Voluntary Disclosure with Evolving News

Cyrus Aghamolla* Byeong-Je An†

February 18, 2018

We study a dynamic voluntary disclosure setting where the manager's information and the firm's value evolve over time. The manager is not limited in her disclosure opportunities but disclosure is costly. The results show (perhaps surprisingly) that the manager discloses even if this leads to a price *decrease* in the current period. The manager absorbs this price drop in order to increase her option value of withholding disclosure in the future. That is, by disclosing today she can improve her continuation value. Further, the results show that firms who disclose more frequently are more likely to be met with a negative market reaction. We extend the model to a continuous-time setting and find that the relative length of delay between disclosures is a salient factor in identifying the type of price movement following disclosure.

1 Introduction

A firm's informational environment is generally characterized by continuous inflows of new information. For example, advances made through research and development could lead to patents and eventual product launches. Similarly, the firm's direction or strategy may change based on current or projected industry conditions. Accordingly, the process of price discovery for the firm generally involves voluntary information disclosures by firm executives regarding the firm's present situation.

The purpose of this paper is to investigate disclosure behavior by a firm manager when the firm's value evolves over time. Our setting is one where the manager privately observes the firm's fundamental value in each of two periods. The firm value is allowed to change over time, and the manager may choose to disclose, at a cost (such as a proprietary cost), her

*University of Minnesota, Carlson School of Management. E-mail: caghamol@umn.edu.
†Nanyang Technological University, Nanyang Business School. E-mail: bjan@ntu.edu.sg.

private information of the firm's value in any present moment. In equilibrium, we find that the manager may disclose her private information even when this leads to a price *decrease* following the disclosure. The manager endures this price drop for the purpose of increasing her continuation value.

To the best of our knowledge, the extant theoretical literature has not captured this kind of disclosure behavior without an additional assumption concerning litigation risk.[1] However, voluntary disclosures which lead to price decreases are pervasive in practice. Indeed, a sizable empirical literature has found that managers typically disclose bad news more often than good news.[2]

We find an endogenous explanation for this anomalous yet enduring empirical regularity. We assume that the firm manager is not limited in her disclosure opportunities. The firm value evolves according to a simple process and the market updates their beliefs on the current firm value based on the past history of dividends and disclosures by the manager (as well as the manager's disclosure strategy). Our main result shows that first-period disclosure by the manager whose value is at the disclosure threshold *always results in a price decrease* (Theorem 1).

A key novelty in the analysis is that, because firm value evolves over time, the manager can influence *tomorrow's* beliefs by disclosing *today*. More specifically, absent disclosure in the first period, the market must determine the second-period threshold using its information set, which includes, at that point, the public news and the manager's disclosure strategy in each period. Interestingly, we find that an increase in the disclosure threshold in the first period increases the second-period disclosure threshold *by a rate less than one*. This equilibrium property implies that there is an additional endogenous downside to withholding disclosure in the first period—as the first-period disclosure threshold does not fully "transfer" to the second period.

Second, we find that, for the threshold-type manager, the second-period disclosure threshold is *always less* if the manager had concealed information in the first period then if she had disclosed information. This implies that the threshold-type manager's non-disclosure price in the second period is strictly higher if she had disclosed her private information in the first period. The reason is that, upon non-disclosure in the first period, the market updates

[1] The empirical evidence on the litigation risk of withholding disclosure has been mixed. Specifically, Francis et al. (1994) and Field et al. (2005) find no evidence of a relation between disclosure and litigation likelihood, while Rogers and Van Buskirk (2009) show that, among firms that had been already subject to class action lawsuits, the firms provided less disclosure after the lawsuit.

[2] For example, see Skinner (1994), Soffer et al. (2000), Matsumoto (2002), Baik and Jiang (2006), Anilowski et al. (2007), and Kross et al. (2011), among others.

its beliefs regarding the evolved second-period value using the conditional expectation for *the set of all non-disclosing types*. The market thus determines the *average* evolved firm value, which results in a strictly lower second-period disclosure threshold level. Hence, by disclosing in the present period, the manager can positively influence the market's belief in the following period by raising that period's disclosure threshold. In other words, disclosure in the present period increases the option value of withholding disclosure in the *following* period.

We note that the first property mentioned above concerns the benefit from withholding disclosure, which includes saving disclosure costs and, more importantly, the possibility that the realized cash flows may overstate firm profitability. The second property concerns the link between present-period disclosures and future-period beliefs. The manager thus faces two competing incentives, each of which resembles an American put option. The observed dividends encourage the manager to withhold disclosure, while the evolving nature of the firm's value induces disclosure. As we show, the evolving nature of the firm leads the option value generated from disclosure to dominate and induces *excessive* disclosure by the manager in the first period. Consequently, the manager is inclined to disclose even if this hurts first-period price, and indeed we find that disclosure always results in a lower first-period price for the threshold-type manager.

We note that the economic forces driving the main result are in contrast to the extant dynamic voluntary disclosure models. Previous models of dynamic disclosure generally involve a manager who can generate a real option from *concealing* information in the present period (e.g., Acharya et al. (2011), Guttman et al. (2014)). These models are dynamic but entail a constant firm value. In contrast, in our setting we find that the manager can improve his option value of disclosure in the future by *revealing* information in the current period. Hence, we find that allowing firm value to change over time leads to significantly different disclosure incentives and behavior. We note that this improved option value from early disclosure prevails even when the manager has a countervailing incentive to withhold information, such as in the form of exogenous positive news which may overstate the firm's value (as in Acharya et al. (2011)).

In further analysis, we extend the discrete-time model to a continuous-time, infinite-horizon setting in order to examine the endogenous length of delay between disclosures. We first show that our main result is preserved in this richer setting. Moreover, the results show that the relative *timing* of when the manager discloses is indicative of the market reaction which ensues. More specifically, disclosures which are made with less delay (i.e., more quickly) since the last disclosure are more likely to result in a price drop following the release of information. Hence, the results identify a salient feature—the amount of delay or time between disclosures—as an important determinant of the market reaction. This is perhaps surprising, as we would not expect that a manager who is more transparent, in the sense of disclosing more often, to be "punished" by the market.

The model provides additional implications which have not been found in previous studies. The results show how positive skewness can arise following joint releases of disclosure and public (news) announcements, which is in dissonance to previous voluntary disclosure models. Interestingly, endogenizing the length of delay upends the implications of Acharya et al. (2011), who find negative skewness when public news announcements are followed by disclosure. In contrast, our model implies that returns can exhibit positive skewness when public news and disclosures are announced in tandem. This occurs since the manager begins disclosure when the belief difference between the fundamental value and the belief difference is sufficiently high. When the public signal improves, this implies that the underlying fundamental is also improving. However, the fundamental may improve in a greater magnitude than the public signal, thus crossing the belief difference threshold and compelling the manager to disclose. This is not possible in Acharya et al. (2011) as the manager always preempts good news announcements in their setting.

The model provides several avenues of future research through novel empirical predictions. Specifically, the model provides predictions concerning disclosure frequency as related to firm properties. The results of the model imply that firms disclose more frequently when: (*i*) there is greater information asymmetry between the firm and the market; (*ii*) the firm's cash flows have relatively higher autocorrelation; (*iii*) there is *less* uncertainty regarding the firm's future value; and (*iv*) the firm has relatively high disclosure costs. These predictions, as well as others, are discussed more thoroughly in Section 3.4.

1.1 Related Literature

Grossman (1981) and Milgrom (1981) first studied static voluntary disclosure and showed that, in the absence of disclosure costs, the agent always reveals her private information in equilibrium.[3] Jovanovic (1982) and Verrecchia (1983) extend this result by examining

[3]This is commonly referred to as the "unraveling result." Grossman (1981) and Milgrom (1981) show that, if disclosure is costless, then another friction, such as lack of common knowledge that the agent received information, must be present in order to prevent unraveling. This latter friction was first explored by Dye (1985) and Jung and Kwon (1988). Voluntary disclosure models typically include either disclosure costs or uncertainty regarding the agent's information endowment to prevent unraveling.

a static disclosure setting where information release is costly. We build from these studies and incorporate disclosure costs as the basic friction which prevents unraveling. We note, however, that this is not the primary economic force which drives our main result.

Our model is related to the literature on dynamic voluntary disclosure. Einhorn and Ziv (2008) and Marinovic and Varas (2016) also consider settings in which the firm value evolves over time. Einhorn and Ziv (2008) examine a repeated game in which disclosures made in the present affect the market's perception that a future-period manager has received material information. Importantly, Einhorn and Ziv (2008) assume that the manager is purely myopic (or short-lived) in the sense that she only seeks to maximize the firm's price in the current period. In contrast, we assume the manager prefers to maximize both short and long-term prices (though we analyze the purely myopic case to establish a benchmark result).

Marinovic and Varas (2016) investigate a continuous-time, binary disclosure model where the firm's value fluctuates according to a Markov process. They assume that the firm faces a risk of litigation when bad news is withheld, and thus *not* disclosing is costly. The model here differs from Marinovic and Varas (2016) primarily in that litigation risk is a fundamental feature of their setting. In contrast, we investigate dynamic disclosure without imposing an exogenous cost of withholding disclosure.

Our setting is also related to a stream of literature in dynamic disclosure where the manager may choose the timing of her disclosure, but the underlying value of the firm does not change. Acharya, DeMarzo, and Kremer (2011, hereafter ADK) investigate a model where an exogenous correlated signal is publicly revealed at a known time. Their results show clustering of announcements in bad times, where the manager discloses immediately if the public signal is sufficiently low. Relatedly, Guttman et al. (2014) consider a two-period model where the manager may receive two independent signals of the firm value in each period. They show that the market value of the firm is higher if one signal is disclosed in the second period rather than if one signal is disclosed in the first period. The main difference in our setting and Acharya et al. (2011) and Guttman et al. (2014) is that we assume that firm value changes over time. Moreover, a driving force in both Acharya et al. (2011) and Guttman et al. (2014) is that the manager can improve his option value by concealing information, whereas we find the opposite force.

Shin (2003, 2006) considers disclosure in a binomial setting where projects may either succeed or fail. The equilibrium constructed is one where the manager follows a "sanitation strategy" where only project successes are disclosed in the interim period. In a similar vein, Goto et al. (2008) extend Shin's (2003) framework to include risk-averse investors. The present setting varies from Shin (2003, 2006) and Goto et al. (2008) in that we are concerned with the timing of disclosures, and characterize the emergence of bad news releases.

Lastly, in our continuous-time framework, we build from the technical methods developed by Scheinkman and Xiong (2003), who investigate a continuous-time, complete information trading model between agents with heterogeneous beliefs. As a methodological contribution, our continuous-time results extend this analysis to a setting with incomplete information (although we do not assume heterogeneous beliefs).

2 Discrete-time Model of Dynamic Disclosure

Our baseline setting is a discrete, two-period model. This parsimonious setting captures the main insight and clearly illustrates the economic forces driving the results. We then extend the discrete setting to a continuous-time framework which allows us to investigate the endogenous length of delay between disclosures.

The firm generates a cash flow s_t in each period ($t = 0, 1$). We assume that a risk-neutral firm manager privately observes the firm's mean cash flow y_0 in time 0, and that (s_0, y_0) is a bivariate normal variable with zero mean and correlation ρ.[4] Specifically, we assume that $\sigma_s = \sigma_y/\rho$, where σ_s and σ_y are volatility parameters of s_0 and y_0, respectively.[5] Thus, conditional on y_0, the cash flow s_0 is given by

$$s_0 = y_0 + w_0,$$

where w_0 is normally distributed with mean zero and variance $(1 - \rho^2)\sigma_s^2$.[6] This may be interpreted such that y_0 is the profitability of the underlying fundamental and w_0 is an industry or macroeconomic shock to cash flows.

Upon learning y_0, the manager may disclose the information to the market, in which case it becomes public information. We assume that disclosure is verifiable in the sense that the manager cannot manipulate the disclosed value. Disclosure is also assumed to be costly for the firm, denoted by a cost $c > 0$. The disclosure cost can be interpreted, for instance, as a certification cost, whereby the manager must hire an auditor to certify

[4]The zero-mean assumption on (s_0, y_0) is without loss of generality.

[5]The results of the model are not qualitatively affected if $\sigma_s \neq \sigma_y/\rho$. We assume this for ease of exposition so that the mean of s_0 can simply be represented by y_0. We later relax this assumption when conducting comparative statics analysis.

[6]Including noise in the cash flow prevents the market from filtering out the mean cash flow perfectly upon observing cash flow in the event that the manager does not disclose.

that the information disclosed is factual. Alternatively, the disclosure may be relevant to proprietary information that could be adopted by competitor firms. Indeed, a wide-scale survey of executives at large public firms finds evidence consistent with this view: "Nearly three-fifths of survey respondents agree or strongly agree that giving away company secrets is an important barrier to more voluntary disclosure" (Graham et al. (2005, p. 62)).[7]

After the manager makes her disclosure decision at time 0, the market, composed of risk-neutral investors, determines the date 0 price of the firm. Then, s_0 is realized and the cash flow net of the disclosure cost (if the manager had disclosed) is distributed to shareholders.

We allow the mean of cash flows to evolve in the sense that new developments may have occurred between time 0 and time 1 such that the underlying firm profitability has improved or declined. This is captured by the time 1 mean cash flow, given by:

$$y_1 = \kappa y_0 + \eta,$$

where $\kappa \in (0, 1]$ denotes autocorrelation of the mean cash flow, and η is a normal variable with mean zero and variance σ_η^2. We assume that η and (s_0, y_0) are independent. Regardless of the time 0 disclosure decision, the manager privately observes y_1. The distribution of η is common knowledge. We assume that the second-period cash flow s_1 is simply given by $s_1 = y_1$.[8] At time 1, after observing y_1 the manager may disclose y_1 to the market. The market then determines the time 1 price of the firm after observing the manager's disclosure decisions at time 0 and at time 1, and the cash flow in the first period. A timeline of model is presented in Figure 1.

The cum dividend price in each period satisfies:

$$\begin{aligned} p_0 &= E[s_0 - cd_0 + s_1 - cd_1 | \Omega_0] \\ p_1 &= E[s_1 - cd_1 | \Omega_1], \end{aligned}$$

where d_t is an indicator equal to one if the manager discloses in time t and zero otherwise. Ω_t denotes the market's information set at time t; Ω_0 includes d_0 and the manager's disclosure strategy, and Ω_1 includes s_0, d_0, d_1, and the manager's disclosure strategy.

The manager is risk neutral and thus her objective is to maximize the sum of current

[7]Empirical evidence of proprietary costs has been documented by Berger and Hann (2007), Bens et al. (2011), and Ellis et al. (2012). Other costs of disclosure—arranging press releases, conference calls, and meetings with analysts—are nontrivial and impose time costs on the manager and monetary costs on the firm.

[8]Allowing (s_1, y_1) to be bivariate normal would not qualitatively affect the results.

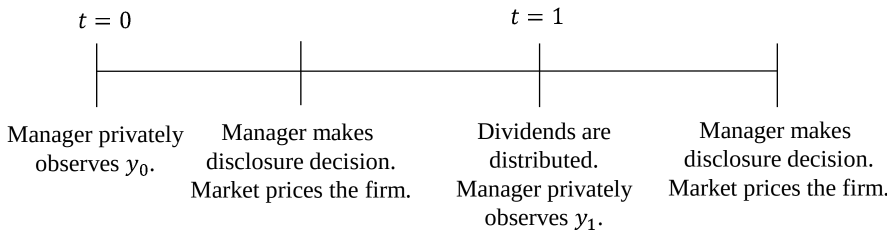


Figure 1: Timeline of the discrete model.

market price and expected market price:

$$\max_{d_0, d_1} p_0 + E[p_1|y_0].$$

The manager is concerned with the market price at all times as it is often the case that an executive's compensation includes bonuses which are determined in part by share price.[9] For simplicity, we assume that there is not discounting by the manager or the market. We note that our results are not qualitatively affected if we incorporate discounting or a scale parameter on the price in the manager's utility, i.e., $\lambda p_0 + (1 - \lambda)p_1$, for $\lambda \in (0, 1)$.

