \) for all \( x \), \( b'(x) \) has to converge to 0 for \( x \to \infty \). As a result, \( \frac{f}{x+1+r} \left( 1 - \frac{b'(x)}{1+b'(x)} \right) \) converges to 0 for \( x \to \infty \). Hence, the marginal cost must also converge to zero and therefore \( b(x) \to 0 \) for \( x \to \infty \).

Finally, we want to show that \( b(x) \) does not have a local minimum. We show that by proving that there does not exist any \( x \) for which \( b(x) \) is weakly increasing and weakly convex. Suppose it were the case that \( b'(x) \geq 0 \) and \( b''(x) \geq 0 \) for a given \( x \). As \( x \) increases, \( b(x) \) weakly increases and hence \( g'(b) \) weakly increases (due to \( b(x) \geq 0 \) and \( b'(x) \geq 0 \)). Moreover, as \( x \) increases, \( b'(x) \) weakly increases and hence \( \frac{b'(x)}{1+b'(x)} \) weakly increases (due to \( b''(x) \geq 0 \)). The latter implies that the marginal benefit, \( \frac{f}{x+1+r} \left( 1 - \frac{b'(x)}{1+b'(x)} \right) \), strictly decreases in \( x \). Since the marginal cost, \( g'(b) \),
weakly increases, this yields a contradiction. ■

4.2.2 Proof of Proposition 1

We start out by showing that \( g(b(x)) - r \) is monotonically decreasing in \( r \). This implies that there exists \( r^* \) such that for \( r \geq r^* \) the disclosure costs \( g(b(x)) \) are (weakly) less than the return on investment \( r \) for all \( x \) and that for \( r < r^* \) there are some types \( x \) for which the disclosure costs \( g(b(x)) \) strictly exceed the return on investment \( r \).

**Lemma 5** For any given \( x \), \( g(b(x)) - r \) is monotonically decreasing in \( r \) where \( b(x) \) is given by (5) with \( b(0) = 0 \) as a boundary condition.

**Proof.** Let \( r_1 < r_2 \). First, we consider the initial value problem of the differential equation in (5) with the boundary condition \( b(0) = \varepsilon \) for \( \varepsilon > 0 \). The slope of the bias function at \( x = 0 \) is \( \frac{1}{g'(\varepsilon)(1+r)} - 1 \) and hence \( b'(0; r_1) > b'(0; r_2) \). Hence, there exists \( \delta > 0 \) such that \( b(x; r_1) > b(x; r_2) \) for all \( x \in (0, \delta) \). Moreover, if there exists an \( x' > 0 \) such that \( b(x'; r_1) > b(x'; r_2) \), then \( b(x; r_1) > b(x; r_2) \) for all \( x \in [x', \infty) \). Suppose this were not true, then there exists \( x'' \) such that \( b(x''; r_1) = b(x''; r_2) \) and \( b'(x''; r_1) \leq b'(x''; r_2) \). However, (5) shows that, for a given \( (x, b) \), \( b'(x) \) is decreasing in \( r \). Hence, for the boundary condition \( b(0) = \varepsilon \) it follows that \( b(x; r_1) > b(x; r_2) \) for all \( x \in (0, \infty) \).

From continuity it follows that \( b(x; r_1) \geq b(x; r_2) \) for the original boundary condition \( b(0) = 0 \). Further, suppose there existed \( x' > 0 \) such that \( b(x'; r_1) = b(x'; r_2) \). Then, it must be that \( b(x; r_1) \) and \( b(x; r_2) \) are tangential, i.e., \( b'(x'; r_1) = b'(x'; r_2) \). However, this cannot be true for \( x' > 0 \). Hence, it follows that \( b(x; r_1) > b(x; r_2) \) for all \( x \in (0, \infty) \). ■

Next, we show that \( r \leq r^* \) such that \( g(b(x)) \leq r \) for all \( x \) is a necessary condition for an equilibrium with full disclosure to exist. Further, \( r > r^* \) implies that there exists some \( x \) for which \( g(b(x)) > r \) which is a necessary condition for an equilibrium with intermediate news undisclosed to exist. In addition, these conditions are sufficient in conjunction with off-equilibrium beliefs that managers that raise capital without issuing a report are of the lowest type \( (x = 0) \).