3 Equilibrium

In this section, we characterize the equilibrium of our baseline setting. Before we begin the analysis of the dynamic model, we first analyze the myopic benchmark, which will be helpful in the ensuing analysis.

3.1 Myopic benchmark

In this special case, we assume that the manager is myopic and simply aims to maximize the price of the current period. This is a variant of the static costly disclosure model studied by Jovanovic (1982) and Verrechia (1983). The main difference is that the non-myopic market must still take into account the expected cash flow of the second period when they price the firm in the first period. This setting provides a point of comparison with the fully dynamic main model and also allows us to more precisely convey how evolving news affects the non-myopic manager's disclosure strategy.

[9] A similar assumption regarding the manager's utility function is made in previous dynamic voluntary disclosure models, such as Acharya et al. (2011) and Guttman et al. (2014).

Since the game ends after the second period, the manager's disclosure strategy in the second period is identical in both the myopic or non-myopic settings. Thus, in this benchmark case we focus on the manager's disclosure strategy in the first period.

We define the function $v(x) \equiv x + \delta(x)$, where $\delta(x)$ can be thought as a "non-disclosure penalty" and is given by

$$\delta(x) = E[\xi|\xi < x] = \phi(x)\Phi(x)^{-1}, \tag{1}$$

where ξ is a standard normal variable, and where $\phi(\cdot)$ and $\Phi(\cdot)$ is the density function and distribution function of the standard normal distribution, respectively. The function $v(x)$ can be thought of as the difference between the true type x and the market price following nondisclosure by the manager. We adopt this notation in order to disentangle these two components (specifically, to isolate the penalty $\delta(x)$ which is a salient feature of the continuous-time setting), as well as to facilitate the analysis in the current section.

We let x^* denote the equilibrium myopic disclosure threshold in the first period, defined whereby the manager discloses if and only if $y_0 \geq x^*$. If the threshold-type manager (i.e., $y_0 = x^*$) discloses at time 0, then the time 0 price $p_0^d(x^*)$ is given by

$$p_0^d(x^*) = E[s_0 - c + s_1 - cd_1|\Omega_0^d] = (1+\kappa)x^* - c(1+\alpha_d), \tag{2}$$

where Ω_0^d is the information available to the market when the manager discloses, and $\alpha_d = E[d_1|\Omega_0^d]$ is the probability of disclosure at time 1 given disclosure at time 0. In the next section, we show that this probability is independent of the myopic threshold x^*. On the other hand, if the disclosure-type manager does not disclose at time 0, the time 0 price is given by

$$p_0^n(x^*) = E[s_0 + s_1 - cd_1|\Omega_0^n] = (1+\kappa)E[y_0|y_0 < x^*] - c\alpha_n(x^*), \tag{3}$$

where Ω_0^n is the information available to the market when the manager does not disclose, and $\alpha_n(x^*) = E[d_1|\Omega_0^n]$ is the probability of disclosure at time 1 given nondisclosure at time 0. In the next section, we show that this probability depends on the myopic threshold. Since the myopic manager is indifferent between disclosure and nondisclosure at x^*, we see that x^* is given by the following condition:

$$\begin{aligned} c(1+\alpha_d) &= (1+\kappa)\{x^* - E[y_0|y_0 < x^*]\} + c\alpha_n(x^*) \\ &= (1+\kappa)\sigma_y v\left(\frac{x^*}{\sigma_y}\right) + c\alpha_n(x^*) \end{aligned}$$

The left-hand side is the expected total disclosure cost when the manager discloses at time 0.

The right-hand side is the size of undervaluation plus the expected disclosure cost at time 1. The myopic disclosure threshold x^* provides a useful benchmark which is frequently used for comparison and in the analysis of the dynamic case. The following proposition establishes existence and uniqueness of this threshold:

Proposition 1 *There exists a unique static disclosure threshold x^* such that the manager discloses if and only if $y_0 \geq x^*$.*

In the Appendix, we also show that $v(x)$ is nonnegative and increasing in x, which implies that the penalty $\delta(x)$ is decreasing in x. This property will be helpful in the following analysis.

3.2 Second-Period Disclosure

We now turn to our main setting where the manager considers both period's prices in the first period. In solving the equilibrium strategy for the dynamic setting, we begin with the manager's decision at time 1 after she has learned y_1. There are two possible paths the manager could have taken prior to time 1: disclosure or nondisclosure in time 0. Below, we analyze each case separately.

Suppose that the time 0 disclosure decision can be characterized by some threshold x_0, such that the manager discloses her private information only if $y_0 \geq x_0$. For now, we keep the time 0 disclosure threshold exogenous and fixed as we analyze the second-period disclosure decision (we endogenize the time 0 decision in the following section). At date 1, the manager will choose to disclose her private information if and only if the expected cash flow at date 1 exceeds the market price absent disclosure plus the disclosure cost.

Time 1 disclosure decision when $d_0 = 1$

First, we consider the case where the manager had disclosed her private information at time 0, i.e., $d_0 = 1$. The manager will also disclose at time 1 if her payoff from disclosure exceeds that from remaining quiet:

$$y_1 - c > E[y_1|\Omega_1^d],$$

where $\Omega_1^d = \{y_0, y_1 < x_d(y_0)\}$ is the information available to the market when the manager had disclosed and she is not disclosing currently, and $x_d(y_0)$ denotes the disclosure threshold at date 1 given that the disclosed value at date 0 is y_0. In the case where the manager had previously disclosed the mean cash flow at time 0, the realization of cash flow s_0 does not deliver additional information to the market that is relevant to y_1. The equilibrium threshold satisfies:

$$x_d(y_0) = c + E[y_1|y_0, y_1 < x_d(y_0)] = \kappa y_0 + \eta^*,$$

where η^* solves

$$c = \eta^* - E[\eta|\eta < \eta^*] = \sigma_\eta v\left(\frac{\eta^*}{\sigma_\eta}\right), \tag{4}$$

and $v(\cdot)$ is defined as in the previous section. The existence and uniqueness of η^* can be shown similarly as in Proposition 1. Based on this threshold, we have that the ex ante likelihood of disclosure at time 1 given that there was disclosure in time 0 is given by $\alpha_d = \Phi\left(-\frac{\eta^*}{\sigma_\eta}\right)$.

Proposition 2 *There exists a unique equilibrium disclosure threshold satisfying equation (4).*

The threshold $x_d(y_0)$ has an intuitive interpretation—when the realized η is sufficiently high, this pushes the new firm value to be above $x_d(y_0)$ and induces disclosure by the manager. Moreover, the disclosure of y_0 in the first period can raise the option value of disclosure in the second period, as the disclosure threshold $x_d(y_0)$ is increasing in y_0. Hence, when the manager discloses a high y_0 in the first period, she has positively influenced the market's belief of y_1 through her disclosure, which carries through as a comparatively higher valuation in the absence of disclosure in the second period. In this sense, early disclosure of positive news in the first period can *increase* the option value of disclosure in the second period. We note that this is a key distinction between the unchanging environment of ADK, as early disclosure in their setting eliminates the option value. As we will see in the following section, this property becomes a salient factor that influences the time 0 disclosure decision.

Time 1 disclosure decision when $d_0 = 0$

We now consider the case where the manager did not disclose at date 0, i.e., $d_0 = 0$. In this case, the manager will disclose at date 1 if and only if

$$y_1 - c > E[y_1|\Omega_1^n], \tag{5}$$

where $\Omega_1^n = \{s_0, y_0 < x_0, y_1 < x_n(x_0, s_0)\}$ is the information available to the market when the manager is not disclosing in both periods, and $x_n(x_0, s_0)$ denotes the disclosure threshold at date 1 given nondisclosure, realized cash flows s_0, and disclosure threshold x_0 at date 0.

Since the manager did not disclose in time 0, the market does not observe y_0. However, the distribution of dividends (which is equal to cash flows s_0) by the firm provides investors with information regarding y_0. As we will see, this signal gives the manager a potential benefit from withholding disclosure in the first period. For example, a positive industry or macroeconomic shock w_0 to cash flows may lead investors to overstate the value of y_0 after observing dividends s_0. Consequently, this may result in a more generous price in the second period absent disclosure through inflated market beliefs of y_1. Hence, this effectively provides the manager with a real option of withholding disclosure in the first period.

From equation (5), we find that the equilibrium threshold satisfies:

$$x_n(x_0, s_0) = \kappa f s_0 + \epsilon^*(g),$$

where $f = \rho\sigma_y/\sigma_s$, $g = x_0 - fs_0$, and $\epsilon^*(g)$ solves

$$c = \epsilon^*(g) - E[\kappa z + \eta | z < g, \kappa z + \eta < \epsilon^*(g)]. \tag{6}$$

Upon observing the first-period cash flows, the market believes that $y_0 = fs_0 + z$, where z is normally distributed with mean zero and variance $\sigma_z^2 = (1-\rho^2)\sigma_y^2$. The information that the manager had not previously disclosed implies that the random variable z is truncated above at $g = x_0 - fs_0$. Thus, $\epsilon^*(g)$ is the mean-adjusted disclosure threshold for the manager.

We see that $x_n(x_0, s_0)$ depends on the realization of cash flows s_0, as well as the manager's time 1 private information, captured by the term $\epsilon^*(g)$. When the cash flow s_0 is high, this raises the disclosure threshold $x_n(x_0, s_0)$. This is intuitive as a high s_0 implies that y_0 and thus y_1 is high. However, a large s_0 also reduces the gap between the first-period threshold and the posterior belief upon observing the cash flow $g = x_0 - fs_0$. This has the additional effect that a higher η is then necessary to induce disclosure by the manager. To see this, note that $g < 0$ implies that $z < 0$, and so η must be sufficiently large to induce $\kappa z + \eta > \epsilon^*(g)$. The following result establishes existence and uniqueness of $\epsilon^*(g)$:

Proposition 3 *There exists a unique fixed point satisfying (6).*

Interestingly, we find that the effect of high cash flows is somewhat mitigated by the fact that the manager did not disclose in the first period. Specifically, even though a high s_0 has a direct effect on $x_n(x_0, s_0)$, it also has an indirect effect through $\epsilon^*(g)$. Intuitively, investors must take into consideration the fact that the manager did not disclose in the first period, and consequently must account for the value of the threshold level of disclosure at time 0, x_0. This implies that, even if period-one cash flows are very high, it is still the case that the manager's information at time 0 was not sufficiently positive to induce disclosure. This is captured by the gap $g = x_0 - fs_0$, which affects $\epsilon^*(g)$. The following proposition provides an important property that is helpful in interpreting the disparate effects of x_0 and s_0:

Lemma 1 *$\epsilon^*(g)$ is increasing in g at a rate less than κ, i.e,*

$$0 < \frac{d\epsilon^*(g)}{dg} < \kappa.$$

Lemma 1 states that $\epsilon^*(g)$ is increasing in g, which implies that $\epsilon^*(g)$ is increasing in x_0 and decreasing in s_0. Consequently, the disclosure threshold $x_n(x_0, s_0)$ is also increasing in the time 0 threshold x_0. This property is straightforward, as less disclosure (higher x_0) at time 0 means that the expected value of a nondisclosing firm in time 1 must also be higher, since y_0 and y_1 are correlated.

However, what is striking is that $\frac{d\epsilon^*(g)}{dg} < \kappa$, which indicates that an increase in x_0 by one results in an increase of $x_n(x_0, s_0)$ by *less* than the autocorrelation (and, hence, by less than one). This implies that the nondisclosure threshold in the first period does not fully "carry over" to the second period. This feature is a significant driving force of the main result that we will see in the following section. To see this intuitively, first note that the manager's primary benefit of withholding disclosure in period one is to save disclosure costs and to take advantage of the possibility that realized cash flow s_0 may be sufficiently favorable such that second-period beliefs overstate the true value y_1. Recall that when s_0 is observed through dividends, this provides information to the market regarding y_0 and thus y_1. The market thus determines it's beliefs regarding y_0 in the first stage of $t = 1$ taking into account dividends s_0 and the manager's strategy x_0.

An increase in the threshold type x_0 overall improves the market's beliefs in the second period, but also increases the set of first-period non-disclosing firms. This latter effect puts an additional disadvantage on the first-period threshold-type x_0. More specifically, the pre-dividend conditional expectation $E(y_0|y_0 < x_0)$ does not increase in line with increases in the threshold x_0. This implies that the threshold-type x_0 becomes relatively more undervalued by the market as x_0 increases. Hence, in determining their beliefs in the second period after observing s_0, the market must take into account the *average* non-disclosing type $E(y_0|y_0 < x_0)$. In this sense, the relatively larger set of first-period non-disclosing firms (or the average $E(y_0|y_0 < x_0)$) "weighs down" even a very favorable dividend signal s_0. Hence, it is comparatively less likely that the threshold-type can take advantage of the observed dividend s_0, even for high values of s_0 (compared to non-disclosing types below x_0). Consequently, the manager with the threshold-type x_0 is relatively more inclined to disclose in the second period as she is unlikely to realize the benefits from an over-stated first-period cash flow s_0.

This leads the second-period threshold $x_n(x_0, s_0)$ to not increase in line with increases in the first-period threshold x_0. In other words, the observed cash flow s_0 becomes less relevant to the non-disclosing manager as x_0 increases. Hence, there is some limitation to the benefits of nondisclosure in the first period, as the threshold level does not fully carry over to the second period.

The effect of the cash flow s_0 on $x_n(x_0, s_0)$ has an analogous effect. As mentioned previously, high cash flows can positively influence the market's belief, but the upside of a high s_0 is limited as a sufficiently high-type firm would have disclosed at time 0. Hence, $\epsilon^*(g)$ is decreasing in s_0, which serves to mitigate the effect of s_0 on the threshold $x_n(x_0, s_0)$. However, the net effect of an increase in s_0 always results in an increase in $x_n(x_0, s_0)$. This can be seen from the property $-\kappa f < \frac{\partial \epsilon^*(g)}{\partial s_0} = -f \frac{d\epsilon^*(g)}{dg} < 0$, which implies that when s_0 increases by one, $x_n(x_0, s_0)$ increases by less than κf. Hence, a high first-period cash flow is always beneficial, but this benefit is also somewhat mitigated by the manager's nondisclosure in the first period.

So far, we have shown two equilibrium disclosure thresholds, $x_n(x_0, s_0)$ and $x_d(x_0)$, which depend on the path that the manager followed to reach time 1. We now present an important equilibrium property which describes the difference in the manager's behavior at time 1 depending on the disclosure history.

Lemma 2 *The threshold-type manager ($y_0 = x_0$) will begin to disclose at a lower value of y_1 in the second period if she had not disclosed at time zero than if she had disclosed, i.e., $x_n(x_0, s_0) < x_d(x_0) \equiv \kappa x_0 + \eta^*$. Moreover, we have that (i) $\epsilon^*(g) - \kappa g \to \eta^*$ and $\frac{d\epsilon^*(g)}{dg} \to \kappa$, as $g \to -\infty$, and (ii) $\epsilon^*(g) \to \bar{\epsilon}$ and $\frac{d\epsilon^*(g)}{dg} \to 0$, as $g \to \infty$, where $\bar{\epsilon}$ is defined in Appendix.*

Lemma 2 indicates that, upon non-disclosure in $t = 0$, the manager *always* begins disclosure at a lower realization of y_1 than if she had disclosed in $t = 0$. This implies that the threshold-type manager's second-period price upon non-disclosure is always *lower* if she had kept quiet in the first period rather than if she had disclosed y_0. In other words, by disclosing in period 1, the threshold-type manager can actually raise her non-disclosure price, and thus her option of keeping quiet, in the second period. Intuitively, this occurs due to the evolving nature of the firm value. To see this more clearly, consider the case where firm value is independent in each period. In this case, past disclosures are irrelevant for the future price, and the manager in $t = 0$ must only weigh the disclosure cost c and the present period's nondisclosure price, e.g., $E[y_0|y_0 < x_0]$. However, when the firm value evolves based on the current value, then the manager must not only consider the present period nondisclosure price, $E[y_0|y_0 \leq x_0]$, but also the fact that her non-disclosure affects market beliefs of the *future* firm value. In this case, nondisclosure results in the market updating their beliefs of y_1 based on the fact that $y_0 \leq x_0$, i.e, the manager's disclosure strategy, and from the observed dividends s_0. In this sense, the market's belief of y_1 considers the evolution from $E[y_0|y_0 \leq x_0; s_0]$, or a value that is *ex ante less than* x_0. Hence, the market is determining the *average* evolved firm value based on its information set, which implies that the market is, in expectation, assigning an evolved value that is less than the threshold type's y_1.

Put differently, nondisclosure by the manager in the present period affects the market's belief of the future value. This is "costly" in the sense that a high-type manager may be leaving money on the table in *future* periods by not disclosing today. The manager can thus positively influence the market's future beliefs, and thus the non-disclosure price in the subsequent period, by disclosing today. In this light, the manager can *increase* his option value of nondisclosure tomorrow by not concealing information in the present period.

We next examine properties of the likelihood of disclosure in $t = 1$. Recall that $\alpha_n(x_0)$ denotes the manager's ex ante probability of disclosing in period two given that she did not disclose in period one, and α_d is the corresponding probability given that she disclosed in period one.

Lemma 3 *The ex ante likelihood of disclosure at time 1 given that there was nondisclosure in time 0 has following properties: (i)* $\alpha_n(x_0) \to \alpha_d$ *and* $\alpha_n(x_0)' < 0$ *as* $x_0 \to -\infty$*, and (ii)* $\alpha_n(x_0) > \alpha_d$ *and* $\alpha_n(x_0)' > 0$ *as* $x_0 \to \infty$*.*

Property (i) of Lemma 3 is intuitive; $x_0 \to -\infty$ implies that the manager always discloses in $t = 0$ and hence the market's belief on the likelihood of disclosure at time 1 approaches α_d. Property (ii) similarly examines the disclosure likelihood as $x_0 \to \infty$, i.e., when the manager never discloses in $t = 0$. Intuitively, two separate effects occur as x_0 increases. First, this increased set of first-period non-disclosers results in a larger set of non-disclosing period-one values y_0 that will ultimately disclose in the second period. This occurs since, as x_0 increases, a larger set of non-disclosing types are, *on average*, being under-valued in the second period. Recall from Lemma 1 that x_n does not increase in line with increases in x_0. This implies that some managers who previously had not disclosed in the first period are more inclined to disclose in the second period. As x_0 increases, we are increasing this set of managers and thus $\alpha_n(x_0)$ increases. Second, as x_0 increases, so does the gap between the market belief of the non-disclosing manager, $E(x|x \leq x_0; s_0)$, and the threshold type x_0. This implies that the gap between x_d and x_n also increases (recall Lemma 2), which consequently implies that $\alpha_n(x_0) > \alpha_d$. Hence, the threshold-type manager starts to disclose "earlier" if she had concealed information in the first period. The market anticipates this and thus the ex ante likelihood of disclosure at time 1 is increasing in the equilibrium disclosure threshold.[10]

The analysis in the second-period disclosure decision shows that the manager must weigh two different real options. The first stems from the fact that the profitability changes over time—by disclosing today, the manager can increase the disclosure threshold, and thus her option value, in the second period. This option enhances the incentive for disclosure in the first period. The second real option arises from the noisy cash flow s_0. The manager can keep quiet in the first period in order to take advantage of a potentially high cash flow. Conversely, this option strengthens the incentive for nondisclosure in the first period. These countervailing forces are salient in the analysis of the time 0 disclosure decision which we examine next.

3.3 First Period Disclosure

We now analyze the manager's time 0 disclosure decision. If the threshold-type manager ($y_0 = x_0$) discloses at time 0 ($d_0 = 1$), the price $p_0^d(x_0)$ in that period is given by equation (2). At date 1, depending on the new mean cash flow, the payoff to the manager is equal to either $y_1 - c$ if $y_1 > x_d(y_0)$ or $x_d(y_0) - c$ if $y_1 \leq x_d(y_0)$. Thus, the expected utility of the threshold-type manager upon initial disclosure is given by:

$$p_0^d(x_0) + E[y_1 - c + (x_d(y_0) - y_1)^+ | y_0 = x_0] = p_0^d(x_0) + \kappa x_0 - c + u_d. \tag{7}$$

The first term in the left-hand side equation (7) is the manager's first-period payoff from disclosure, which is simply the time 0 market price. The second term is the manager's expected second-period payoff, which includes the option value of disclosure, given by:

$$u_d = E[(\eta^* - \eta)^+]. \tag{8}$$

[10] In terms of the derivation, as the manager withholds disclosure for all realizations of y_0, in the first stage of the second period the market believes that y_1 is normally distributed with mean $\kappa f s_0$ and variance $\sigma_\epsilon^2 = \kappa^2 \sigma_z^2 + \sigma_\eta^2$ (see Appendix). Hence, the mean-adjusted disclosure threshold $\epsilon^*(g)$ approaches the limit threshold $\bar{\epsilon}$ (defined in the Appendix). This leads to the property that $-\bar{\epsilon}/\sigma_\epsilon > -\eta^*/\sigma_\eta$, which implies that $\alpha_n(x_0) > \alpha_d$.

Observe that equation (8) is similar to that of an American put option, where the manager can exercise the option to disclose when the realization of η exceeds η^*. Or, equivalently, the manager exercises the option to hide information when the realization of η is lower than the threshold.

Conversely, if the threshold-type manager does not disclose at time 0, the market price $p_0^n(x_0)$ in that period is given by equation (3). At time 1, the market price is either $y_1 - c$ from disclosure or $x_n(x_0, s_0) - c$ from nondisclosure. Thus, the expected utility of the manager upon nondisclosure in the first period is given by:

$$p_0^n(x_0) + E[y_1 - c + (x_n(x_0, s_0) - y_1)^+ | y_0 = x_0] = p_0^n(x_0) + \kappa x_0 - c + u_n(x_0),$$

where the option value upon nondisclosure in the first period, denoted by $u_n(x_0)$, is given as:

$$u_n(x_0) = E[(x_n(x_0, s_0) - \kappa x_0 - \eta)^+]. \tag{9}$$

Similar to equation (8), the above equation also resembles an American put option, where the manager exercises the disclosure option when the realization of η exceeds the threshold $x_n(x_0, s_0) - \kappa x_0$. The difference between the put option we have developed in equation (9) and the classic put option model is that the equivalent of the strike price in our put option is itself a random variable. Thus, we can clearly see that the manager does not disclose initially in hopes of taking advantage of either a high realization of cash flow s_0, which increases the strike price, or a low realization of η, which decreases the mean cash flow. The equilibrium first-period disclosure threshold thus satisfies:

$$p_0^d = p_0^n(x_0) + u_n(x_0) - u_d. \tag{10}$$

We have two possible cases:

- Case 1: $p_0^n(x_0) < p_0^d(x_0)$. In this case, the market price upon disclosure at the first-period disclosure threshold is higher than the non-disclosure market price. In order for this to be the case, the value of the put option upon non-disclosure in time 0 is higher than the value of the put option upon disclosure, i.e., $u_n(x_0) > u_d$. Hence, the option value of delay in the first period is sufficiently high such that the manager withholds disclosures comparatively more often in the first period. As a result, the price *increases* upon disclosure, as the manager bears additional undervaluation due to the put option from non-disclosure in time 0. This is similar to the excessive delay result presented in Proposition 4 of ADK.

- Case 2: $p_0^n(x_0) > p_0^d(x_0)$. Here, the market price upon disclosure is below the non-disclosure market price in the first period. This occurs when the value of the put option upon non-disclosure is *lower* than the value of the put option upon initial disclosure, i.e., when $u_n(x_0) < u_d$. Hence, by disclosing at time 0, the manager can *increase* the option value in the second period. This follows from the analysis in Section 3.2; by disclosing in time 0, the manager can raise the threshold $x_d(y_0)$. Interestingly, in this case, the market price at time 0 *decreases* upon disclosure by the manager. This implies that the manager is disclosing excessively in time 0, and does so even in cases in which the market price drops after disclosure. In other words, to improve the option value in the second period, the manager delays less and even sacrifices a higher market price in the first period. This is in contrast to the result in ADK, as the manager's ex ante disclosure can only improve the market price in their setting.

To further investigate conditions under which Case 2 occurs, we examine the equilibrium condition (10). We find that Case 2 always occurs.

Theorem 1 *There exists a unique fixed point satisfying equation (10). Moreover, Case 2 always occurs. Also, the first-period dynamic disclosure threshold is lower than the myopic disclosure threshold:* $x_0 < x^*$.

Theorem 1 states that the price always decreases upon disclosure in the first period by the threshold-type manager. This statement has a natural interpretation. By disclosing at time 0, the manager obtains the put option u_d whose strike price is η^*. On the other hand, the threshold-type manager ($y_0 = x_0$) can obtain the potential gain from not disclosing at time 0: $u_n(x_0)$ with the strike price $x_n(x_0, s_0) - \kappa x_0$. Since the value of the put option price is increasing in its strike price, and since $x_n(x_0, s_0) < \kappa x_0 + \eta^* = x_d(x_0)$ by Lemma 2, the option value upon disclosure is *always* greater than the option value upon nondisclosure. Hence, we find that disclosure by the threshold-type always results in a *decrease* in the time 0 market price.

Notably, Theorem 1 shows that, when the firm value evolves over time, the manager discloses even though this results in a lower period 1 price. In others words, by keeping quiet at time 0, the manager's price would have been higher. Note that the evolution of the firm value is essential for this result; under the unchanging environment, the option value upon disclosure is *always* zero. Hence, we have identified the key mechanism—time-varying firm value—which *endogenously* generates excessive disclosure or, in other words, disclosure which results in a price drop.

We note also that the option value of withholding disclosure in the future is so strong that the public signal, s_0, never induces excessive disclosure in the first period under any condition. This implies that the firm's changing environment fundamentally affects disclosure decisions. Below, we discuss several empirical implications that arise from this setting.

3.4 Equilibrium Properties

Our model provides a theoretical link between the equilibrium disclosure threshold and the price jump at disclosure. In this section, we illustrate how these endogenous variables respond when an exogenous variable shifts. First, we establish the following result regarding the volatility of cash flow s_0.

Proposition 4 *The first-period disclosure threshold x_0 is independent of the volatility of actual cash flow, σ_s.*

We find that the first-period disclosure threshold does not vary in changes in the volatility of the first-period cash flow. This is perhaps counter-intuitive, as we would expect the option value from nondisclosure, $u_n(x_0)$, to be more valuable for the manager when σ_s is higher. However, an increase in σ_s also has the opposing effect whereby investors place comparatively less weight on the realization of cash flow when it conveys relatively less information about the firm's mean cash flow. We find that these two effects off-set each other and lead x_0 to be unaffected by changes in σ_s.

More precisely, at $t = 0$, from the perspective of the threshold-type manager ($y_0 = x_0$), the gap between the previous firm value and the investors' posterior belief, $g = x_0 - fs_0 = (1 - f)x_0 - fw_0$, is normally distributed with mean and variance:

$$
\begin{aligned}
E[g|y_0 = x_0] &= (1 - f)x_0, && (11)\\
Var(g|y_0 = x_0) &= \rho^2(1 - \rho^2)\sigma_y^2. && (12)
\end{aligned}
$$

Note that the variance of the gap is independent of the volatility of actual cash flow since the variance of w_0 is $(1 - \rho^2)\sigma_s^2$ and $f = \rho\sigma_y/\sigma_s$. That is, when actual cash flow is more volatile, investors place less weight on the announcement of s_0 and the manager anticipates this. This implies that the option value upon non-disclosure, $u_n(x_0)$, and thus the first-period disclosure threshold, is independent of the volatility of the first-period cash flow. We next examine the limiting behavior of the first-period threshold.

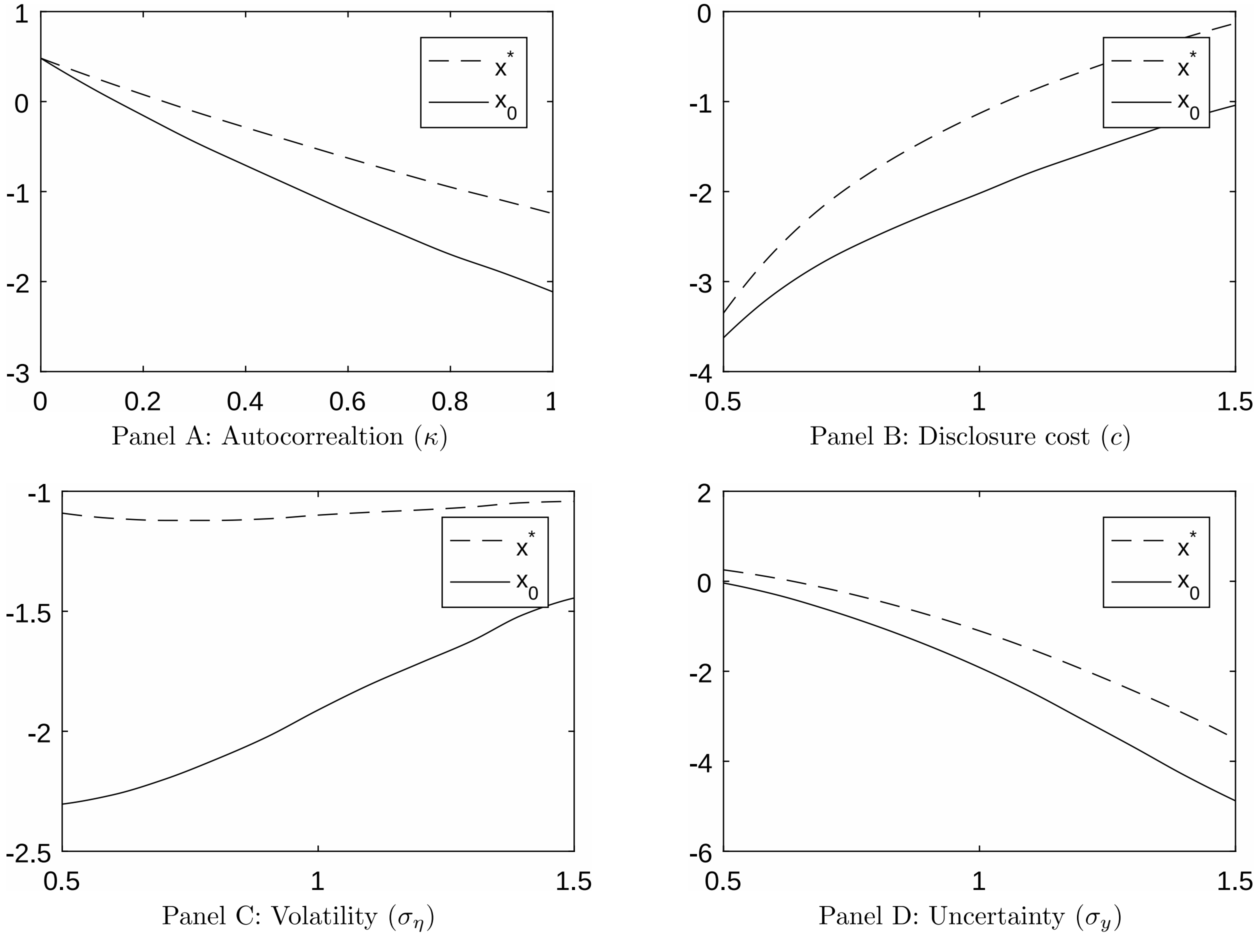


Figure 2: Effect of changes in parameters on disclosure threshold. The baseline parameters are: $\sigma_y = 1,\ \sigma_\eta = 1,\ \sigma_s = 2,\ c = 1,\ \rho = 0.5$, and $\kappa = 0.9$.