**Lemma 6** A necessary and sufficient condition for existence of a full disclosure equilibrium is that for any value of the assets in place, \( x \), the following inequality holds

\[
    r \geq g(b(x)).
\]  
(7)

31
Proof. In a full disclosure equilibrium, the manager always discloses, the firm equity is correctly priced and hence the firm invests in the positive NPV project. Hence, a firm’s payoff is \( x + r - g(b(x)) \). We need to show that none of the types has incentives to deviate. One potential deviation in a full disclosure equilibrium is for a type not to disclose and not to invest which yields a payoff of \( x \). A necessary condition to preclude such deviation is that the disclosure cost of all types are lower than their return on the investment, \( r \). So, a necessary condition for the existence of a full disclosure equilibrium is that condition (7) holds for any \( x \), or equivalently, \( g^{-1}(r) > b(x) \).

Condition (7) is not only necessary, but also sufficient for existence of a full disclosure equilibrium. To show the sufficiency of this condition one needs to preclude any form of deviation. There are two other types of potential deviations: disclosure of a report other than the type’s equilibrium report and investment and non-disclosure and investment. By construction of the bias function \( b(x) \), which is the solution of the differential equation (5), any deviation to a disclosure of another type is precluded. To preclude deviation to non-disclosure, we need to assign off equilibrium beliefs that guarantee that no type wants to deviate to a non-disclosure and investment strategy. The only off-equilibrium beliefs that guarantee no such deviation is that all types that do not disclose but do raise capital are perceived as the lowest type. Note that these off-equilibrium beliefs are natural, since in equilibrium the only type who is always indifferent between disclosing and investing and not disclosing and investing is the lowest type. Moreover, the beliefs are robust to D1 and D2 refinements (see Cho and Kreps 1987; Banks and Sobel 1987; Fudenberg and Tirole 1991).

Lemma 7 A necessary and sufficient condition for existence of an equilibrium with intermediate news undisclosed is that there exists a value of assets in place, \( x \), such that

\[
r < g(b(x)).
\]

Proof. Condition (8) implies that there are exactly two values of assets in place, \( x_1^D \) and \( x_2^D \), such that \( b(x_1^D) = b(x_2^D) = g^{-1}(r) \). We want to show that the following equilibrium, which relies on condition (8) always exists: For all \( x \in [0, x_1^D) \) and \( x \geq x_2^D \) the manager discloses according to the full disclosure equilibrium and invests and for all \( x \in [x_1^D, x_2^D) \) the manager does not disclose and does not invest. If investors observe an off-equilibrium report they believe that the manager reported according to the full disclosure equilibrium. Lemma 8 guarantees that for all \( x \in [x_1^D, x_2^D) \), the manager prefers no investment over investment when he does not disclose. The off-equilibrium beliefs guarantee that all types that disclose in equilibrium do not deviate to
a different off-equilibrium report. As before, we assume that if a firm invests without disclosing investors believe that the firm is of the lowest type, i.e., the value of the assets in place is zero. Together, this shows that condition (8) is sufficient to have an equilibrium with intermediate news undisclosed. To show that condition (8) is also necessary, note that if condition (8) does not hold then any type that is supposed not to disclose and not to invest would deviate to his full disclosure report following which he invests.19

4.2.3 Proof of Corollary 1

Recall that \(x_1^D\) and \(x_2^D\) are given by \(g(b(x_1^D)) = g(b(x_2^D)) = r\). Differentiating this condition with respect to \(x_i^D\) and \(r\) for \(i = 1, 2\) yields

\[
g'(b(x_i^D)) b'(x_i^D) dx_i^D + \left( g'(b(x_i^D)) \frac{\partial b(x_i^D)}{\partial r} - 1 \right) dr = 0.
\]

Rearranging yields

\[
dx_i^D \over dr = -{g'(b(x_i^D)) \over g'(b(x_i^D))} \frac{\partial b(x_i^D)}{\partial r} - 1.
\]

\(g'(b(x_i^D))\) is positive and Lemma 5 implies that \(\frac{\partial b(x_i^D)}{\partial r}\) is negative. Hence, the numerator is always negative and the denominator takes the sign of \(b'(x_i^D)\). For \(x_1^D\) (\(x_2^D\)) the bias is increasing and hence \(dx_i^D \over dr > 0\) (\(dx_i^D \over dr < 0\)).