Proposition 5 *We have following limiting behavior of the first-period disclosure threshold: as* $|\rho| \to 1$, $x_0 \to x^*$*; and as* $\kappa \to 0$, $x_0 \to x^*$*.*

We see that the first-period disclosure threshold is equal to the myopic one as $|\rho| \to 1$. This occurs since the manager's option upon non-disclosure becomes less relevant for the first-period disclosure decision since investors have more precise information about y_0 as $|\rho|$ increases. Consequently, the market eventually recovers the non-disclosed mean firm value if s_0 and y_0 are perfectly correlated and thus there's no incentive to preempt excessively relative to the myopic one while incurring the disclosure cost. This implies that $x_0 = x^*$. Similarly, when the mean cash flows are independent of each other, i.e., $\kappa = 0$, the first-period disclosure decision is irrelevant for the second-period decision and hence the manager becomes myopic effectively.

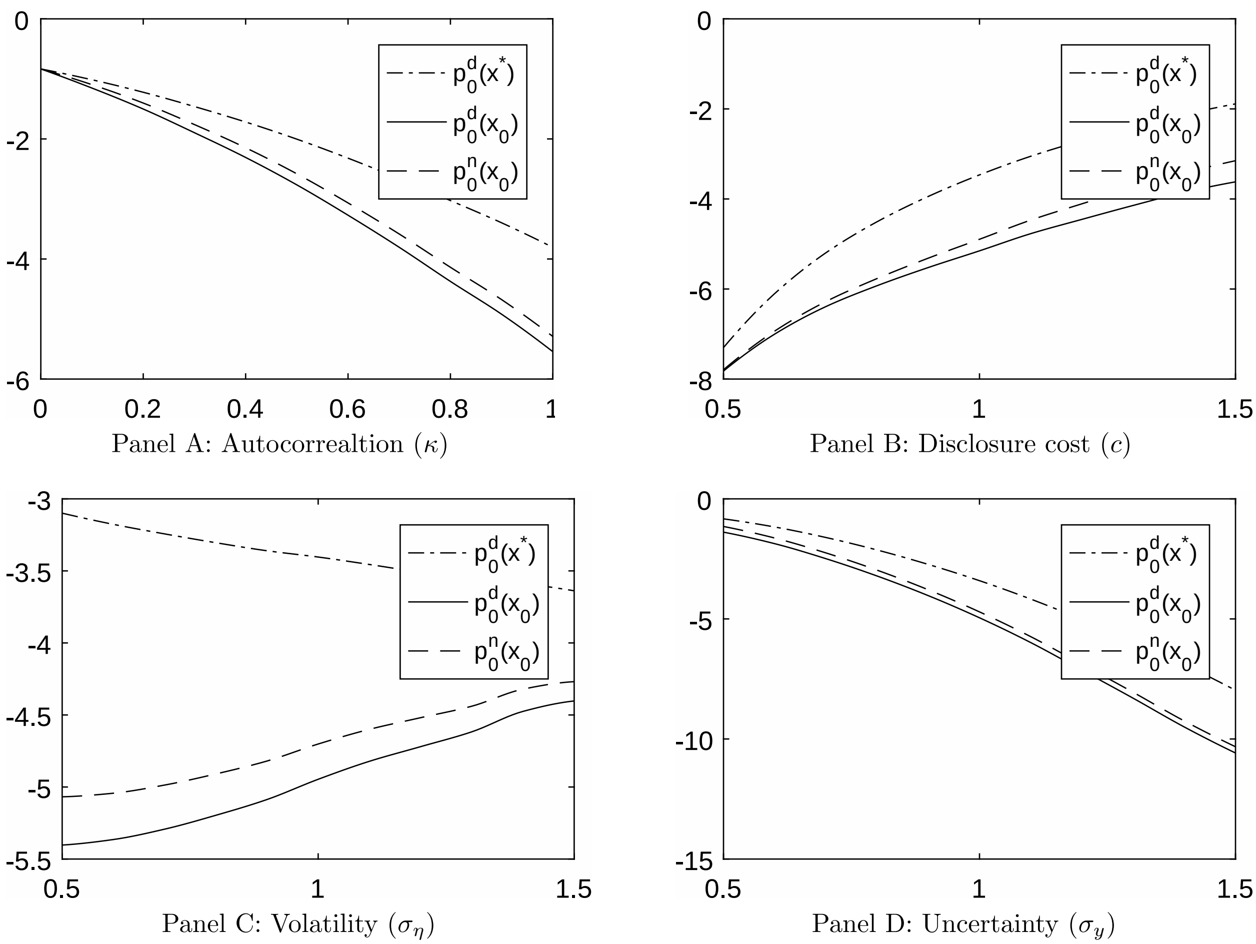


Figure 3: Effect of changes in parameters on threshold-type price. The baseline parameters are: $\sigma_y = 1,\ \sigma_\eta = 1,\ \sigma_s = 2,\ c = 1,\ \rho = 0.5$, and $\kappa = 0.9$.

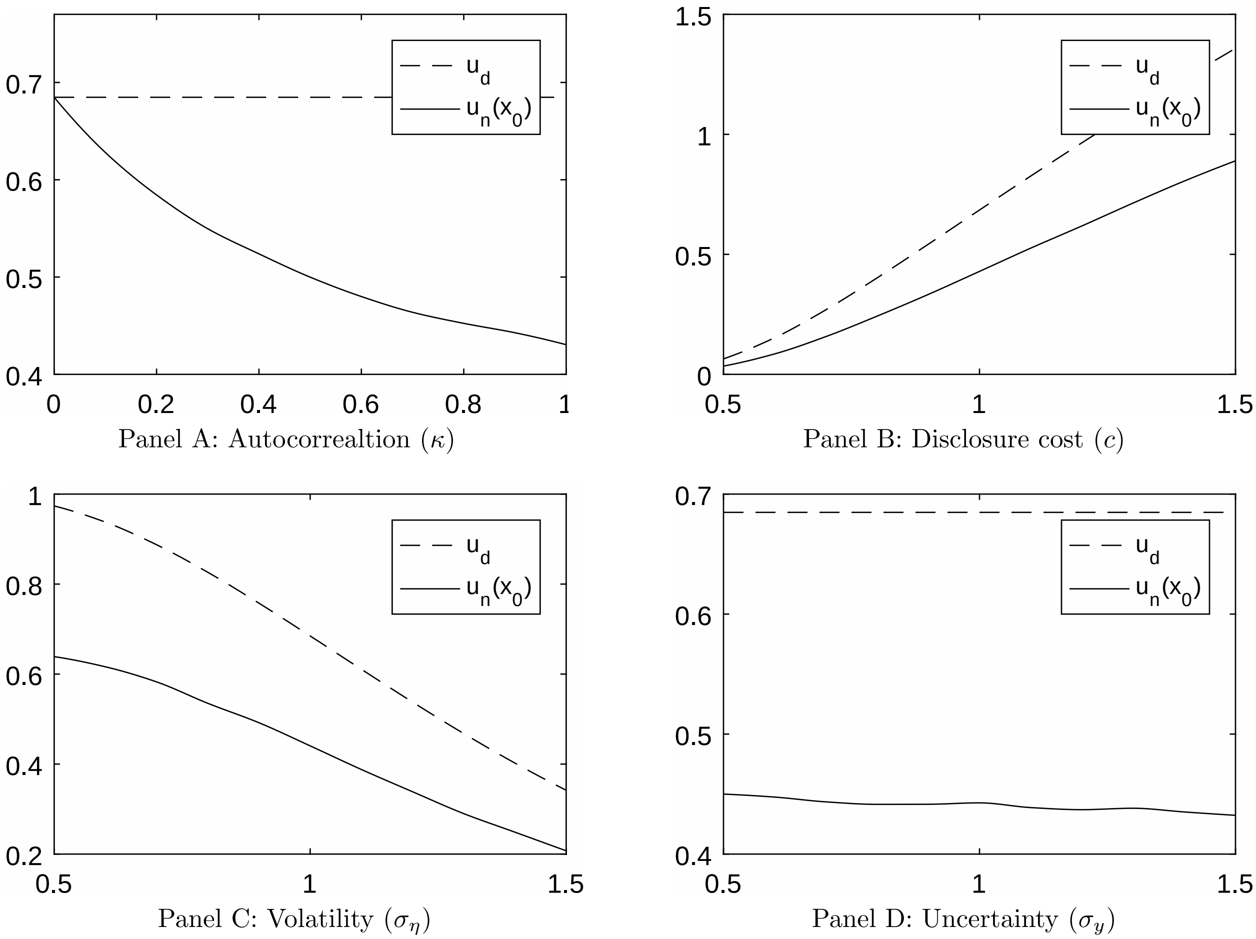


Figure 4: Effect of changes in parameters on option value. The baseline parameters are: $\sigma_y = 1,\ \sigma_\eta = 1,\ \sigma_s = 2,\ c = 1,\ \rho = 0.5$, and $\kappa = 0.9$.

Autocorrelation

We see in Panel A of Figure 2 that the first-period threshold is decreasing in κ. This occurs since the first-period mean cash flow y_0 becomes more salient for the market beliefs in the second period as κ increases. This results in a decreased benefit for the manager from withholding disclosure in $t = 0$, as the gap between x_n and x_d increases in κ (as indicated by Panel A of Figure 3. Hence, the results of the model imply that we should expect more frequent voluntary disclosure when there is greater autocorrelation.

Cost of disclosure

In Panel B, we show the effect of changes in the cost of disclosure on the dynamic threshold. When c is low, it is less costly for the manager to take advantage of the option value from disclosure, u_d, in the first period. This leads to a lower dynamic threshold. Interestingly, as c increases, u_d becomes more valuable for the manager as first-period disclosure has a greater impact on the market's beliefs in the second period. This is due to the direct effect that a rising c has on lowering the disclosure threshold x_0. Moreover, the rising disclosure cost has direct effects on the price absent disclosure in the first period. We thus predict that firms in industries with relatively low proprietary costs will have more frequent voluntary disclosures.

Firm volatility

Next, we examine the effect of changes in the volatility of the initial firm value, σ_y, in Panel D. We begin with a discussion of the effect of σ_y on the purely myopic threshold x^*, as this will be useful in understanding the effects of firm volatility on the dynamic threshold. We see that the myopic threshold is decreasing in σ_y. Intuitively, this occurs because the market has greater uncertainty regarding y_0 when σ_y is high. This implies that the market is more likely to be over-valuing the firm during non-disclosure, and thus imposes a greater penalty. Hence, the purely myopic manager is compelled to disclose more often as the benefits of non-disclosure diminish as σ_y increases.

The first-period dynamic threshold is similarly decreasing in σ_y, however, the effect on x_0 is more prominent when compared to the myopic threshold. As in the myopic case, an increase in σ_y increases the non-disclosure penalty in the first period. However, in the dynamic case, an increase in σ_y increases the non-disclosure penalty in the second period as well, conditional on the manager not disclosing in the first period. This decreases the option value from non-disclosure, $u_n(x_0)$, and thus lowers the first-period threshold x_0.

We can interpret σ_y as the market's level of uncertainty or as the information asymmetry between the manager and the market at time 0. We predict that firms whose information environments generally involve greater information asymmetry or greater uncertainty will also have more frequent voluntary disclosures. Some evidence of this has been found by Anantharaman and Zhang (2011) and Balakrishnan et al. (2014), who document that firms increase their frequency of earnings guidance in response to decreases in analyst following of the firm.

Interestingly, we find that an increase in the volatility of the change in firm value, σ_η, increases the first-period dynamic threshold x_0. To see this, first consider the effect of an increase in σ_η on the second-period threshold. As discussed previously, when the volatility is greater, the non-disclosure penalty is also more severe, and thus a higher σ_η induces more disclosure in the second period. Consequently, in the first period, the manager's option value from disclosure, u_d, is now relatively *less* valuable when σ_η is higher. This implies that the manager has a relatively stronger incentive to take advantage of the option value from non-disclosure in the first period and thus x_0 increases.

We thus have the following prediction: Firms which are characterized as having relatively greater uncertainty in their long-run or future value (such as firms with high R&D expenses) will have *less* frequent voluntary disclosures.

3.5 Discussion

We now discuss our results in the context of the related extant literature. By definition, the threshold type is indifferent between keeping quiet and disclosure in equilibrium. As we showed in Section 3.1, the disclosure threshold in the myopic case satisfies $p_0^d(x^*) = p_0^n(x^*)$. This implies that the price assigned to non-disclosing firms is equal to the price upon disclosure by the threshold type as there is no option value in the myopic case. Hence, the threshold type in the myopic case receives the same price under disclosure and non-disclosure at time 0.

In the model of ADK, the manager has an additional incentive to withhold disclosure as the news announcement may overstate the value of the firm. In the context of the costly disclosure setting, ADK is the limiting case when $\sigma_\eta \to 0$, such that the firm's mean cash flow remains the same. Now, suppose that the threshold-type discloses at time zero. Then, the time 1 price is given by

$$p_1^d(x_0) = x_0 - c.$$

At time 0, the firm's mean cash flow becomes $x_0 - c$ due to disclosure cost and remains the same at time 1. On the other hand, if the threshold-type hides at time 0, then the time 1 price is given by

$$p_1^n(x_0) = x_0 - c + u_n(x_0),$$

Thus, the time zero price upon disclosure can be expressed in a simplified form (for illustrative purposes), as[11]:

$$p_0^d(x_0) = p_0^n(x_0) + u_n(x_0).$$

Since the option value is always positive, we will have $p_0^d(x_0) > p_0^n(x_0)$, or, in other words, the market price always *increases* upon disclosure in the model of ADK.[12]

In contrast, in the present setting, the equilibrium condition can be expressed as (10). Unlike ADK, in the current setting it is possible to have $u_d > u_n$ as the firm value is evolving. Moreover, Theorem 1 states that we always have $u_d > u_n$, which implies that the market price always *decreases* upon disclosure by the threshold-type x_0. This occurs since the manager can increase her option value by disclosing in the first period. Although she endures a price drop in period one, she still prefers to disclose as this increases her continuation utility. Note that this is not possible in the setting of ADK as there is no option value once disclosure has occurred.

Moreover, an immediate implication of a relatively lower threshold x_0 is that there is less delay or a greater frequency of disclosure. As we see above, a low x_0 often occurs when u_d is high relative to u_n. Correspondingly, disclosures which are made under a low first-period threshold level are met with price decreases. Hence, the results of the model predict that firms which are more frequent with their voluntary disclosures are more frequently met with a market reaction which is *negative*, or that these disclosures are often "bad news" in nature. This also helps to reconcile numerous results in the empirical literature which have found that bad news disclosures are more frequent than disclosures of good news (see, e.g., Soffer et al. (2000), Matsumoto (2002), Baik and Jiang (2006), Anilowski et al. (2007), Kross et al. (2011)). In the following section, we explore further the model's empirical implications.

4 Continuous-time Setting

We now extend the model to a continuous-time framework to investigate additional properties of dynamic disclosure where the firm value evolves over time. The key advantage of

[11]See equation (12) of ADK.
[12]This is also noted on page 2965 of ADK.

the continuous-time setting is that we can analyze the endogenous length of delay *between* disclosures, which was not feasible in the discrete setting. We ultimately show that the length of delay is a salient factor in the market reaction following disclosure.

As in the discrete setting, the manager privately observes the mean cash flow, y_t, at every point in time. We assume that the mean cash flow is driven by the following process, which is analogous to that in Section 2:

$$dy_t = \lambda \left(\bar{y} - y_t \right) dt + \sigma_y dZ_t, \tag{13}$$

where $\bar{y}$ is the long-run mean, λ is the rate of mean-reversion, and σ_y is the constant volatility, all of which are commonly known by the manager and investors. The manager can disclose at any time and as often as she prefers.

Actual cash flow is revealed via a Brownian diffusion process. The actual cash flow is publicly observable and satisfies:

$$ds_t = y_t dt + \sigma_s dB_t - c dF_t, \tag{14}$$

where σ_s is the constant volatility parameter, and B_t is a standard Brownian motion. We assume that the correlation between dZ and dB is ρdt. The shock to the true cash flow, B, can be thought of as a macroeconomic or industry shock, which is correlated with an idiosyncratic shock to the firm's mean cash flow, Z. Finally, F_t is the counting process for disclosures. During nondisclosure, $dF_t = 0$, and a discontinuous increase by one in F_t (i.e., $dF_t = 1$) corresponds to disclosure with an atom of probability mass. The discrete cost of disclosure c, incurred each time the manager discloses, are borne entirely by the firm (i.e., shareholders receive net of c).[13]

We continue to assume that there is a continuum of risk-neutral investors with unit mass. The market price of the firm at every point in time is set by the investors, which is given by their belief of the firm's mean cash flow conditional on the history of actual cash flow, disclosures, and the manager's disclosure strategy. As is in the discrete setting, the risk-neutral manager is concerned with the firm's market valuation at every point in time. The manager's objective is to maximize the following:

$$E \left[\int_0^\infty e^{-rt} p_t dt \right], \tag{15}$$

[13]The results would be qualitatively unchanged if rather the manager incurred a portion or all of these costs.

where p_t is the market price of the firm at time t, and r is the manager's rate of time preference.

4.1 Investors' Beliefs

Investors form beliefs regarding the firm value V_t through the history of public news observations, disclosures, and the manager's disclosure strategy. During times of non-disclosure, we separate the two sources of information. We use $\mathcal{F}_t$ to denote the information filtration generated by the history of cumulative news $\{Y_s : 0 \leq s \leq t\}$ and the investors' prior. We define the investors' posterior estimates of the mean and variance of the firm's value conditional on $\mathcal{F}_t$ as follows:

$$\hat{p}_t = E[V_t|\mathcal{F}_t], \text{ and } \gamma_t = E[(V_t - \hat{p}_t)^2|\mathcal{F}_t],$$

with initial values $\hat{p}_0 = V_0$ and $\gamma_0 = 0$. This implies that investors know the true firm value at time 0 (or, equivalently, we begin in a disclosure state).[14] Using the Kalman filter technique, the dynamics of the investors' belief using only the public news is derived as:

$$d\hat{p}_t = \lambda\left(\bar{V} - \hat{p}_t\right)dt + \frac{s(\gamma_t)}{\sigma_Y}\left(dY_t - \hat{p}_t dt\right), \tag{16}$$

where $s(\gamma) = \frac{\gamma + \rho\sigma_V\sigma_Y}{\sigma_Y}$. We commonly refer to the naive belief $\hat{p}_t$ as the investors' *filtered value*. The conditional variance γ_t satisfies the following Riccati equation:

$$d\gamma_t = \left(-2\lambda\gamma_t + \sigma_V^2 - s(\gamma_t)^2\right)dt. \tag{17}$$

As time goes to infinity, the posterior variance reaches the following steady-state:

$$\gamma_\infty = \sigma_Y^2(\phi - \lambda) - \rho\sigma_V\sigma_Y, \tag{18}$$

where

$$\phi = \sqrt{(\lambda + \rho\sigma_V/\sigma_Y)^2 + (1-\rho^2)\sigma_V^2/\sigma_Y^2}. \tag{19}$$

The market price during non-disclosure must also account for another important source of information—the manager's disclosure strategy. We introduce the following class of disclosure strategies. A disclosure strategy we consider is a stopping time, which can be represented as a stochastic process F_t along the sample path $\{V_t, \hat{p}_t, \gamma_t\}$. We construct the equilibrium

[14] This is primarily for parsimony. The results are not sensitive to this assumption.

of interest by defining a class of candidate equilibria with a unique disclosure threshold and non-manipulable disclosure. First, we define the gap g_t as the difference between the firm value and the filtered value, $g_t = V_t - \hat{p}_t$. We conjecture that the manager follows a disclosure threshold strategy, such that the manager discloses the firm value if and only if this difference is sufficiently large, i.e., the manager discloses whenever $g_t \geq g^*$.[15] Intuitively, the manager is willing to bear some undervaluation in the price in order to economize disclosure costs. However, when this undervaluation is sufficiently severe (i.e., when $g > g^*$), she is better off incurring the disclosure cost and raising the investors' valuation. This implies that the threshold should be positive since the gap is greater than the magnitude of actual mis-pricing: $V_t - p_t < V_t - \hat{p}_t = g_t$, as we will show below. Hence, for any $g^* > 0$, we define the disclosure strategy as

$$F_t = \begin{cases} F_s + 1 & if\ there\ exists\ s \leq t\ such\ that\ V_s - \hat{p}_s \geq g^* \\ F_s & otherwise. \end{cases}$$

Define a random variable $v = V_t | \mathcal{F}_t$. Then, v is normally distributed with a mean of $\hat{p}_t$ and a variance of γ_t from the perspective of investors. Based on the manager's disclosure strategy perceived by investors, the information from non-disclosure implies that $v < \hat{p}_t + g^*$. Thus, after incorporating the manager's disclosure strategy, we can compute the market price during non-disclosure as:

$$\begin{aligned} p_t &= E[v | v < \hat{p}_t + g^*] \\ &= \hat{p}_t - \sqrt{\gamma_t}\delta\left(\frac{g^*}{\sqrt{\gamma_t}}\right), \end{aligned}$$

where $\delta(x) = \phi(x)\Phi(x)^{-1}$ denotes part of the non-disclosure penalty as in the discrete setting. The market price is the sum of two components, which separate the two sources of information available to the market during non-disclosure. The first component, $\hat{p}_t$, is the market price for a Bayesian who updates only based on the public news. The second component, $\sqrt{\gamma_t}\delta(g^*/\sqrt{\gamma_t})$, is the non-disclosure penalty based on the manager's disclosure strategy. Note that at time zero and any point in which the manager discloses, there is no uncertainty about the firm value and the non-disclosure penalty is zero: $p_0 = \hat{p}_0$. As time

[15] This implies that manager's disclosure decision does not depend on the posterior variance. There are two reasons for us to consider this strategy. First, with this strategy we can have a similar characterization of the disclosure threshold as in the discrete-time setting. Second, any other strategy where the threshold depends on the posterior variance must prevent unraveling by imposing $\lim_{\gamma \to 0} g^* > 0$, since upon disclosure both the gap g and the variance become zero.

passes, both the posterior variance and the non-disclosure penalty are increasing, conditional on no disclosure having occurred since the last disclosure event. In equilibrium, the manager's disclosure strategy F_t maximizes (15) given the firm's value V_t and the market price p_t.

4.2 Recursive Formulation for the Manager's Problem

The market price is a time-homogeneous Markov process, meaning that the candidate equilibrium has a stationary structure, with $(V, \hat{p}, \gamma)$ serving as the state variable. We can equivalently use (V, g, γ) as the state variable, which allows the analysis to be more tractable. During non-disclosure, from the perspective of the privately informed manager, the gap g evolves as:

$$dg_t = -\left(\lambda + \frac{s(\gamma_t)}{\sigma_Y}\right) g_t dt + \sigma_V dZ_t - s(\gamma_t) dB_t.$$

4.2.1 Disclosure Option Value

Let $W(V, g, \gamma)$ denote manager's value function. During non-disclosure, the posterior variance is always between 0 and γ_∞, the gap is less than the threshold $g < g^*$, and $dF_t = 0$. Thus, for $0 \leq \gamma \leq \gamma_\infty$ and $g \leq g^*$, the value function satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} rW(V, g, \gamma) \quad = \quad & V - g - \sqrt{\gamma}\delta\left(\frac{g^*}{\sqrt{\gamma}}\right) + \lambda\left(\bar{V} - V\right) W_V - \left(\lambda + \frac{s(\gamma)}{\sigma_Y}\right) gW_g \\ & + \left(-2\lambda\gamma + \sigma_V^2 - s(\gamma)^2\right) W_\gamma + \frac{1}{2}\sigma_V^2 W_{VV} \\ & + \sigma_V(\sigma_V - \rho s(\gamma))W_{Vg} + \frac{1}{2}(\sigma_V^2 - 2\rho\sigma_V s(\gamma) + s(\gamma)^2)W_{gg}. \end{aligned} \tag{20}$$

In the RHS, the first three terms are the market price during non-disclosure. The remaining terms are the change in the manager's value function due to changes in the firm's value V, the gap g, and the posterior variance γ. We see two benefits of withholding disclosure. First, there is the possibility that investors may overvalue the firm, i.e., $g + \sqrt{\gamma}\delta(g^*/\sqrt{\gamma}) < 0$. Second, even though the market may undervalue the firm in the current moment, the market may begin to overvalue the firm or the firm value may improve in the future.

A key simplification that arises in our setup is that the manager's value function can be decomposed into the value function absent the option to disclose and the option value to disclose by exploiting the fact that the disclosure option value arises only from the gap, g

and the posterior variance γ. Thus, a natural conjecture for the solution to equation (20) is:

$$W(V,g,\gamma) = \underbrace{\frac{\bar{V}}{r} + \frac{V-\bar{V}}{r+\lambda}}_{\substack{\text{Value function under}\\ \text{no private information}}} + \underbrace{u(g,\gamma)}_{\substack{\text{Disclosure}\\ \text{option}}}. \tag{21}$$

The first two terms in (21) are the manager's value function when the firm value is observed perfectly by investors. A deviation from the long-run mean at the current moment is discounted with $r+\lambda$ since the firm value follows a mean-reverting process with mean-reversion speed λ. The second term is the option value of disclosure.

Now, we only need to solve for the disclosure option value $u(g,\gamma)$. By substituting the conjectured solution into the HJB equation, we obtain the following equation for $u(g,\gamma)$:

$$\begin{aligned} ru(g,\gamma) &= -g - \sqrt{\gamma}\delta\left(\frac{g^*}{\sqrt{\gamma}}\right) - \left(\lambda + \frac{s(\gamma)}{\sigma_Y}\right) g u_g \\ &\quad + \left(-2\lambda\gamma + \sigma_V^2 - s(\gamma)^2\right) u_\gamma + \frac{1}{2}(\sigma_V^2 - 2\rho\sigma_V s(\gamma) + s(\gamma)^2) u_{gg}. \end{aligned} \tag{22}$$

4.2.2 Boundary Conditions

In this section, we analyze the boundary conditions in order to solve the partial differential equation in (22). Denote by τ as a time in which the gap exceeds g^*. At that point, the manager discloses the new firm value $V_{\tau^+} = V_\tau - c$. This event moves the investors' belief towards the new firm value. Specifically, they now believe that the firm value is V_{τ^+} without uncertainty, i.e., $\gamma_{\tau^+} = 0$. This implies that, upon disclosure, the market price is equal to the new firm value, there is no disclosure penalty, and the gap is now zero: $p_{\tau^+} = \hat{p}_{\tau^+} = V_{\tau^+}$ and $g_{\tau^+} = 0 < g^*$.

In sum, by disclosing the new firm value, the manager moves to the state where the firm value is $V - c$, the gap is zero, and there is no uncertainty at that moment. The manager's value function must be continuous before and after disclosure, which implies the following condition at the disclosure threshold g^*:

$$W(V,g^*,\gamma) = W(V-c,0,0) \iff \frac{c}{r+\lambda} = u(0,0) - u(g^*,\gamma). \tag{23}$$

Next, since g^* is optimally chosen, the following smooth-pasting condition should hold at g^*:

$$W_g(V,g^*,\gamma) = u_g(g^*,\gamma) = 0. \tag{24}$$

Finally, since the posterior variance γ can never exceed the steady-state level γ_∞, we also have a boundary condition at (g, γ_∞). Since the drift of γ is zero and the drift of the gap is $-\phi$ at γ_∞, we have the following ODE at (g, γ_∞):

$$ru(g, \gamma_\infty) = -g - \sqrt{\gamma_\infty}\delta\left(\frac{g^*}{\sqrt{\gamma_\infty}}\right) - \phi g u_g + \frac{1}{2}\sigma_g^2 u_{gg}, \tag{25}$$

where $\sigma_g^2 = \sigma_V^2 - 2\rho\sigma_V s(\gamma_\infty) + s(\gamma_\infty)^2$. Using techniques developed in Scheinkmen and Xiong (2003), we characterize the solution of $u(g, \gamma)$ at $\gamma = \gamma_\infty$ in the following proposition:

Proposition 6 *Let*

$$h(g) = \begin{cases} U\left(\frac{r}{2\phi}, \frac{1}{2}, \frac{\phi}{\sigma_g^2}g^2\right) & if\ \ g \leq 0 \\ \frac{2\pi}{\Gamma\left(\frac{1}{2}+\frac{r}{2\phi}\right)\Gamma\left(\frac{1}{2}\right)} M\left(\frac{r}{2\phi}, \frac{1}{2}, \frac{\phi}{\sigma_g^2}g^2\right) - U\left(\frac{r}{2\phi}, \frac{1}{2}, \frac{\phi}{\sigma_g^2}g^2\right) & if\ \ g > 0, \end{cases} \tag{26}$$

where $\Gamma(\cdot)$ is the Gamma function, and $M(\cdot, \cdot, \cdot)$ and $U(\cdot, \cdot, \cdot)$ are two Kummer functions described in the Appendix. Then, any solution to (25) must satisfy

$$u(g, \gamma_\infty) = -\frac{g}{r+\phi} - \frac{\sqrt{\gamma_\infty}}{r}\delta\left(\frac{g^*}{\sqrt{\gamma_\infty}}\right) + \beta h(g). \tag{27}$$

The first two terms are the impact of the investors' mis-pricing on the manager's value function. The last term is the present value of the flow payoff that the manager receives whenever the gap reaches the threshold, given that the current gap is $g < g^*$. An additional technical result we use to characterize the solution is that $h(g)$ is positive, increasing, convex, and that $\lim_{g \to -\infty} h(g) = 0$ (see Appendix). Thus, when g_t is sufficiently low (negative), i.e., when investors are severely overvaluing the firm, then the last term $\beta h(g)$ is close to zero and the option value of disclosure is just the impact of the investors' overvaluation. As g_t increases, $\beta h(g)$ also increases. The manager would prefer to remain quiet as long as the option value is greater than the option value upon disclosure net of the long term impact of the disclosure cost.

To further illustrate the intuition behind the boundary condition at (g^*, γ_∞), we can substitute equation (27) into (23). Then we have:

$$\frac{c}{r+\lambda} = \frac{g^*}{r+\phi} + \frac{\sqrt{\gamma_\infty}}{r}\delta\left(\frac{g^*}{\sqrt{\gamma_\infty}}\right) + u(0,0) - \beta h(g^*). \tag{28}$$

We can compare equation (28) above with equation (10), which pins down the first-period

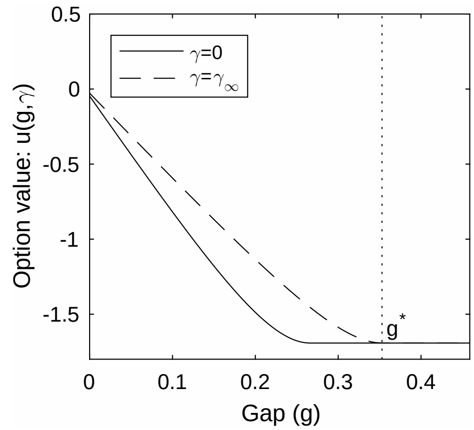


Figure 5: Option value. The baseline parameters are: $\bar{V} = 0.5,\ \sigma_V = 0.1,\ \sigma_Y = 0.4,\ c = 0.4,$ $\rho = -0.8,\ \lambda = 0.2231$, and $r = 0.02$.

disclosure threshold x_0 in the discrete-time setting. The left-hand side in (28) is the cost of disclosure in terms of manager's value function. Although the firm value decreases by c at the time of disclosure, its long term impact on the manager's value function is scaled by $1/(r+\lambda)$, since the manager's time preference is r and the firm value is mean-reverting with a speed of λ. In the discrete-time setup, it can be expressed as $c\left(1 + \frac{\kappa c}{R}\right)$ as in equation (10). Similarly, the first two terms in the right-hand side of (28) are the benefits of disclosure in terms of the manager's value function. By disclosing, the manager can remove the undervaluation: $V - p = g^* + \sqrt{\gamma_\infty}\delta\left(\frac{g^*}{\sqrt{\gamma_\infty}}\right)$. The manager's benefit from bridging the gap g^* is scaled by $1/(r+\phi)$, since the gap is mean-reverting with a speed of ϕ when $\gamma = \gamma_\infty$. The manager's benefit from removal of the non-disclosure penalty is discounted by r since the same constant penalty would be imposed unless the manager did not disclose. These first two terms on the right-hand side of equation (28) are analogous to $x_0 + \sigma_V\delta\left(\frac{x_0}{\sigma_V}\right)$ of equation (10), which represents the manager's benefit of disclosure in the discrete-time setting.