4.2.4 Proof of Lemma 3

Suppose, for a given \(X_{nd}\), \(E[\bar{x} + I + r|x \in X_{nd}] = \bar{\mu}_{nd}\). Further, if \(\{x|x < x_I\} \cap X_{nd} = \emptyset\) then \(E[\bar{x} + I + r|x < x_I, x \in X_{nd}] = 0\). Hence, \(\alpha_{nd}(x_I = 0) = \frac{I}{I + r}\) and \(\lim_{x_I \to \infty} \alpha_{nd}(x_I) = \frac{I}{\bar{\mu}_{nd} + I + r}\). Hence, \(\alpha \in \left( \frac{I}{I + r}, \frac{I}{\bar{\mu}_{nd} + I + r}\right) \subset (0, 1)\).

Substituting equation (2) into equation (3) and rearranging yields

\[
{I \over E[\bar{x} + I + r|x < x_I, x \in X_{nd}] + I + r} + \max \{0, r - g(b(x_I))\} = \left(1 - {I \over E[\bar{x} + I + r|x < x_I, x \in X_{nd}] + I + r}\right) (I + r).
\]

Both the LHS and the RHS are continuous in \(x_I\). Moreover, for \(x_I = 0\) the LHS equals \(0\) and the RHS equals \(r\) while for \(x_I \to \infty\) the LHS approaches \(\infty\) and the RHS equals \(\frac{\bar{\mu}_{nd} + r}{\bar{\mu}_{nd} + I + r} (I + r)\) which is finite. Hence, there exists \(x_I \in (0, \infty)\) such that the equation (3) hold when \(\alpha_{nd}\) is given by \(\alpha_{nd} = I/E[\bar{x} + I + r|x < x_I, x \in X_{nd}]\).

19This argument relies on off-equilibrium beliefs that equal to the full disclosure beliefs, however, it is easy to show that if condition (8) does not hold an equilibrium with intermediate news undisclosed cannot exist for any off-equilibrium beliefs.
4.2.5 Proof of Lemma 4

As an intermediate step, we prove the following lemma.

**Lemma 8** In any fully revealing disclosure equilibrium if there exists a non-disclosure interval, \((x_1^D, x_2^D)\) such that there exists \(x'\) which is \(\epsilon\) to the left of \(x_1^D\) that invests and discloses then any \(x \in (x_1^D, x_2^D)\) does not invest.

**Proof.** Let \(y \in \{0, 1\}\) be the investment decision where \(y = 1\) indicates that the firm pursues the investment opportunity and \(y = 0\) otherwise. Suppose type \(x_1^D\) were indifferent between disclosing \(x_R(x_1^D)\) and not disclosing and investing, i.e.,

\[
\left(1 - \frac{I}{E(\hat{x}|x_R(x_1^D)) + I + r}\right) (x_1^D + I + r) - g(x_R(x_1^D) - x_1^D) = \left(1 - \frac{I}{E(\hat{x}|nd, y = 1) + I + r}\right) (x_1^D + I + r)
\]

(9)

For \(x_1^D\) to be indifferent investors’ beliefs have to be such that \(E(\hat{x}|x_R(x_1^D)) > E(\hat{x}|nd, y = 1)\) because the disclosure costs are strictly positive (this follows from the bias being strictly positive; note that zero bias is not feasible for finite \(x\)). We also know that type \(x'\) prefers to invest and disclose over non-disclosure and investment, i.e.,

\[
\left(1 - \frac{I}{E(\hat{x}|x_R(x')) + I + r}\right) (x' + I + r) - g(x_R(x') - x') = \left(1 - \frac{I}{E(\hat{x}|nd, y = 1) + I + r}\right) (x' + I + r) > 0
\]

(10)

Since \(x'\) is sufficiently close to \(x_1^D\) it is the case that \(E(\hat{x}|x_R(x_1^D)) > E(\hat{x}|x_R(x')) > E(\hat{x}|nd, y = 1)\). In order to arrive at a contradiction we compute the payoff of type \(x_1^D\) from mimicking \(x'\) over his payoff from investing without disclosing, i.e.,

\[
\left(1 - \frac{I}{E(\hat{x}|x_R(x')) + I + r}\right) (x_1^D + I + r) - g(x_R(x') - x_1^D) = \left(1 - \frac{I}{E(\hat{x}|nd, y = 1) + I + r}\right) (x_1^D + I + r)
\]

(11)
In order to arrive at a contradiction we want to show that the expression in (11) is positive (which implies that type $x_1^D$ wants to deviate and mimic $x'$)

\[
\left(1 - \frac{I}{E(\tilde{x}|x_R(x')) + I + r}\right) (x_1^D + I + r) - g(x_R(x') - x_1^D) - \left(1 - \frac{I}{E(\tilde{x}|nd, y = 1) + I + r}\right) (x_1^D + I + r) \\
= \left(1 - \frac{I}{E(\tilde{x}|x_R(x')) + I + r}\right) (x' + \varepsilon + I + r) - g(x_R(x') - x' - \varepsilon) \\
- \left(1 - \frac{I}{E(\tilde{x}|nd, y = 1) + I + r}\right) (x' + \varepsilon + I + r) \\
= A + \left(1 - \frac{I}{E(\tilde{x}|x_R(x')) + I + r}\right) \varepsilon + g(x_R(x') - x') - g(x_R(x') - x' - \varepsilon) \\
- \left(1 - \frac{I}{E(\tilde{x}|nd, y = 1) + I + r}\right) \varepsilon \\
= A + \left(\frac{I}{E(\tilde{x}|nd, y = 1) + I + r} - \frac{I}{E(\tilde{x}|x_R(x')) + I + r}\right) \varepsilon + g(x_R(x') - x') - g(x_R(x') - x' - \varepsilon)
\]

where $A$ denotes the LHS expression in (10). Note that

\[
\left(\frac{I}{E(\tilde{x}|nd, y = 1) + I + r} - \frac{I}{E(\tilde{x}|x_R(x')) + I + r}\right) \varepsilon > 0
\]

because $E(\tilde{x}|x_R(x')) > E(\tilde{x}|nd, y = 1)$ and

\[
g(x_R(x') - x') - g(x_R(x') - x' - \varepsilon) > 0
\]

which follows from the bias being positive and

\[
x_R(x') - x' > x_R(x') - x' - \varepsilon \\
x_R(x') > x_R(x') - \varepsilon
\]

which implies that the expression in (12) is positive. As a result, type $x_1^D$ prefers mimicking the report of $x'$ which contradicts the assumption that we are in equilibrium. ■

To complete the proof of Lemma 4, we have to show that if there exists a type $x$ that prefers to invest without disclosing then all types to the left of $x$ also prefer to invest without disclosing. Based on Lemma 8 type $x$ belongs to the left-most non-disclosure interval. Hence, all types to the left of $x$ do not disclose. We need to show that all types to the left of $x$ prefer investing without disclosure to non-investing without disclosure if $x$ prefers to invest without disclosure to non-investing without disclosure. This follows from the fact that lower types give up the same fraction of their firm which is worth less in return for the same return on investment.
4.2.6 Proof of Proposition 2

We start by showing that for sufficiently small \( x \), \( b(x) \) is strictly greater than \( b_I(x) \). From this it follows that \( b(x) \) and \( b_I(x) \) intersect at least once.