Finally, the last two terms of (28) are the difference between the option value after disclosure and the flow payoff to the manager at $g = g^*$. By disclosing, the manger obtains the option value evaluated at $g = 0$ and $\gamma = 0$. In (10), these two terms are expressed as $\frac{1}{R}(u_D - u_N(x_0))$. Overall, the right-hand side is the difference in the option value after and before disclosure. Note that $u(0,0)$ is an endogenous solution to equation (22) given the boundary condition at $\gamma = \gamma_\infty$, i.e., $(g^*,\ \beta)$. Similar to the results in the discrete-time setting, we show that the market price drops at the time of disclosure under some conditions in the following section. To the best of our knowledge, there is no analytical solution to (22) and thus we solve it numerically. The methodology we utilize is provided in the Appendix.

Figure 5 illustrates the equilibrium option value as a function of the gap g. We vary the gap while keeping the true firm value at the long-run mean $V = \bar{V}$ and the posterior variance at zero and γ_∞. First, the option value is decreasing in the gap. As the gap increases, investors undervalue the firm more. The manager is willing to bear undervaluation as long as the option value is greater than the continuation value upon disclosure. At the optimal disclosure threshold, the option value is exactly equal to the continuation value and the marginal option value is zero. The option value is smaller when the posterior variance is smaller. This is because for the same gap the size of undervaluation is greater when the posterior variance is larger.

4.3 Market Price upon Disclosure

The manager optimally chooses the disclosure boundary so that her value function is smooth upon disclosure. However, there will be a jump in the market price at the time of disclosure. We can compute this jump size in the market price when the manager discloses ($g_\tau = g^*$) as:[16]

$$p_{\tau^+} - p_\tau = \sqrt{\gamma_\tau} v \left(\frac{g^*}{\sqrt{\gamma_\tau}} \right) - c. \tag{29}$$

The first term of the right-hand side of (29) is the market's reaction to the disclosed firm value. Upon disclosure, investors fairly price the firm, which implies that the gap becomes zero and there is no non-disclosure penalty. The second term is the cost of disclosure. We show below that, similar to the discrete setting, the manager may disclose even if this results in a decrease of the current market price. The following result characterizes the price movement upon disclosure:

Theorem 2 *The price jump is increasing in the posterior variance. If $g^* > c$, then the price jump is always positive. If $g^* < g_0$, then the price jump is always negative, where g_0 is the static disclosure threshold solving $c = \sqrt{\gamma_\infty} v \left(\frac{g_0}{\sqrt{\gamma_\infty}} \right)$. If $g_0 < g^* < c$, there exists a unique posterior variance $\bar{\gamma} \in (0, \gamma_\infty)$ such that for $\gamma < \bar{\gamma}$ the price jump is negative and for $\gamma > \bar{\gamma}$ the price jump is positive.*

Theorem 2 above shows conditions under which the price jump is upward or downward

[16] This follows from:

$$\begin{aligned} p_{\tau^+} &= V_\tau - c \\ p_\tau &= \hat{p}_\tau - \sqrt{\gamma_\tau} \delta(g^*/\sqrt{\gamma_\tau}) = V_\tau - g^* - \sqrt{\gamma_\tau} \delta(g^*/\sqrt{\gamma_\tau}). \end{aligned}$$

upon disclosure. Interestingly, we find that, under certain conditions, disclosure may result in either a price increase or decrease depending on the posterior variance at the time of disclosure. This implies that the *timing* of disclosure is indicative of the market reaction which occurs upon disclosure. For example, if

At the instant right before the manager discloses, the posterior variance γ is between 0 and γ_∞ and the gap is g^*. The non-disclosure penalty is larger when the posterior variance is larger at the time immediately preceding disclosure. Thus, the upward movement in the price is increasing in the size of the posterior variance, since a larger non-disclosure penalty is removed upon disclosure.

To more clearly illustrate the intuition behind Theorem 2, it is useful to consider the problem in the context of the static model. Recall the equilibrium condition of the static disclosure threshold in the discrete-time setting given by equation (4), and suppose that the posterior variance is at the steady state. If the manager is concerned only about the *current* price, then her equilibrium strategy would entail a threshold gap g_0 at which the size of undervaluation is exactly equal to the disclosure cost:

$$c = g_0 + \sqrt{\gamma_\infty}\delta\left(\frac{g_0}{\sqrt{\gamma_\infty}}\right).$$

Since the size of undervaluation is greater than the gap, which is the difference between the firm value and the filtered value, the static disclosure threshold must always be less than the cost of disclosure: $g_0 < c$. Then, the optimal dynamic disclosure threshold g^* must be in one of three distinct regions: $g^* > c$, $g^* < g_0$, or $g_0 < g^* < c$.

In the case where $g^* > c$, the price jump is positive when $\gamma \to 0$. This implies that the price jump is always positive when $g^* > c$, as the price jump is increasing in the posterior variance. Likewise, if the price jump is negative when $\gamma \to \gamma_\infty$, then the jump is always negative: $\sqrt{\gamma_\infty}v\left(\frac{g^*}{\sqrt{\gamma_\infty}}\right) < c = \sqrt{\gamma_\infty}v\left(\frac{g_0}{\sqrt{\gamma_\infty}}\right)$, which implies that $g^* < g_0$. If $g_0 < g^* < c$, we find that the price jump is zero when $\gamma = \bar{\gamma}$. This implies that the price jump is positive when $\gamma > \bar{\gamma}$, as the jump is increasing in the posterior variance. Similarly, the price jump is negative when $\gamma < \bar{\gamma}$. Moreover, since the posterior variance is a deterministic function of time, we can pin down the time $\bar{t}$ such that the posterior variance is $\bar{\gamma}$ at $t = \bar{t}$. Hence, if the gap reaches the threshold level before $\bar{t}$ is reached since the time of the last disclosure, then disclosure results in a downward price jump.

Theorem 2 result implies that the manager discloses whenever the gap g exceeds the optimal disclosure threshold g^*, even though the market price may drop by doing so. This

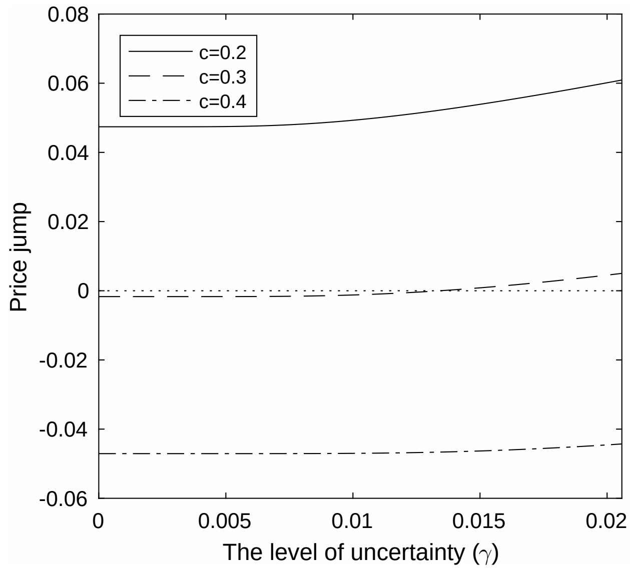


Figure 6: Price jump at disclosure when $c = 0.2,\ 0.3$, and 0.4. The baseline parameters are: $\bar{V} = 0.5,\ \sigma_V = 0.1,\ \sigma_Y = 0.4,\ \rho = -0.8,\ \lambda = 0.2231$, and $r = 0.02$.

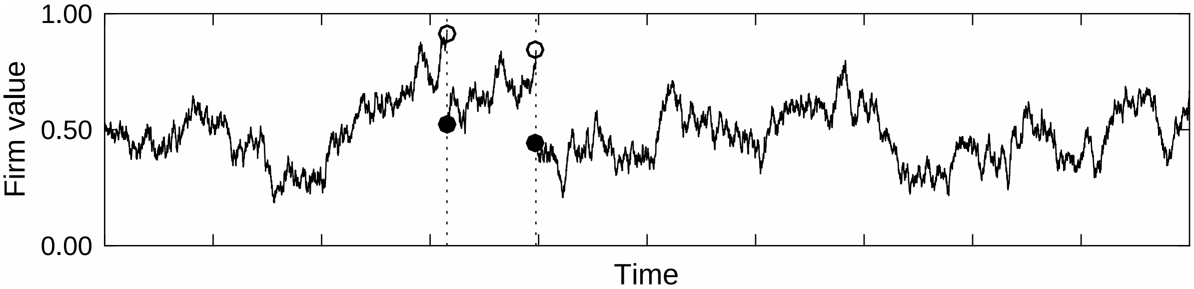
is analogous to Theorem 1 of the discrete case, except for the important difference that the delay between disclosures is now a salient determinant of the market reaction which follows. This is a seemingly paradoxical result—we would not expect the manager to disclose to obtain a lower market price. However, this is possible because, as shown in the previous section, the manager considers her long-term value function when deciding the disclosure threshold. Even if the market price drops upon disclosure in the present moment, the manager can remove the market's undervaluation and obtain the disclosure option value again. The long-term impact on the manager's value function dominates the cost of disclosure and compensates for the decrease in the manager's utility from the price decrease after disclosure.

Figure 6 shows the size of the price jump as a function of the posterior variance at the moment before disclosure for three different disclosure costs. For a low disclosure cost, the price jump is always positive. Since disclosure cost is low, the manager will not bear undervaluation and thus the disclosure threshold is lower. The market also takes into account manager's strategy and thus the non-disclosure penalty is lower. When disclosure cost is low enough, the disclosure threshold might be even higher than disclosure cost and price jump is always positive. For intermediate disclosure cost, both positive and negative price jump can be observed depending on posterior variance right before disclosure. When disclosure is made at low posterior variance, i.e. little time has elapsed since the last disclosure, then accordingly non-disclosure penalty is smaller and the price jump might be negative. For high disclosure cost, the manager is willing to save disclosure cost by delaying disclosure or bearing undervaluation as mush as she can. Since the disclosure threshold is higher, for the same posterior variance the non-disclosure penalty is smaller, which implies that price jump is also smaller. When disclosure cost is high enough, price jump at γ_∞ might be even negative and thus price jump is always negative.

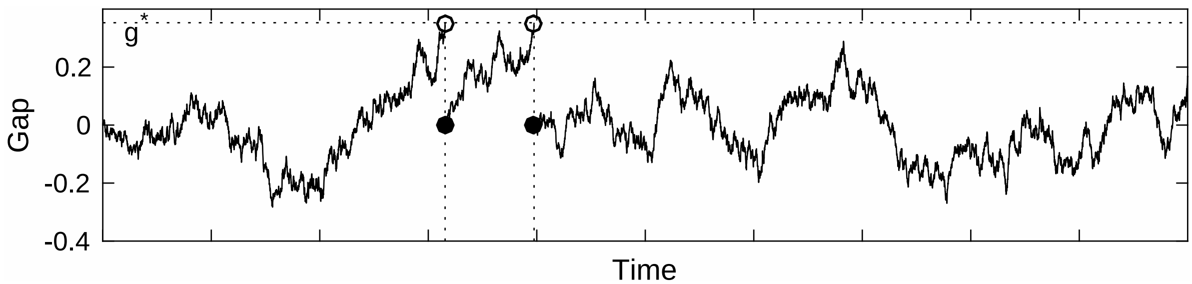
In Figure 7, we generate a sample path of the firm value and news and then plot the firm value, gap, market price, and posterior variance. With parameters we use, the equilibrium disclosure threshold is lower than the static disclosure threshold so that we can observe a negative price jump at disclosure. At the beginning, the firm value is at the long-run mean and the market knows the firm value for sure. There are two disclosures along the sample path. At the first disclosure, enough time has passed since time zero and thus the posterior variance is at the steady state level. At the time of disclosure, we can see that the firm value is decreased by c, the gap is reset to zero, the market price is also decreased by $c - \sqrt{\gamma_\tau} v \left(\frac{g^*}{\sqrt{\gamma_\tau}} \right)$, and the posterior variance drops to zero. Then, the game starts again and continues until gap hits the threshold again. The second disclosure occurs after little time has elapsed since the first disclosure and thus the posterior variance right before the second disclosure is smaller than the steady state level. This implies that the price decrease at the second disclosure is smaller than the price decrease at the first disclosure.

4.4 Relation to Empirical Literature

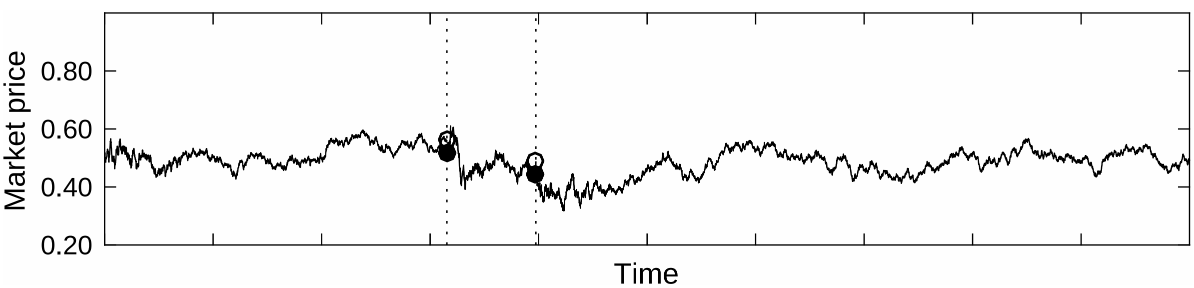
There is a sizable empirical literature on voluntary disclosure. The present model helps to shed light on some of the documented empirical regularities. The large-scale survey of executives by Graham et al. (2005) finds evidence in support of voluntary disclosure as embedding a real option. We formalize this notion and find a unique equilibrium disclosure strategy. In terms of the observed delay in the release of voluntary disclosures, the model helps to explain the patterns found in Kothari et al. (2009) and Sletten (2012). Kothari et al. (2009) find evidence supporting the hypothesis that managers withhold information over time up to a threshold before issuing a disclosure. In a similar vein, Sletten (2012) documents that firms disclose information more often following negative shocks to share prices. This is formally captured in our model, as Theorem 1 shows that the manager delays disclosure until the difference between the market's belief and fundamental value reaches a unique threshold, at which point the manager is compelled to disclose. Moreover, Kothari et al. (2009) document that share prices fluctuate downward during periods of non-disclosure, but then jump upward upon disclosure. This pattern corresponds closely



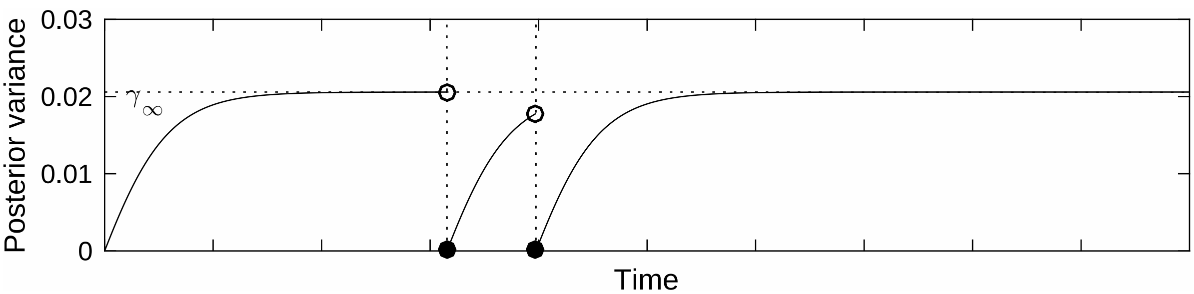
Panel A: Firm value (V)



Panel B: Gap (g)



Panel C: Market price (p)



Panel D: Posterior variance (γ)

Figure 7: Equilibrium dynamics for a fixed sample path. The baseline parameters are: $\bar{V} = 0.5$, $\sigma_V = 0.1$, $\sigma_Y = 0.4$, $c = 0.4$, $\rho = -0.8$, $\lambda = 0.2231$, and $r = 0.02$.

to the results presented in Propositions 1 and 6 and, where the market price is depressed during periods of non-disclosure and increased upon disclosure. Our numerical analysis in the preceding sections also provides additional predictions concerning the magnitude of the market reaction upon disclosure and the length of delay as related to characteristics of the firm and its informational environment.