**Lemma 9** There exists \( \varepsilon > 0 \) such that \( b(x) > b_I(x) \) for \( x \in (0, \varepsilon) \).

**Proof.** From Definition 1, it follows that

\[
b_I(x) = g^{-1}\left( \frac{x - E(\tilde{x}|\tilde{x} < x)}{E(\tilde{x}|\tilde{x} < x) + I + r} \right)
\]

and

\[
b'_I(x) = \frac{1}{g'(b_I(x))} \frac{I}{E(\tilde{x}|\tilde{x} < x) + I + r} \left( 1 - \frac{x + I + r}{E(\tilde{x}|\tilde{x} < x) + I + r} (x - E(\tilde{x}|\tilde{x} < x)) \frac{f(x)}{F(x)} \right).
\]

We want to show that \( \lim_{x \to 0} \frac{b'(x)}{b'_I(x)} > 1 \) where \( b'(x) \) is given by equation (1).

\[
\lim_{x \to 0} \frac{b'(x)}{b'_I(x)} = \lim_{x \to 0} \frac{\frac{I}{g'(b_I(x))} \left( 1 - \frac{x + I + r}{E(\tilde{x}|\tilde{x} < x) + I + r} (x - E(\tilde{x}|\tilde{x} < x)) \frac{f(x)}{F(x)} \right)}{\frac{I}{I + r} - 0} \lim_{x \to 0} \frac{g'(b_I(x))}{g'(b(x))}
\]

where the last equality follows from

\[
\lim_{x \to 0} x \frac{f(x)}{F(x)} = \lim_{x \to 0} \frac{f(x) + xf'(x)}{f(x)} = 1 + \lim_{x \to 0} \frac{xf'(x)}{f(x)} = 1 \text{ for } f(0) > 0 \text{ and } |f'(0)| < \infty
\]

\[
\lim_{x \to 0} E(\tilde{x}|\tilde{x} < x) \frac{f(x)}{F(x)} = \lim_{x \to 0} \frac{f(x)}{F^2(x)} \int_0^x zf(z) dz = \lim_{x \to 0} \frac{f'(x) \int_0^x zf(z) dz + f(x)xf(x)}{2F(x)f(x)} = \lim_{x \to 0} \frac{f'(x)\int_0^x zf(z) dz + f(x)xf(x)}{2F(x)f(x)} = \lim_{x \to 0} \frac{f'(x)\int_0^x zf(z) dz + f(x)xf(x)}{2F(x)f(x)}
\]

We know that

\[
\lim_{x \to 0} \frac{b'(x)}{b'_I(x)} = 2.
\]

Recall that \( b_I(0) = b(0) \). First, consider the case when \( \lim_{x \to 0} \frac{b'(x)}{b'_I(x)} \leq 1 \). Then, \( \lim_{x \to 0} \frac{g'(b(x))}{g'(b_I(x))} \geq 2 \). Which implies that there exists an interval \((0, \varepsilon)\) for which \( b(x) > b_I(x) \) (which can only hold when \( \lim_{x \to 0} \frac{b'(x)}{b'_I(x)} = 1 \)). Next, consider the case when \( \lim_{x \to 0} \frac{b'(x)}{b'_I(x)} > 1 \). Then, there must be an interval for which \( b(\cdot) \) is steeper than \( b_I(\cdot) \) because both functions are differentiable for \( x > 0 \),
and, as a result there exists an interval \((0, \varepsilon)\) for which \(b(x) > b_I(x)\). Hence, there always exists an interval \((0, \varepsilon)\) for which \(b(x) > b_I(x)\) \(\blacksquare\).

Next, we show that there always exists an equilibrium in which low types do not disclose but invest. The other characteristics of the equilibrium in Proposition 2 follow immediately from Proposition 1 and Lemma 4.

**Lemma 10** There always exists an equilibrium in which low types do not disclose but invest.

**Proof.** First, we consider the case in which the parameters are such that \(b(x) \leq g^{-1}(r)\) (i.e., \(r\) sufficiently high). Let

\[
u_1(x') = \frac{E[\tilde{x}|x<x'] + r}{E[\tilde{x}|x<x'] + I + r}(x' + r + I) - (x' + r - g(b(x')))
\]

We want to show that there exists \(x' > 0\) such that \(u_1(x') = 0\). From the definition of \(u_1(x')\) it follows that \(\lim_{x' \to 0} u_1(x') = 0\) and \(\lim_{x' \to \infty} u_1(x') = -\infty\). Moreover,

\[
\frac{\partial u_1(x')}{\partial x'} \bigg|_{x'=0} = \frac{(E[\tilde{x}|x<x'] + I + r) \frac{\partial E[\tilde{x}|x<x']}{\partial x} - (E[\tilde{x}|x<x'] + r) \frac{\partial E[\tilde{x}|x<x']}{\partial x}}{(E[\tilde{x}|x<x'] + I + r)^2}(x' + r + I) \bigg|_{x'=0}
\]

\[+\left(\frac{E[\tilde{x}|x<x'] + r}{E[\tilde{x}|x<x'] + I + r} - (1 - g'(b(x))b'(x'))\right)\bigg|_{x'=0}
\]

\[= \left(\frac{I(x' + r + I)}{(E[\tilde{x}|x<x'] + I + r)^2} \frac{\partial E[\tilde{x}|x<x']}{\partial x'}\right)\bigg|_{x'=0}
\]

\[+ \frac{E[\tilde{x}|x<x'] + r}{E[\tilde{x}|x<x'] + I + r} - \left(1 - \frac{I}{x' + I + r} \frac{b'(x')}{1 + \theta(x')}\right)\bigg|_{x'=0}
\]

\[= \frac{I}{I + r} \frac{\partial E[\tilde{x}|x<x']}{\partial x'} + \frac{r}{I + r} - \left(1 - \frac{I}{I + r}\right)\]

Hence, continuity of \(u(x')\) implies that there exists at least one \(x' > 0\) such that \(u_1(x') = 0\). Let

\[x_I = \min \{x'| u_1(x') = 0, x' > 0\}\]

We have shown that \(\lim_{x' \to 0} u_1(x') = 0\), \(\lim_{x' \to \infty} u_1(x') = -\infty\), and that \(u_1(x')\) is increasing in \(x'\). By construction of \(x_I\), for all \(x' \in [0, x_I]\) we have \(u_1(x') \geq 0\). Since \(\frac{E[\tilde{x}|x<x'] + r}{E[\tilde{x}|x<x'] + I + r}\) is increasing in \(x'\) it follows that

\[\frac{E[\tilde{x}|x<x_I] + r}{E[\tilde{x}|x<x_I] + I + r}(x' + r + I) - (x' + r - g(b(x'))) \geq 0.
\]
Hence, all \( x' \in [0, x_I) \) prefer investment without disclosure to investment with disclosure. We further need to show that no type \( x' > x_I \) wants to deviate and invest without disclosure. We know that \( x' > x_I \) does not mimic \( x_I \) by issuing the report \( x_R (x_I) \) and investing.

\[
x' + r - g (b(x')) \geq \frac{x_I + r}{x_I + I + r} (x' + r + I) - g (b(x_I) - (x' - x_I))
\]

We want to show that type \( x' > x_I \) prefers issuing the report \( x_I \) to investment without disclosure, i.e., we want to show that

\[
\frac{x_I + r}{x_I + I + r} (x' + r + I) - g (b(x_I) - (x' - x_I)) > \frac{E [\tilde{x} | x < x_I] + r}{E [\tilde{x} | x < x_I] + I + r} (x' + r + I)
\]

(13)

First, note that we can restrict the analysis to \( x' \) that do not need to bias their report downwards in order to mimic \( x_I \). The reason is that \( x' \) that would have to bias its report downward in order to mimic \( x_I \) can mimic a higher type without incurring any biasing costs. Hence, we restrict attention to \( x' \in (x_I, x_I + b(x_I)) \). Taking the derivative of the LHS of (13) net of the RHS of (13) with respect to \( x' \) yields

\[
\frac{x_I + r}{x_I + I + r} + g' (b(x_I) - (x' - x_I)) - \frac{E [\tilde{x} | x < x_I] + r}{E [\tilde{x} | x < x_I] + I + r}
\]

which is positive for \( x' \in (x_I, x_I + b(x_I)) \). We know that (13) holds with equality for \( x_I \). Hence, the inequality in (13) holds for all \( x' \in (x_I, x_I + b(x_I)) \). This proves that no type \( x' > x_I \) wants to deviate and invest without disclosure. The manager’s equilibrium strategy is as follows: \( x \in [0, x_I) \) invest but do not disclose and \( x \in [x_I, \infty) \) invest and disclose.