The results of the model also have implications for the skewness of observed returns. As documented by Beedles (1979), as well as several other studies, individual stock returns tend to have a positive skewness. In environments where information is learned by the market over time, the model helps to explain how positive skewness can arise when there is disclosure after public news releases. Proposition 6 and Theorem 2 show that the manager discloses her private information when the difference between the fundamental value and the filtered value becomes sufficiently large. When news is bad, or when market sentiment deteriorates, this triggers disclosure by the manager. This results in a *negative* skewness following a disclosure. A similar implication is made in Acharaya, DeMarzo, Kramer (2011, "ADK"). However, in ADK's model, the release of private information after public news cannot generate *positive* skewness. In contrast, in the present model, when news is good or market sentiment improves, this implies that the fundamental firm value is also improving. However, the fundamental value may be improving in a magnitude faster than the public signal, so that the belief threshold for disclosure is exceeded and the manager is compelled to disclose. This implies that even though the public signal is releasing good news, there is also disclosure of good news by the firm, which thus leads to positive skewness in the stock return after disclosure following public news announcements. This feature cannot arise in ADK as good news announcements are always preempted by the manager in the equilibrium of their setting.

Although most of our predictions have not been previously tested, there is some evidence concerning the prediction regarding the precision of public information and the frequency of disclosures. We predict that firms with relatively less precise public information will have less frequent disclosures and greater delay in their release of information. Recent studies investigating the relation between short selling and disclosure provide evidence consistent with this prediction. Specifically, Clinch et al. (2016) find causal evidence that firms who were subject to less short-sale regulation (which resulted in increased short selling) increased their frequency of disclosure. Similarly, Hu (2016) documents that firm voluntary disclosures increased following greater transparency of firm-level short interest. In both studies, firms subject to the new regulation saw improvements in their informational environments.[17] Hence, the increase in the precision of public information led to more frequent disclosures, consistent with our empirical prediction.

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[17] There is a vast literature examining the effect of short selling on information asymmetry and price efficiency (e.g., Bris et al. (2007), Saffi and Sigurdsson (2011), Beber and Pagano (2013), Boehmer et al. (2013), and Boehmer and Wu (2013), among others). These studies generally find that short selling improves price efficiency and liquidity, and helps mitigate information asymmetry.

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Appendix

A Proofs

A.1 Proof of Proposition 1 and 2

To prove Proposition 1, we first prove the following Lemma:

Lemma A1 *$v(x)$ is non negative and increasing in x. Furthermore, $\delta(x)$ is weakly decreasing in x. Finally, $\lim_{x\to-\infty}\delta(x) = -x$, $\lim_{x\to\infty}\delta(x) = 0$, $\lim_{x\to-\infty}\delta(x)v(x) = 1$, and $\lim_{x\to\infty}\delta(x)v(x) = 0$.*

Proof of Lemma A1. First, we want to show that $\delta(x) \geq -x$ so that $v(x) \geq 0$. When $x \geq 0$, clearly it holds. For $x < 0$, define $R(x) = \delta(x)^{-1}$. Then, we want to show that $R(x) \leq -\frac{1}{x}$ for $x < 0$. The first derivative is

$$R'(x) = 1 + xR(x) \tag{A.1}$$

and we also have

$$\lim_{x\to-\infty} xR(x) = -1 \tag{A.2}$$

Suppose that at any point $x_1 < 0$, $R(x_1) > -\frac{1}{x_1}$, i.e. $x_1R(x_1) < -1$ by contradiction. Then, by (A.1) $R'(x) < 0$ and $R(x)$ would continue to increase with decreasing x. This also implies that $xR(x)$ would continue to decrease, hence we should have $xR(x) < -1$ for any $x \leq x_1$, which contradicts (A.2). Therefore we show that $R(x) \leq -\frac{1}{x}$, i.e. $\delta(x) \geq -x$ for $x < 0$ too.

Next, we want to show that $v'(x) > 0$. The first derivative of $v(x)$ is given by

$$v'(x) = 1 - \delta(x)v(x)$$

Notice that this is variance of a standard normal variable ϵ conditional on $\epsilon < x$. Since this must be positive, we have $v'(x) > 0$. This also implies that $\delta(x)v(x) < 1$ and $-1 < \delta'(x) = -\delta(x)v(x) \leq 0$ since $\delta(x) > 0$ and $v(x) \geq 0$.

Finally, since $\delta(x)$ is mean of a standard normal variable with one sided truncation of upper tail at x, we have that $\delta(x) \to -x$ as $x \to -\infty$ and $\delta(x) \to 0$ as $x \to \infty$. This also implies that $\delta'(x) = -\delta(x)v(x) \to -1$ as $x \to -\infty$ and $\delta'(x) = -\delta(x)v(x) \to 0$ as $x \to \infty$. ■

Proposition 1 immediately follows from Lemma A1 and Lemma 3 and Proposition 2 follows from Lemma A1.

A.2 Proof of Proposition 3 and Lemma 1

Since the baseline model is a special case that the correlation between news and time zero cash flow is zero, for the rest of proof we consider a general case. We first establish the following Lemma:

Lemma A2 *Define a function $k(x, y; s)$ for any $s > 0$*

$$k(x, y; s) = \int_{-\infty}^{y} v(x - sz)\frac{\phi(z)}{\Phi(y)}dz$$

We have following properties of $k(x, y; s)$:

$$k_x(x, y; s) > 0, \quad and \quad k_x(x, y; s) + \frac{1}{s} k_y(x, y; s) > 0.$$

Proof of Lemma A2. The first derivative with respect to x is given by

$$k_x(x, y; s) = \int_{-\infty}^{y} (1 - \delta(x - sz)v(x - sz)) \frac{\phi(z)}{\Phi(y)} dz > 0$$

The second inequality is due to lemma A1. Take the first derivative with respect to y:

$$\begin{aligned} k_y(x, y; s) &= \delta(y) \left[v(x - sy) - \int_{-\infty}^{y} v(x - sz) \frac{\phi(z)}{\Phi(y)} dz \right] \\ &= \delta(y) \left[v(x - sy) - v(x - sz) \frac{\Phi(z)}{\Phi(y)} |_{-\infty}^{y} - s \int_{-\infty}^{y} (1 - \delta(x - sz)v(x - sz)) \frac{\Phi(z)}{\Phi(y)} dz \right] \\ &= -s\delta(y) \int_{-\infty}^{y} (1 - \delta(x - sz)v(x - sz)) \frac{\Phi(z)}{\Phi(y)} dz \end{aligned}$$

The second equality holds since we have

$$\Phi(z) = \left(1 - \frac{1}{z^2} + \mathcal{O}\left(\frac{1}{z^2}\right)\right) \frac{\phi(z)}{|z|}$$

and thus

$$\lim_{z \to -\infty} v(x - sz)\Phi(z) = \lim_{z \to -\infty} \frac{x - sz}{|z|} \left(1 - \frac{1}{z^2} + \mathcal{O}\left(\frac{1}{z^2}\right)\right) \phi(z) = 0$$

Finally, we have

$$k_x(x, y; s) + \frac{1}{s} k_y(x, y; s) = \int_{-\infty}^{y} (1 - \delta(x - sz)v(x - sz))(\delta(z) - \delta(y)) \frac{\Phi(z)}{\Phi(y)} dz > 0$$

The second inequality is due to $\delta(z) > \delta(y)$ for $z < y$ and $\delta(\cdot)v(\cdot) < 1$. ■

Now, we can express (6) using $k(x, y; s)$:

$$\begin{aligned} c &= \epsilon^*(g) - E\left[E[\kappa z + \eta | z, \eta < \epsilon^*(g) - \kappa z] | z < g\right] \\ &= \epsilon^*(g) - E\left[\kappa z - \sigma_\eta \frac{\phi(\frac{\epsilon^*(g) - \kappa z}{\sigma_\eta})}{\Phi(\frac{\epsilon^*(g) - \kappa z}{\sigma_\eta})} | z < g\right] \\ &= \epsilon^*(g) + \sigma_\eta \int_{-\infty}^{g} \left[-\frac{\kappa z}{\sigma_\eta} + \delta\left(\frac{\epsilon^*(g) - \kappa z}{\sigma_\eta}\right)\right] \frac{\phi(\frac{z}{\sigma_z})}{\sigma_z \Phi(\frac{g}{\sigma_z})} dz \\ &= \sigma_\eta k\left(\frac{\epsilon^*(g)}{\sigma_\eta}, \frac{g}{\sigma_z}; \frac{\kappa \sigma_z}{\sigma_\eta}\right), \end{aligned} \tag{A.3}$$

where $g = x_0 - \rho\sigma_y s/\sigma_s$. It can be immediately seen that when the correlation is zero (baseline model), we have $g = x_0$ and $x_n(x_0) = \epsilon^*(x_0)$. By Lemma A2, given g the right hand side of (A.3) is increasing in $\epsilon^*(g)$

so that there exists a unique fixed point. Next, totally differentiate (A.3), then we have

$$0<\frac{d\epsilon^*(g)}{dg}=-\frac{\frac{\sigma_\eta}{\sigma_z}k_y\left(\frac{\epsilon^*}{\sigma_\eta},\frac{g}{\sigma_z};\frac{\kappa\sigma_z}{\sigma_\eta}\right)}{k_x\left(\frac{\epsilon^*}{\sigma_\eta},\frac{g}{\sigma_z};\frac{\kappa\sigma_z}{\sigma_\eta}\right)}<\kappa$$

by Lemma A2.

A.3 Proof of Lemma 2 and 3

Notice that as $g\to-\infty$, $\epsilon^*(g)$ solves

$$\begin{aligned}
c &= \lim_{g\to-\infty}\left[\sigma_\eta\int_{-\infty}^{g}v\left(\frac{\epsilon^*(g)-\kappa z}{\sigma_\eta}\right)\frac{\phi(\frac{z}{\sigma_z})}{\sigma_z\Phi(\frac{g}{\sigma_z})}dz\right]\\
&= \lim_{g\to-\infty}\left[\sigma_\eta v\left(\frac{\epsilon^*(g)-\kappa z}{\sigma_\eta}\right)\frac{\Phi(\frac{z}{\sigma_z})}{\Phi(\frac{g}{\sigma_z})}|_{-\infty}^{g}-\kappa\int_{-\infty}^{g}\left\{1-\delta\left(\frac{\epsilon^*(g)-\kappa z}{\sigma_\eta}\right)v\left(\frac{\epsilon^*(g)-\kappa z}{\sigma_\eta}\right)\right\}\frac{\Phi(\frac{z}{\sigma_z})}{\Phi(\frac{g}{\sigma_z})}dz\right]\\
&= \lim_{g\to-\infty}\sigma_\eta v\left(\frac{\epsilon^*(g)-\kappa g}{\sigma_\eta}\right)
\end{aligned}$$

Thus, we have $\epsilon^*(g)-\kappa g\to\eta^*$ as $g\to-\infty$. This also implies that $\lim_{g\to-\infty}\frac{d\epsilon^*(g)}{dg}=\kappa$.

Now, suppose that $g\to\infty$. This implies that the manager always hide time zero mean cash flow. Thus, after observing realization of cash flow investors believe that time one mean cash flow $y_1=\kappa y_0+\eta=\kappa\rho\sigma_y s_0/\sigma_s+\kappa z+\eta$ is normally distributed with mean $\kappa\rho\sigma_y s_0/\sigma_s$ and variance $\sigma_\epsilon^2=\kappa^2\sigma_z^2+\sigma_\eta^2$. Thus, as $g\to\infty$, we should have that $\epsilon^*(g)\to\bar\epsilon$, where $\bar\epsilon$ solves

$$c=\sigma_\epsilon v\left(\frac{\bar\epsilon}{\sigma_\epsilon}\right). \tag{A.4}$$

This also implies that $\lim_{g\to\infty}\frac{d\epsilon^*(g)}{dg}=0$. We can now show that

$$x_n(x_0,s_0)=\kappa f s_0+\epsilon^*(g)<\kappa x_0+\eta^*=x_d(x_0)\leftrightarrow\epsilon^*(g)-\kappa g<\eta^*,$$

for any g since we have that $\lim_{g\to-\infty}\epsilon^*(g)-\kappa g=\eta^*$ and that $\frac{d}{dg}(\epsilon^*(g)-\kappa g)<0$.

Next, to prove Lemma 3 we define the following function for any $s>0$:

$$F(x,y;s)=\int_{-\infty}^{y}\Phi\left(-x+sz\right)\frac{\phi(z)}{\Phi(y)}dz$$

and establish the following properties.

Lemma A3 $F_x(x,y,s)<0$, $sF_x(x,y,s)+F_y(x,y,s)<0$, $\lim_{y\to-\infty}F(x,y,s)=\Phi(-x+sy)$.

Proof of Lemma A3. Take the partial derivative with respect to x:

$$F_x=-\int_{-\infty}^{y}\phi\left(-x+sz\right)\frac{\phi(z)}{\Phi(y)}dz<0$$

Take the partial derivative with respect to y:

$$\begin{aligned} F_y &= \delta(y)\left[\Phi(-x+sy) - \int_{-\infty}^{y} \Phi\left(-x+sz\right)\frac{\phi(z)}{\Phi(y)}dz\right] \\ &= s\delta(y)\int_{-\infty}^{y} \phi\left(-x+sz\right)\frac{\Phi(z)}{\Phi(y)}dz \end{aligned}$$

Thus, we have

$$sF_x(x,y,s) + F_y(x,y,s) = s\int_{-\infty}^{y} \phi\left(-x+sz\right)\left(\delta(y)-\delta(z)\right)\frac{\Phi(z)}{\Phi(y)}dz < 0$$

Finally, $F(x,y,s)$ can be expressed as

$$F(x,y,s) = \Phi(-x+sy) + s\int_{-\infty}^{y} \phi\left(-x+sz\right)\frac{\Phi(z)}{\Phi(y)}dz$$

which implies that $\lim_{y\to-\infty} F(x,y,s) = \lim_{y\to-\infty}\Phi(-x+sy)$. ■

Now, the ex ante likelihood of disclosure at time 1 given nondisclosure at time 0 can be expressed using the function $F(x,y,s)$:

$$\begin{aligned} \alpha_n(x_0) &= \Pr(y_1 > x_n(x_0,s_0)|y_0 < x_0) \\ &= E\left[E[1(y_1 > x_n(x_0,s_0))|s_0, z < g]\right] \\ &= E\left[\int_{-\infty}^{g} \Phi\left(-\frac{\epsilon^*(g)-\kappa z}{\sigma_\eta}\right)\frac{\phi\left(\frac{z}{\sigma_z}\right)}{\sigma_z\Phi\left(\frac{g}{\sigma_z}\right)}dz\right] \\ &= E\left[F\left(\frac{\epsilon^*(g)}{\sigma_\eta}, \frac{g}{\sigma_z}, \frac{\kappa\sigma_z}{\sigma_\eta}\right)\right], \end{aligned}$$

where $g = x_0 - fs_0$ and the last expectation is done with respect to s_0. Taking x_0 to $-\infty$, then we have

$$\alpha_n(x_0) = E\left[\lim_{g\to-\infty} F\left(\frac{\epsilon^*(g)}{\sigma_\eta}, \frac{g}{\sigma_z}, \frac{\kappa\sigma_z}{\sigma_\eta}\right)\right] = E\left[\lim_{g\to-\infty}\Phi\left(-\frac{\epsilon^*(g)-\kappa g}{\sigma_\eta}\right)\right] = \Phi\left(-\frac{\eta^*}{\sigma_\eta}\right) = \alpha_d.$$

The second equality is due to Lemma A3 and the third one is due to Lemma 2. Taking x_0 to ∞, then the manager is always hiding y_0 and from the perspective of the market z becomes just a normal variable. The manager will disclose at time 1 if $\kappa z + \eta > \bar{\epsilon}$, where $\bar{\epsilon}$ solves (A.4). As $x_0 \to \infty$, we have

$$\alpha_n(x_0) \to \Phi\left(-\frac{\bar{\epsilon}}{\sigma_\epsilon}\right) > \alpha_d.$$

The last inequality holds since $\sigma_\epsilon > \sigma_\eta$. Lastly, we can take the first derivative of $\alpha_n(x_0)$:

$$\alpha_n(x_0)' = \frac{1}{\sigma_z}E\left[\frac{\sigma_z}{\sigma_\eta}\frac{d\epsilon^*(g)}{dg}F_x + F_y\right].$$

By Lemma 2 and A3, we have $\alpha_n(x_0)' \to \frac{1}{\sigma_z}E\left[\frac{\kappa\sigma_z}{\sigma_\eta}F_x + F_y\right] < 0$ as $x_0 \to -\infty$ and $\alpha_n(x_0)' \to \frac{1}{\sigma_z}E\left[F_y\right] > 0$ as $x_0 \to \infty$.

A.4 Proof of Theorem 1

We first compute the option values. The option value upon the initial disclosure is given by

$$u_d = \int_{-\infty}^{\eta^*} (\eta^* - \eta)\frac{1}{\sqrt{2\pi\sigma_\eta^2}} e^{-\eta^2/2\sigma_\eta^2} d\eta = \eta^*\Phi\left(\frac{\eta^*}{\sigma_\eta}\right) + \sigma_\eta \delta\left(\frac{\eta^*}{\sigma_\eta}\right)\Phi\left(\frac{\eta^*}{\sigma_\eta}\right) = \sigma_\eta v\left(\frac{\eta^*}{\sigma_\eta}\right)\Phi\left(\frac{\eta^*}{\sigma_\eta}\right) = c\Phi\left(\frac{\eta^*}{\sigma_\eta}\right).$$

The last equality holds by the definition of η^*. Next, similarly we can compute the option value upon the initial nondisclosure:

$$\begin{aligned} u_n(x_0) &= E\left[(\epsilon^*(g) - \kappa g - \eta)^+\right] \\ &= E\left[E\left[(\epsilon^*(g) - \kappa g - \eta)^+ | w_0\right]\right] \\ &= E\left[\sigma_\eta \Phi\left(\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta}\right) v\left(\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta}\right)\right], \end{aligned}$$

where the last expectation is done with respect to w_0 and $g = (1-f)x_0 - f w_0$. The equilibrium condition (10) for the first-period disclosure threshold can be rewritten as

$$c(1+\alpha_d) = (1+\kappa)\sigma_y v\left(\frac{x_0}{\sigma_y}\right) + u_d - u_n(x_0) + c\alpha_n(x_0). \tag{A.5}$$

Define a function

$$f(x) = (1+\kappa)\sigma_y v\left(\frac{x}{\sigma_y}\right) + u_d - u_n(x) + c\alpha_n(x).$$

Take derivative

$$f'(x) = (1+\kappa)\left[1 - \delta\left(\frac{x}{\sigma_y}\right) v\left(\frac{x}{\sigma_y}\right)\right] - (1-f)E\left[\Phi\left(\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta}\right)\left(\frac{d\epsilon^*(g)}{dg} - \kappa\right)\right] + c\alpha_n(x)'.$$

Note that we use $(\Phi(x)v(x))' = \Phi(x)$. Take x to $-\infty$, then by Lemma A1 and 3

$$\begin{aligned} \lim_{x\to-\infty} f(x) &= c\alpha_d \\ \lim_{x\to-\infty} f'(x) &= \lim_{x\to-\infty} c\alpha_n(x)' < 0 \end{aligned}$$

Take x to ∞, then we have

$$\begin{aligned} \lim_{x\to\infty} f(x) &= \lim_{x\to\infty} (1+\kappa)\sigma_y v\left(\frac{x}{\sigma_y}\right) + u_d + c\alpha_n(x) = \infty \\ \lim_{x\to\infty} f'(x) &= 1 + \kappa + \lim_{x\to\infty} c\alpha_n(x)' > 0 \end{aligned}$$

Thus, there exists a unique x solving $c(1+\alpha_d) = f(x)$. Suppose that x_0 is such x. Then, we have

$$\begin{aligned} u_d - u_n(x_0) &= c\Phi\left(\frac{\eta^*}{\sigma_\eta}\right) - E\left[\sigma_\eta\Phi\left(\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta}\right) v\left(\frac{\epsilon^*(g) - \kappa g}{\sigma_\eta}\right)\right] \\ &> c\left[\Phi\left(\frac{\eta^*}{\sigma_\eta}\right) - \Phi\left(\frac{\eta^*}{\sigma_\eta}\right)\right] \\ &> 0, \end{aligned}$$

since $\epsilon^*(g) - \kappa g < \eta^*$ by Lemma 2 and $\Phi(x)v(x)$ is an increasing function. This implies that the myopic threshold x^* should be higher than x_0 since $f(x)$ is increasing at $x = x_0$.

A.5 Proof of Proposition 4 and 5

The mean and variance of g conditional on the initial mean cash flow is independent of σ_s. Thus, $\epsilon^*(g)$ is also independent of σ_s and so does $u_n(x_0)$.

When $|\rho| \to 1$, upon observing s_0 investors can recover y_0 perfectly. Thus, two option values are identical, which implies $x_0 = x^*$. When $\kappa \to 0$, the information of nondisclosure is irrelevant for the second period decision. This implies $x_0 = x^*$.

A.6 Proof of Proposition 6

The proof is constructed following Scheinkman and Xiong (2003, Proposition 2 in the Appendix). Two Kummer functions are defined as

$$M(a, b, y) = 1 + \frac{ay}{b} + \frac{(a)_2 y^2}{(b)_2 2!} + \cdots, \tag{A.6}$$

with $(a)_n = a(a+1)(a+2)\cdots(a+n-1)$ and $(a)_0 = 1$, and

$$U(a, b, y) = \frac{\pi}{\sin(\pi b)}\left[\frac{M(a, b, y)}{\Gamma(1+a-b)\Gamma(b)} - y^{1-b}\frac{M(1+a-b, 2-b, y)}{\Gamma(a)\Gamma(2-b)}\right]. \tag{A.7}$$

These two Kummer functions have the following properties: $M_y(a, b, y) > 0$ for all $y > 0$, $M(a, b, y) \to \infty$, and $U(a, b, y) \to 0$ as $y \to \infty$. Consider the following differential equation

$$yv''(y) + \left(\frac{1}{2} - y\right)v'(y) - \frac{\delta}{2\rho}v(y) = 0. \tag{A.8}$$

It is straightforward to verify that

$$u(g, \gamma_\infty) = -\frac{g}{r+\phi} - \frac{\sqrt{\gamma_\infty}}{r}\delta\left(\frac{g^*}{\sqrt{\gamma_\infty}}\right) + \tilde{u}(g)$$

satisfies (25) with $\tilde{u}(g) = v\left(\phi g^2/\sigma_g^2\right)$. Then, a general solution to (A.8) is (see Abramowitz and Stegun, 1964, chapter 13)

$$v(y) = \alpha M\left(\frac{r}{2\phi}, \frac{1}{2}, y\right) + \beta U\left(\frac{r}{2\phi}, \frac{1}{2}, y\right). \tag{A.9}$$

We can construct two solutions $v(\phi g^2/\sigma_g^2)$: $v(\phi g^2/\sigma_g^2) = \alpha M + \beta U$ for $g < 0$ and $v(\phi g^2/\sigma_g^2) = \alpha' M + \beta' U$ for $g > 0$. This gives us four unknowns $(\alpha, \alpha', \beta, \beta')$. As $g \to -\infty$, the manager always withhold the information, i.e. $\tilde{u}(g) = 0$, which implies that α must be zero. Therefore,

$$\tilde{u}(g) = \beta U\left(\frac{r}{2\phi}, \frac{1}{2}, y\right) \quad \text{if } g \leq 0 \tag{A.10}$$

Also at $g = 0$ two solutions should have same values and first-order derivatives. From the definition of the two Kummer functions, we have

$$g \to 0-, \qquad \tilde{u}(g) \to \frac{\beta\pi}{\Gamma\left(\frac{1}{2}+\frac{r}{2\phi}\right)\Gamma\left(\frac{1}{2}\right)} \qquad \tilde{u}'(g) \to \frac{\beta\pi\sqrt{\phi}}{\sigma_g \Gamma\left(\frac{r}{2\phi}\right)\Gamma\left(\frac{3}{2}\right)}, \tag{A.11}$$

$$g \to 0+, \qquad \tilde{u}(g) \to \alpha' + \frac{\beta'\pi}{\Gamma\left(\frac{1}{2}+\frac{r}{2\phi}\right)\Gamma\left(\frac{1}{2}\right)}, \qquad \tilde{u}'(g) \to -\frac{\beta'\pi\sqrt{\phi}}{\sigma_g \Gamma\left(\frac{r}{2\phi}\right)\Gamma\left(\frac{1}{2}\right)}. \tag{A.12}$$

By matching the values and first-order derivatives of $\tilde{u}(g)$, we have

$$\beta' = -\beta, \quad \alpha' = \frac{2\beta\pi}{\Gamma\left(\frac{1}{2}+\frac{r}{2\phi}\right)\Gamma\left(\frac{1}{2}\right)}.$$

Define $h(g)$ as follows: $h(g) = v(\phi g^2/\sigma_g^2)/\beta$. Then, any solution to (22) must satisfy $\tilde{u}(g) = \beta h(g)$.

Lemma A4 *Consider a function $h(g)$ defined in equation (26). Then, $h(g) > 0$, $\lim_{g\to-\infty} h(g) = 0$, $h'(g) > 0$, $h''(g) > 0$, and $h'''(g) > 0$.*

Proof. From the solution constructed in Proposition 6, we have $\lim_{g\to-\infty} h(g) = 0$, $h(0) = \frac{\pi}{\Gamma(\frac{1}{2}+\frac{r}{2\phi})\Gamma(\frac{1}{2})} > 0$ and for $g < 0$, $h'(g) > 0$. Thus, $h(g)$ is strictly positive and increasing when $g < 0$. If $h(g') > 0$ and $h'(g') = 0$ for some g', (22) implies that $h''(g') > 0$. Hence, $h(g)$ has no local maximum while it is positive, and thus it is always positive and monotonically increasing. Since $h'(g) > 0$ for $g \leq 0$ and $h''(g) \geq 0$ for $g \geq 0$ by (22), we have also $h'(g) > 0$ for $g \geq 0$.

Next, we prove the convexity of $h(g)$. For $g > 0$, $h''(g) > 0$ by (22). Let us assume that there exists $g'' < 0$ such that $h''(g'') \leq 0$. Then, by (22)

$$h'''(g'') = \frac{2\phi g^* h''(g'')}{\sigma_g^2} + \frac{2(r+\phi)h'(g'')}{\sigma_g^2} > 0 \tag{A.13}$$

This implies that $h''(g) < 0$ for $g < g''$ and $\lim_{g\to-\infty} h'(g) = \infty$. It contradicts that $h(-\infty) = 0$. Finally, we can show that $h'''(g) > 0$ by repeating the proof that we use for $h''(g)$. ■

A.7 Proof of Theorem 2

If $g^* > c$, the price jump when $\gamma \to 0$ is $g^* - c > 0$, which implies that the price jump is always positive since it is increasing in γ. If $g^* < g_0$, the price jump when $\gamma \to \gamma_\infty$ is $\sqrt{\gamma_\infty} v\left(\frac{g^*}{\sqrt{\gamma_\infty}}\right) - c < \sqrt{\gamma_\infty} v\left(\frac{g_0}{\sqrt{\gamma_\infty}}\right) - c = 0$. Thus, the price jump is always negative. Finally, if $g_0 < g^* < c$, then we can find $\bar{\gamma} \in (0, \gamma_\infty)$ such that the price jump at this variance is zero since the price jump when $\gamma \to 0$ is $g^* - c < 0$, when $\gamma \to \gamma_\infty$ is $\sqrt{\gamma_\infty} v\left(\frac{g^*}{\sqrt{\gamma_\infty}}\right) - c > \sqrt{\gamma_\infty} v\left(\frac{g_0}{\sqrt{\gamma_\infty}}\right) - c = 0$, and the price jump is increasing in γ.

B Numerical Methodology

We solve the option value in the continuous-time model numerically. Consider a grid $g_i = ia$ for $i = -l, \cdots, m$ and $\gamma_j = jb$ for $j = 0, \cdots, n$, where a and b are the size of grid. We impose the following condition: $-la = -\underline{g}$

for some large $\underline{g}$ and $nb = \gamma_\infty$. First, take g^* as given. This is equivalent to choose $g^* = i^*a$ $(-l < i^* < m)$ and $\beta = \frac{1}{h'(i^*)(r+\phi)}$. Then, the solution at γ_n is

$$u(i,n) = \begin{cases} -\frac{g_i}{r+\phi} - \frac{\sqrt{\gamma_n}\delta\left(\frac{g^*}{\sqrt{\gamma_n}}\right)}{r} + \beta h(i) & i \leq i^* \\ u(i^*,n) & i > i^* \end{cases}$$

We can solve (25) using finite difference method (FDM) and then verify the boundary condition:

$$-\frac{g^*}{r+\phi} - \frac{\sqrt{\gamma_n}\delta\left(\frac{g^*}{\sqrt{\gamma_n}}\right)}{r} + \beta h(i^*) = u(0,0) - \frac{c}{r+\lambda}$$

If it does not hold, try again with a new i^* until it holds. We can approximate partial derivatives:

$$\begin{aligned} u_\gamma(i,j) &= \frac{u(i,j+1) - u(i,j)}{b} \\ u_g(i,j) &= \frac{u(i+1,j) - u(i-1,j)}{2a} \\ u_{gg}(i,j) &= \frac{u(i+1,j) - 2u(i,j) + u(i-1,j)}{a^2} \end{aligned}$$

Substitute these into (25), then we obtain

$$c_{ij}u(i-1,j) + d_{ij}u(i,j) + e_{ij}u(i+1,j) = f_{ij} + k_{ij}u(i,j+1),$$

for $i = -l, \cdots, m-1$, and $j = 0, \cdots, n-1$, $u(i-1,j)$. The parameters c_{ij}, d_{ij}, e_{ij}, f_{ij}, and k_{ij} are straightforward to derive. We make two additional assumptions at the boundary, g_l: $u_{gg}(l,j) = 0$. We can solve for $u(i,j)$ backward given the solution at $j+1$ until $j = 0$ and verify the boundary condition.