Next, we consider the case in which \( b(x) > g^{-1} (r) \) for some \( x \). We define

\[
u_1 (x') = \frac{E [\tilde{x} | x < x'] + r}{E [\tilde{x} | x < x'] + I + r} (x' + r + I) - (x' + \max \{0, r - g(b(x'))\})
\]

By the same argument as above, there exists at least one \( x' > 0 \) such that \( u_1 (x') = 0 \). If \( \tilde{x} = \min \{x' | u_1 (x') = 0, x' > 0\} \) is such that \( b(\tilde{x}) \leq g^{-1} (r) \), then \( x_I = \min \{x' | u_1 (x') = 0, x' > 0\} \), then \( x \in [0, x_I) \) invest but do not disclose and \( x \in [x_I, \infty) \) invest and disclose. If \( \tilde{x} = \min \{x' | u_1 (x') = 0, x' > 0\} \) is such that \( b(\tilde{x}) > g^{-1} (r) \), then \( x_I = \min \{x' | b(x') = g^{-1} (r)\} \). Let \( x_D = \max \{x' | b(x') = g^{-1} (r)\} \). In equilibrium, \( x \in [0, x_I) \) invest but do not disclose; \( x \in [x_I, x_D) \) do not invest and do not disclose and \( x \in [x_D, \infty) \) invest and disclose. ■

**Proof of Corollary 2**

The cumulative distribution function of \( h(x) = \frac{x f(x)}{\int_0^x y f(y) dy} \) is given by

\[
H(x) = \int_0^x h(z) dz = \int_0^x \frac{zf(z)}{\int_0^\infty y f(y) dy} dz = \frac{F(x) E_f [\tilde{x} | \tilde{x} < x]}{\int_0^\infty y f(y) dy}
\]

38
We want to show that $E[f|\tilde{x}<x] < E[h|\tilde{x}<x]$. We can rewrite $E[h|\tilde{x}<x]$ as

$$E[h|\tilde{x}<x] = \frac{1}{H(x)} \int_0^x z^2 f(z) \, dz = \frac{1}{F(x) E[f|\tilde{x}<x]} \int_0^x z^2 f(z) \, dz$$

Hence, $E[f|\tilde{x}<x] < E[h|\tilde{x}<x]$ is equivalent to

$$(E[f|\tilde{x}<x])^2 < E[f|\tilde{x}<x] < 0 < Var[f|\tilde{x}<x]$$

which always holds.

### 4.2.7 Proof of Proposition 3

**Proposition 3** In any equilibrium in which the equilibrium reporting strategy is given by the bias in (1) with the boundary condition $b(0) = 0$, there are at most two non-disclosure intervals.

We start by proving the following lemma.

**Lemma 11** If the highest type of a non-disclosure interval, $x'$, does not invest then there does not exist an additional non-disclosure interval to the right of $x'$.

**Proof.** $x'$ must be indifferent between disclosing and investing and not disclosing and not investing. This requires $b(x') = g^{-1}(r)$. Suppose there exists an additional non-disclosure interval, $(x_1, x_2)$, where $x' < x_1 < x_2$. Then, the bias must be decreasing over the interval $[x', x_1]$ because the bias can not exceed $g^{-1}(r)$ and the bias never increases in $x$ once it has decreased. This implies that the bias at $x_1$ is strictly lower than $g^{-1}(r)$. Since $x'$ prefers not to invest if he does not disclose, $x_1$ must strictly prefer not to invest if he does not disclose. $x_1$ is indifferent between disclosing and investing and not disclosing and not investing only if $b(x_1) = g^{-1}(r)$. This yields a contradiction because we argued that $b(x_1) < b(x') = g^{-1}(r)$. □

From Lemma 8 it follows that the highest type of the second non-disclosure interval, $x_2$, never invests. Following an argument similar to the proof of Lemma 11 it follows that there are at most two non-disclosure intervals.
References


